Competition and Manipulation in Derivative Contract Markets*

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September 2020

Abstract

This paper studies manipulation in cash-settled derivative contract markets. When traders hedge factor risk using cash-settled derivatives, which are settled based on the price of a spot good, traders can manipulate settlement prices by trading the spot good. In equilibrium, manipulation can make all agents worse off. I define two measures of manipulation-induced welfare losses, which can be estimated using commonly observed market data. Using these measures, I estimate how large manipulation-induced distortions would be if COMEX gold futures were cash-settled using the London Bullion Market Association gold price benchmark.

Keywords: derivatives, manipulation, regulation

JEL classifications: D43, D44, D47, G18, K22, L40, L50

*I am grateful for data support from the Fama Miller Center. I appreciate comments from Mohammad Akbarpour, Yu An, Sam Antill, Anirudha Balasubramanian, Markus Baldauf (discussant), Alex Bloedel, Eric Budish, Jeremy Bulow, Shengmao Cao, Gabe Carroll, Juan Camilo Castillo, Scarlet Chen, Wanning Chen, Yiwei Chen, Yuxin (Joy) Chen, Cody Cook, Doug Diamond, Peter DeMarzo, Rob Donnelly, Tony Qiaofeng Fan, Winston Feng, Deeksha Gupta (discussant), Robin Han, Benjamin Hebert, Hugh J. Hoford, Emily Jiang, David Kang, Zi Yang Kang, Anil Kashyap, Aryan Kejriwal, Fuhito Kojima, Nadia Kotova, Pete Kyle, Eddie Lazear, Wenhao Li, Yucheng Liang, Jacklyn Liu, Jialing Lu, Carol Hengheng Lu, Danqi Luo, Hanno Lustig, Anthony Lynch, Suraj Malladi, Giorgio Martin, Sophie Moinas (discussant), Negar Matoorian, Ellen Muir, Evan Munro, Mike Ostrofsky, Paul Oyer, Agathe Pernoud, Peter Reiss, Sharon Shiao, John Shim, Ryan Shyu, Andy Skrzypacz, Paulo Somaini, Takuo Sugaya, Cameron Taylor, Jia Wan, Lulu Wang, Bob Wilson, Milena Wittwer, Alex Wu, David Yang, Kerry Yang Siani, Jessica Yu, Ali Yurukoglu, Becky Zhang, Jeffrey Zhang, and Mingxi Zhu, anonymous referees, and many seminar participants. I am very grateful to my advisors, Lanier Benkard and Paul Milgrom, as well as my committee members, Darrell Duffie, Glen Weyl and Brad Larsen, for their continuous guidance and support. I especially thank Darrell Duffie, who originally inspired this work.

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1 Introduction

This paper studies manipulation in cash-settled derivative contract markets. If a trader wants to buy or sell exposure to some risk factor, such as US equities, interest rates, volatility, or energy, the simplest way to do so is often to use a cash-settled derivative contract. Some examples of these contracts are S&P 500 futures, LIBOR and SOFR derivatives, VIX futures and options, and various oil, gas and electricity derivatives. Cash-settled derivatives constitute some of the world’s largest markets: the total notional size of the interest rate derivatives market alone is over $100 trillion USD.

Practically, a cash-settled derivative is simply a contract whose payoff is determined based on some price benchmark, which is constructed based on the trade price of some spot good. If the spot price benchmark accurately reflects some risk factor, contracts settled using the benchmark can be used to trade exposure to this risk factor. For example, a long VIX futures position, at settlement, pays its holder some multiple of the CBOE Volatility Index, which is calculated based on prices of S&P 500 options set in a settlement auction. VIX futures are useful for trading volatility to the extent that the option settlement prices are representative of US equity volatility.

Derivative contract holders who are also active in spot markets have incentives to distort price benchmarks in order to increase contract payoffs. A trader who is long VIX futures can increase her futures payoffs by buying S&P 500 options at the settlement auction to raise the VIX settlement value. If the trader’s futures position is large, her increased futures profits may outweigh any losses incurred by buying S&P 500 options at elevated prices. If many traders bid this way, however, their bids would add noise to the VIX at settlement, creating nonfundamental risk for all agents holding VIX futures contracts.

