De Loecker and Warzynski (2012) Markup Formula Derivation

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1 Introduction

[1] derive a formula for firms' markups in terms of expenditure shares and output elasticities. Their derivation uses duality and the expenditure function associated with a production function. I found this formulation somewhat unintuitive, so this is an alternative derivation which doesn't use duality.

2 Model

A firm produces output according to the production technology $q(x)$. The firm can purchase inputs x at cost cx, and receives revenue which is some function $R(q(x))$ of her total output $q(x)$; hence the firm's optimization problem is:

$$\max_x R(q(x)) - cx$$

The econometrician observes a firm's total revenues $R(q(x))$ and wage bill cx. We will show that, in addition to these, the production function elasticity $\epsilon_x = \frac{dq}{dx}\frac{x}{q}$ is all that we need to identify "markups" (which we will define).

2.1 Defining markups

Taking derivatives, the firm's FOC is:

$$R'(q(x))\, q'(x) = c$$

We will define markups as the ratio between average revenue per unit output $\frac{R(q(x))}{q(x)}$ and marginal production cost per unit output. Note marginal cost is:

$$\frac{d[\text{Cost}]}{dq} = \frac{d[\text{Cost}]}{dx}\frac{dx}{dq} = (c)\left(\frac{1}{q'(x)}\right) = R'(q(x))$$

Where the first two equalities are definitions, and the last quality uses the firm's FOC. So:

Definition 1. We define the markup μ as:

$$\mu \equiv \frac{R(q(x))/q(x)}{R'(q(x))} \tag{1}$$

in words, the ratio of the average revenue per unit production $R(q(x))/q(x)$ to the marginal cost per unit production $R'(q(x))$.

Example. (Monopoly markup) A simple example of a revenue function $R(\cdot)$ is monopoly profits; suppose there is a market inverse demand function $p(x)$, so that $R(q(x)) = p(q(x))\,q(x)$, then the firm solves $\max_x p(q(x))\,q(x)$. So the derivative of revenue $R'(q)$ is the standard marginal revenue function $p(q) + qp'(q)$. Again, in the classical monopoly problem $R(q(x)) = p(q(x))\,q(x)$, we have:

$$\mu = \frac{p(x)}{R'(q(x))} = \frac{p(x)}{c/q'(x)}$$

which is the standard ratio of price to marginal cost. But, note that this markup is well-defined for arbitrary revenue functions $R(\cdot)$.

Remark. Note that these markups μ capture information about the profits of the producing firm, but not necessarily about market power distortions in downstream markets. For example, suppose that the producer perfectly price discriminates in downstream markets, so that $R'(q(x))$ is decreasing because the firm sells to marginal consumers with progressively lower values. Then markups are positive because the firm makes profits from selling at high prices to high-value inframarginal consumers, thus attaining average revenues above marginal costs; but, the marginal consumer faces price equal to marginal cost, so the market outcome is socially efficient.

2.2 Solving for markups

Take the first-order condition:

$$R'(q(x))\,q'(x) = c$$

Multiply both sides by x:

$$R'(q(x))\,q'(x)\,x = cx$$

Now multiply and divide by q:

$$R'(q(x))\,q(x)\left(q'(x)\,\frac{x}{q(x)}\right) = cx$$

The quantity $q'(x)\,\frac{x}{q(x)}$ is the elasticity of production with respect to x, which we will call ϵ_x. So we can write this as:

$$R'(q(x))\,q(x)\,\epsilon_x = cx \tag{2}$$

Remark. For some intuition, suppose that markets are competitive so that marginal revenue is the constant market price, $R'(q(x)) = p$. Then we have $pq(x)\,\epsilon_x = cx$. This is the standard expenditure share equation for CRS production functions: $\epsilon_x = \frac{cx}{pq}$. But note that we never assumed CRS – this then says that for arbitrary smooth production functions, under price-taking, the ratio of total input expenditures cx to total revenue pq is the production elasticity ϵ_x. I was surprised to learn this.

We don't know $R'(q(x))$, so we want to replace it by the observed total revenue $R(q(x))$ and the markup μ. Multiply and divide by the average revenue per unit output $R(q(x))/q(x)$:

$$\left(\frac{R'(q(x))}{R(q(x))/q(x)}\right)(R(q(x))/q(x))\,q(x)\,\epsilon_x = cx$$

Now we can apply (1):

$$R(q(x))\left(\frac{1}{\mu}\right)\epsilon_x = cx$$

Now we are done:

$$\mu = \epsilon_x \frac{R(q(x))}{cx} \tag{3}$$

2.3 Generalization

- If there are multiple inputs, some of which face production costs, we can use any input which doesn't fact production costs. In particular, if the firm solves:

$$\max_{x_1 \dots x_N} R\left(q\left(x_1 \dots x_N, y_1 \dots y_M\right)\right) - \sum_i c_i x_i$$

  where x_i's (labor, materials) are fully adjustible and y_i's (capital) are not, note that the FOC's for each flexible x_i is

$$\frac{dR}{dq}\frac{\partial q}{\partial x_i} = c_i$$

  and all the algebra goes through as before.

- Any single flexible input is sufficient to identify markups, so we can get multiple estimates with multiple flexible inputs.

- We don't need to observe prices of inputs or outputs, only expenditure shares (though will generally use quantities for production function estimation).

References

[1] Jan De Loecker and Frederic Warzynski. Markups and firm-level export status. *American Economic Review*, 102(6):2437–71, May 2012.