Legally, trading spot goods to move settlement prices and influence derivative payoffs is considered contract market manipulation, and is illegal in the US and many other jurisdictions. Regulators have imposed billions of dollars of fines on market participants for manipulation in the past two decades alone. However, manipulation is poorly understood from the perspective of economic theory. We do not know how manipulation affects the welfare of different classes of market participants: whether it is Pareto disimproving, or
simply creates transfers from non-manipulators to manipulators. We do not understand what makes contract markets vulnerable to manipulation, so we do not know how to empirically measure manipulation risk in contract markets.

In this paper, I build a simple model to illustrate the effects of contract market manipulation on market participants’ welfare. Using the model, I develop two measures of manipulation-induced welfare losses, which can be estimated using market data that is often collected by regulators.

I assume that a large number of risk-averse agents have exogeneous exposures to a common risk factor. Agents cannot contract on the risk factor directly, but can trade cash-settled derivative contracts in a competitive market, which are tied to the auction price of a spot good. The spot good is traded by a finite number of spot traders, and I assume all spot traders’ marginal value for the spot good is equal to the risk factor. Thus, if spot traders behaved competitively, the spot good market would clear with no trade, the spot auction price would be exactly equal to risk factor, and all agents could perfectly share factor risk using derivative contracts.

In equilibrium, however, first-best risk sharing is not attainable because the spot market is not perfectly competitive. Spot traders have price impact, so they have incentives to trade the spot good in a way that increases their derivative contract payoffs. For example, a spot trader with a long contract position has higher incentives to buy the spot good, since her purchases increase the spot auction price and thus her contract payoffs.

Manipulation causes auction prices to become noisy signals of the risk factor, creating non-fundamental basis risk for all contract holders. This affects spot traders’ contract purchasing decisions: traders tend to buy less contracts because of increased basis risk, but tend to buy more because of anticipated profits from manipulating the settlement price. In equilibrium, spot traders may even over-hedge, purchasing larger derivative positions than their total factor risk exposures. Manipulation also distorts outcomes in spot markets: spot traders trade positive quantities of the spot good, even though they perfectly agree on its value, in order to move prices.

Market structure in the spot market determines how vulnerable the contract market is to manipulation. Manipulation-induced distortions are larger when spot markets are less competitive, and when traders have high spot good holding costs, so that the price
impact of spot good trades is larger. Distortions are also larger when spot traders hold larger contract positions, as this gives them larger incentives to manipulate settlement prices.

There are two main effects of manipulation on spot traders’ welfare. Spot traders receive positive transfers in expectation, because they can move prices in favor of their contract positions on average, but they also face increased risk due to settlement price variance created by other spot traders. The negative effects can be strong enough that spot traders, as a group, are worse off in equilibrium, relative to a world in which all agents behaved competitively. In some settings, a regulator could increase all market participants’ welfare by imposing taxes or limits on spot traders’ contract positions.

Pure hedgers – agents who hold contract positions, but cannot trade the spot good – are unambiguously harmed by manipulation. Hedgers’ welfare losses from manipulation can be summarized by two quantities: basis risk, which is manipulation-induced variance in contract settlement prices; and manipulation rents, which are the net transfers from pure hedgers to spot traders created by spot traders’ price impact. Both metrics depend on the size of spot traders’ contract positions, and the market structure of spot markets.

I then generalize the spot market model, in order to bring it to data realistically. I allow spot traders to have asymmetric holding costs for the spot good, to receive arbitrarily distributed inventory shocks for spot goods, and to have arbitrarily distributed derivative contract positions. In the general model, basis risk and manipulation rents can be expressed in terms of the slopes of agents’ auction bid curves, and the variances and covariances of spot traders’ inventory shocks and contract positions. Thus, both metrics can in principle be estimated using market data observed by regulators.

I apply the model to answer the following question: how large would manipulation-induced distortions be, if COMEX gold futures were cash-settled using the London Bullion Market Association (LBMA) gold price benchmark? The LBMA gold price is set in a static auction which uses a dynamic multi-round price-setting mechanism. I show that, in a simple model of this dynamic auction, there exists an equilibrium equivalent to the unique equilibrium of a static auction. Assuming agents play this equilibrium, using data on auction round prices and volumes, I can recover the slope of aggregate demand in each auction I observe.
Since I only observe aggregate bidding data, not data on individual bidders, I use a bounds approach to estimate both metrics. I simulate a large number of individual bid slopes and inventory variances which are consistent with observed aggregate bid slopes and volumes, and I calculate the maximum and minimum values of basis risk and manipulation rents across all simulated individual parameters. I also calculate both risk metrics under a variety of assumptions on auction aggregate slopes and volumes, the number of auction participants, and variances and covariances between contract positions and inventory positions.

Across all these sources of uncertainty, I find that both manipulation risk metrics are fairly low. Even if all spot traders held contract positions with standard deviation equal to half the COMEX futures position limit, manipulation rents would be at most 2.88 basis points per dollar of hedgers’ notional gold exposures, and basis risk would be at most only 2.09% of total monthly gold price variance. These results imply that, if the LBMA gold price were used to settle COMEX gold futures, manipulation-induced distortions would be relatively small, under many possible assumptions about market conditions.

1.1 Implications for contract market regulators

Contract market manipulation is currently regulated using a primarily behavioral approach. Manipulation is defined as a certain pattern of behavior – trading with the intent to move prices – and regulators bring cases against market participants based on evidence, often taken from emails and phone calls, that a trader intentionally created price impact for profit. But traders know that their trades move prices; the role that they play in markets is precisely to manage and optimize the price impact of their trades. In the current regulatory environment, a trader who acknowledges this fact can face billions of dollars in fines.

The basic problem with the behavioral approach is that the precise boundaries of what constitute illegal manipulation have never been clear. The law itself does not define manipulation, and there is substantial disagreement in the legal, economic, and regulatory literatures on how manipulation should be defined, creating substantial regulatory uncertainty for market participants. Moreover, even if manipulation can be
defined in an internally consistent and testable way, it is unclear that the cases in which regulators can catch traders acknowledging their manipulative behavior are in fact cases where manipulation is most harmful for social welfare.

Manipulation could instead be regulated using a primarily structural approach. Antitrust regulators, for example, do not punish firms for setting prices above marginal costs; regulators acknowledge firms’ incentives to maximize profits, and aim to keep industries competitive enough that market competition disciplines firms’ pricing behavior, without the need for case-by-case regulatory intervention. Practically, regulators use metrics such as the Herfindahl-Hirschman index (HHI) to monitor trends in industry concentration. If markets are concentrated enough that consumer welfare losses may be large, regulators intervene, reducing concentration by blocking mergers or forcing divestitures. If markets are relatively competitive, so welfare losses are likely to be small, regulators largely leave firms to their own devices.

Contract market regulators have a variety of policy tools for controlling contract market structure. However, these tools are difficult to apply effectively, because we do not have a strong theoretical understanding of contract market manipulation. We do not even know whether manipulation should be regulated at all, from a theoretical perspective: we do not yet know whether manipulation creates welfare losses for market participants which regulation can mitigate. Even if we believe manipulation should be regulated, we do not have metrics like the HHI to determine how large manipulation-induced distortions are likely to be in any given market, so regulators have little theoretical guidance in deciding how to apply the tools at their disposal.

This paper attempts to address both of these issues. Theoretically, my results show that manipulation is indeed a market failure deserving of regulation: when contract holders have price impact in spot markets, equilibrium outcomes can be Pareto dominated, and regulatory intervention can be Pareto improving. Practically, I develop two simple model-based measures of hedgers’ welfare losses from manipulation; these metrics can be used by regulators to identify contract markets which are vulnerable to manipulation, and to predict how policy interventions will influence market outcomes and market participants’ welfare.

The results of this paper could thus be used to regulate contract market manipulation
in a more structural manner, which could both reduce regulatory uncertainty for market participants, and direct scarce regulatory attention towards settings in which manipulation is likely to be most harmful to social welfare.

1.2 Related literature

To my knowledge, this is the first paper to analyze the welfare effects of contract market manipulation, microfounding behavior in both the contract and spot markets. This is also the first paper to propose empirically implementable measures of contract market manipulation risk.


Technically, the spot market auction model of this paper builds on the literature on linear-quadratic double auctions; some papers in this literature are Kyle (1989), Vayanos (1999), Vives (2011), Rostek and Weretka (2015), Du and Zhu (2017), Duffie and Zhu (2017), and Lee and Kyle (2018). In particular, the proof of equilibrium in the generalized spot market model of section 5 builds on a result in Du and Zhu (2012). To my knowledge, the contract market model of this paper, in which agents hedge factor risk using contracts settled based on an auction price, is new to the literature.

price manipulation, and Nozari, Pascutti and Tookes (2019) study the related phenomenon of profitable price impact in convertible bond markets. There is also a large regulatory and legal literature on contract market manipulation, which I discuss briefly in subsection 2.2 below.

Some recent papers analyze the question of optimal benchmark design, such as Duffie, Dworczak and Zhu (2017), Duffie and Dworczak (2018), Eisl, Jankowitsch and Subrahmanyam (2017), Coulter, Shapiro and Zimmerman (2018) and Baldauf, Frei and Mollner (2018). Duffie and Dworczak (2018) and Baldauf, Frei and Mollner (2018) propose using volume-weighted average price schemes, whereas Coulter, Shapiro and Zimmerman (2018) proposes an incentivized announcement scheme somewhat similar to an auction. In contrast to this literature, I do not attempt to find an optimal mechanism for benchmark determination in this paper; instead, I aim to quantify manipulation risk, taking as given that the benchmark is determined in a uniform-price auction.

1.3 Outline

The remainder of the paper proceeds as follows. Section 2 discusses some history and institutional details of contract market manipulation. Section 3 introduces the baseline model, and section 4 derives the main theoretical results. Section 5 derives two measures of hedgers’ welfare losses from manipulation, and shows how to measure both in a more general model of spot markets. Section 6 estimates both measures using data on the LBMA gold price. Section 7 discusses implications of my findings, and section 8 concludes. Proofs, derivations and other supplementary material are presented in the appendix.

2 Institutional background

2.1 Cash-settled derivative contracts and manipulation

A trader who holds a cash-settled derivative contract is entitled to a payment tied to a price benchmark, which are set based on trade prices of some spot (or underlying) good.
Cash-settled derivative contracts allow market participants to trade exposure to prices of spot goods, without incurring the various costs – capital costs, physical transportation and storage costs – associated with trading the spot goods directly.

The first cash-settled contract was the Eurodollar futures contract, introduced on the Chicago Mercantile Exchange in 1981. Futures and options contracts for livestock and many dairy products are financially settled based on USDA-published average transaction prices for underlying commodities. Many energy derivatives, such as oil, gas, and electricity, are settled using price benchmarks calculated based on government or industry sources. Many financial derivatives are also cash-settled using benchmarks, such as LIBOR and SOFR for interest rates, the WM/Reuters fix as well as other FX benchmarks, indices for equities, commodities, FX, and volatility.

An important stylized fact is that there is often a liquidity mismatch between contract markets and spot market: the volume of trade in derivatives settled using a given price benchmark is often much larger than the volume of trade in spot assets used to determine the price benchmark. For example, the Platts Inside FERC Houston Ship Channel benchmark for natural gas prices is based on around 1.4 million MMBtus of natural gas trades per week; open interest in the ICE HSC basis future, which is financially settled based on the Platts benchmark, is more than 75 million MMBtus for many delivery months. The Secured Overnight Financing Rate (SOFR), designed to...
replace USD LIBOR as an interest rate benchmark, is based on average daily volumes of approximately $1 trillion in overnight treasury-backed repo loans\textsuperscript{16} as of 2014, the total notional volume of contracts linked to USD LIBOR was estimated to be greater than $160 trillion\textsuperscript{17}.

There are two main explanations for this liquidity mismatch. The first is speculation: some market participants may wish to trade exposure to price risk simply to express views on the direction of future prices. Derivatives have lower logistical and capital costs to trade than spot goods, so speculators will tend to prefer trading using derivatives. The second is cross-hedging: if prices of a group of spot goods are very correlated, derivative activity often concentrates in one or a few contracts for liquidity reasons. For example, a gas company located in Texas may choose to hedge risks using Henry Hub, Louisiana gas futures contracts, even if Texas gas futures are available, if the Louisiana contract is more liquid.

As a result of this liquidity mismatch, if a spot market participant holds a large derivative position, she may have incentives to trade spot assets non-fundamentally, in a way that moves price benchmarks and increases the payoffs on her contract position. For example, if a gas trader holds a large position in ICE Houston Ship Channel (HSC) basis futures, she can increase her futures payoff by buying spot gas at HSC to increase the benchmark price. The trader may incur losses on her spot market trades, but could generate much larger profits on her futures position.

2.2 Legal background

In the US, manipulation and attempted manipulation of contract markets are illegal under the Commodity Exchange Act of 1936. Regulators have policed contract market manipulation aggressively in the last few decades. UBS was fined $15 million by the CFTC in 2018 for attempting to manipulate gold futures contracts. The CFTC and the FERC have fined traders millions of dollars for manipulating oil, gas, electricity, precious

\textsuperscript{16}NY Fed’s Secured Overnight Financing Rate Data.

\textsuperscript{17}Financial Stability Board (2018).
metals, and propane derivatives.\textsuperscript{18} Fines for financial derivative manipulation are orders of magnitude larger: banks have been fined over $10 billion for FX manipulation,\textsuperscript{19} over $8 billion USD for manipulation of LIBOR and other interest rate benchmarks,\textsuperscript{20} and over $500 million for manipulation of the ISDAFIX interest rate swap benchmark.\textsuperscript{21}

Despite its importance in practice, manipulation law and policy is a contentious topic. The Commodity Exchange Act outlaws manipulation, but does not define it in any way. The CFTC’s operational definition of manipulation essentially states that trades made with the intent to create artificial prices are manipulative.\textsuperscript{22} This definition is still vague, and there has been substantial disagreement in both the economic and legal literatures, both on what can reasonably be defined as manipulation under current law, and on how manipulation should be regulated from a social planner’s perspective.

In recent times, the legal literature has largely moved away from the “artificial price” notion, focusing instead on “intent” as the standard of proof for manipulation. Perdue (1987) argues that manipulation should be defined as conduct which “would be uneconomic or irrational, absent an effect on market price.” Fischel and Ross (1991), similarly, argue that manipulation can only reasonably be defined based on the intent of the trader.

Regulatory authorities have also largely relied on proof of intent as the primary basis for prosecuting manipulators. Charges are brought based on “smoking gun” evidence that trades were made with the intention of moving benchmark prices, rather than for any fundamental reason. Levine (2014) quotes a number of trader chat messages used in FX manipulation lawsuits. Other examples include the CFTC’s lawsuits brought against Parnon Energy, Inc. and others for crude oil manipulation, Energy Transfer Partners, L.P. and others for natural gas manipulation, and Barclays for ISDAFIX manipulation.\textsuperscript{23}

Even if it is internally consistent, the standard of intent seems difficult to justify from an economic perspective: any given pattern of trades should have the same positive or negative effects on market quality regardless of the intent of the trader. The need to

\textsuperscript{18}See, for example, CFTC Press Release 6041-11, 128 FERC 61,269, CFTC Press Release 4555-01, CFTC Press Release 7683-18, and CFTC Press Release 5405-07.
\textsuperscript{19}Levine (2015).
\textsuperscript{20}Ridley and Freifeld (2015).
\textsuperscript{21}Leising (2017).
\textsuperscript{22}17 CFR Part 180.
\textsuperscript{23}See the CFTC’s website for Parnon Energy, Inc, Energy Transfer Partners, L.P, and Barclays.
prove intent also imposes a high bar of proof on regulators, as it is difficult to prosecute manipulators in settings where regulators cannot easily access traders’ communication records. Moreover, legally sophisticated market participants could avoid being implicated in manipulation lawsuits, simply by taking care not to make statements about the intentions behind their trades in recorded media.

There is also some disagreement in the literature as to whether manipulation should be regulated at all. [Hieronymus (1977) pg. 328] argues that contract market manipulation will not survive under market competition. [Fischel and Ross (1991)] argue that “actual trades should not be prohibited as manipulative regardless of the intent of the trader”, and that market competition is likely to deter manipulation. [Markham (1991)] similarly proposes to abandon the concept of manipulation, and to instead empower the CFTC to take a broader set of actions to maintain fair and orderly markets.

3 Model

I assume that agents have exogeneous exposures to a common risk factor, and can trade derivative contracts to share factor risk. The model has two core frictions. First, agents cannot contract on the risk factor directly: they must instead contract on the auction price of a spot good, whose price is informative about the risk factor. Second, the spot market is imperfectly competitive, so spot good trades move prices. Together, these two frictions imply that spot traders who buy contracts to hedge factor risk have incentives to trade non-fundamentally in spot markets, to influence the payoffs on their derivative contract positions. This creates nonfundamental risk and prevents agents from perfectly sharing risk.

There are two kinds of agents. There is a continuum of pure hedgers, who buy derivative contracts, but do not participate in spot markets. There are also \( n \) identical spot traders, who can trade both derivative contracts and spot goods. I assume that spot traders are negligibly small relative to pure hedgers. All agents are risk-averse with CARA utility over wealth, with identical risk aversions; that is, agent \( i \)’s utility if she attains wealth \( W \)

\footnote{A few overviews of market manipulation are [Putninš (2012)], [Markham (2014)], and [Putninš (2020)].}
is:
\[ U_i(W) = -e^{-\alpha W} \] (1)

Agents play a 5-stage game:

1. Agents draw their factor risk exposures, \( x_i \).
2. Agents buy derivative contracts, \( c_i(x_i) \).
3. The risk factor, \( \psi \), is realized.
4. Spot traders bid to trade the spot good in an auction.
5. Derivative contracts pay all contract holders based on the spot auction price.

In stage 1, each spot trader and each pure hedger is endowed with \( x_i \) net units of a productive asset, where
\[ x_i \sim N(0, \sigma_x^2) \]

The productive asset pays its holder \( x_i \psi \) net units of wealth. Thus, the value of each unit of the productive asset depends on a normally distributed risk factor \( \psi \), where:
\[ \psi \sim N(\mu_\psi, \sigma_\psi^2) \]

I thus refer to \( x_i \) as agents’ factor exposures. The risk factor \( \psi \) represents, for example, future oil prices, volatility, stock prices, or interest rates; agents may be positively or negatively exposed to each of these sources of risk. I assume that factor exposures \( x_i \) are privately observed. For analytical convenience, I normalize wealth by assuming that an agent who has factor exposure \( x_i \) is also endowed with \(-x_i \mu_\psi \) units of wealth, so that the expectation of all agents’ wealth is 0, regardless of \( x_i \).

In stage 2, agents can trade derivative contracts to hedge their factor exposures. I use \( c_i \) to denote the contract position of agent \( i \), and I assume that agents’ contract positions are private information. I assume agents can purchase an arbitrary number of contracts at price \( p_c = \mu_\psi \), and I assume it is costless to hold derivative contracts.

In stage 3, the risk factor \( \psi \) is drawn and commonly observed by agents. In stage 4 agents bid to trade the spot good in a uniform-price double auction. In stage 5, the
auction clearing price $p$ is used to settle agents’ derivative contracts: that is, each agent is paid $c_ip$.

Derivative contracts are useful because agents can purchase them prior to the realization of $\psi$, so contracts can be used to hedge factor exposures. If agents could purchase contracts directly tied to the risk factor – that is, contracts which pay $\psi$ per contract in stage 5 – an agent with contract position $c_i$ would receive total wealth:

\[
\frac{x_i\psi - x_i\mu}{\psi c_i - p c_i}
\]

At price $p_c = \mu\psi$, all agents would choose to hedge perfectly, setting $c_i = -x_i$. Since there is a continuum of agents and factor exposures are mean-0, markets for derivative contracts would clear, and agents would achieve the first-best outcome of perfect risk sharing.

The first core friction of the model is that agents cannot contract directly on $\psi$. In practice, traders cannot contract on abstract risk factors, such as oil prices, volatility, stock prices, or interest rates, directly. Instead, traders can buy and sell contracts which are settled based on price benchmarks set in markets for spot goods, such as the Brent or WTI oil price indices, the VIX, the S&P 500 and Russell 2000, and LIBOR or SOFR. To model this, I assume there is a single homogeneous spot good, traded by a finite number of spot traders. A spot trader who purchases $z_i$ net units of the spot good attains a monetary payoff:

\[
\psi z_i - \frac{1}{2k}z_i^2
\]

The $\psi z_i$ component of (2) implies that the marginal value of each spot trader for the spot good depends on the risk factor $\psi$. The quadratic component, $\frac{1}{2k}z_i^2$, implies that spot traders have decreasing marginal values for the spot good; this can be thought of as a kind of holding cost for the spot good.

For physical goods such as oil or gas, the quadratic term might represent physical storage and infrastructure costs; for financial assets such as FX, interest rate swaps, or repo, these costs may correspond to capital or balance sheet costs. Alternatively, these costs may arise from anticipated price impact from liquidating spot positions in the future.
The parameter $\kappa$ measures the size of these costs: when $\kappa$ is lower, spot traders are less willing to take on large spot good positions, so the price impact of spot trades will be larger.

Combining all terms, a spot trader’s total wealth, over her factor exposures and outcomes in both the contract and spot markets, is:

$$W_{\text{spot}}(x_i, c_i, z_i, p, \psi) = \psi x_i - \mu_{\psi} x_i - \mu_{\psi} c_i + pc_i + z_i\psi - \frac{1}{2\kappa}z_i^2 - pz_i$$  \hspace{1cm} (3)

A pure hedger’s total wealth is:

$$W_{\text{hedger}}(x_i, c_i, p, \psi) = \psi x_i - \mu_{\psi} x_i - \mu_{\psi} c_i + pc_i$$  \hspace{1cm} (4)

### 3.1 The competitive-bidding benchmark

Since all spot traders agree that the marginal value of the spot good is equal to $\psi$ when $z_i = 0$, a benchmark for $\psi$ can be constructed using any market mechanism which elicits spot traders’ marginal values for spot goods. If all traders behaved competitively, cash-settled derivatives would then implement the first-best outcome of perfect risk sharing. To see this, note that in stage 4, each spot trader’s marginal value of the spot good is the derivative of (2) with respect to $z_i$:

$$\psi - \frac{z_i}{\kappa}$$  \hspace{1cm} (5)

If spot traders submitted bid curves competitively, equating their marginal values of the spot good to prices, all traders would bid:

$$z_{B_i}(p) = \kappa (\psi - p)$$  \hspace{1cm} (6)

The unique auction clearing price would be $p = \psi$. There would be no trade of the spot good; this is efficient, since traders’ valuations for the spot good are identical. Anticipating
that \( p = \psi \), all agents would buy contracts to fully hedge their factor exposures, so agents would be able to perfectly share factor risk.

The second core friction in the model is that the spot market is imperfectly competitive. Spot traders are strategic, and recognize that their spot good trades move \( p \). Thus, suppose a spot trader has a negative factor exposure \( x_i \), and buys a positive contract position \( c_i \) to hedge her factor risk exposure; in the spot market, the trader has an incentive to increase her bid in the spot auction, in order to increase \( p \) and thus her contract payoffs. This makes \( p \) a noisy signal of \( \psi \), and prevents perfect risk sharing; I analyze how these incentives play out in equilibrium in the following section.

Note that a cash-settled derivative contract is simply a bet on the price benchmark \( p \), so the fair price of a derivative contract at settlement is always equal to \( p \). Derivative contracts are only linked to the risk factor, \( \psi \), to the extent that \( p \) is informative about \( \psi \). If, for example, spot traders are able to move \( p \) higher than \( \psi \), there is no way that pure hedgers, who can only trade derivative contracts, to fix this supposed mispricing: there is no arbitrage trade, involving only the derivative contract, that profits from the fact that \( p \) differs from \( \psi \).

### 3.2 Discussion of model assumptions

#### 3.2.1 Assumptions on the contract market

I assume that agents can purchase an arbitrary number of contracts at price equal to \( \mu_\psi \). Thus, there is no factor risk premium, and spot traders’ contract positions have no price impact. This is internally consistent: given the assumptions of my model – there are a continuum of hedgers, spot traders are negligibly small, all agents have no information about \( \psi \), and factor exposures \( x_i \) have mean 0 – \( \mu_\psi \) is the unique market-clearing price in the contract market.

The assumption that spot traders are small, so contract purchases have no price impact, is likely a reasonable approximation for many (but not all) contract markets. Subsection 2.1 shows that contract markets are often much larger than spot markets, and section 6 shows that, in the gold market, position limits imply that spot traders in the LBMA gold auction could not hold more than around 2% of total contract volume. Appendix 15
B.4 relaxes this assumption, analyzing spot traders’ optimal behavior when their contract purchases have some exogenously specified effect on contract prices. This causes spot traders to buy less contract per unit of their factor exposures.

I assume that agents have no holding costs for their contract positions. This is a reasonable approximation in many settings: derivatives are useful because they are highly leveraged financial assets, so they have effectively no physical holding costs, and low capital costs relative to spot goods. Agents’ contract positions are still limited, however, by their ability to bear factor risk: an agent who purchases a very large contract position holds a very large exposure to $\psi$.

Contract purchases may be motivated by speculation rather than hedging. In appendix B.3 I assume agents have heterogeneous beliefs about the mean $\mu_{\psi}$ of the risk factor; this leads agents to purchase nonzero contract positions, because agents think contracts are mispriced in the first period. The implications for manipulation in the spot market are unchanged.

Finally, while the model of the contract market is quite stylized, assumptions about the contract market only matter for the results in section 4 regarding spot traders’ contract purchasing decisions and welfare. As I discuss in section 5 in a general model of the spot market, the econometrician can measure hedgers’ welfare losses from manipulation if she observes the structure of spot markets and spot traders’ contract positions at settlement. Details of why and how spot traders entered into their contract positions do not affect these calculations. Thus, my metrics for hedger welfare losses are valid in a variety of different models of the contract market.

3.2.2 Assumptions on the spot market

The baseline spot market model is very stylized; in subsections 5.2 and 5.3 I generalize the model, allowing spot traders to have heterogeneous spot holding capacities, inventory shocks, and arbitrarily distributed contract positions.

Throughout the paper, I take the number of spot market participants, $n$, and their spot good holding capacities, $\kappa$, as exogeneous; I am essentially analyzing contract market manipulation holding fixed spot market structure. Spot market structure can change,
as agents enter, exit, and adjust their holding costs, but these changes tend to be costly and take time. For physical spot goods, such as oil and gas, holding costs depend on costly infrastructure such as pipelines and storage facilities; for financial assets, many benchmarks are set in inter-dealer markets, which do not allow free entry. The assumption that spot market structure is essentially fixed in the short run seems reasonable in many settings; however, the metrics I propose would need to be updated over time, as spot market structure changes.

Relatedly, I do not allow spot traders to buy and store the spot good prior to the realization of $\psi$; this is obviously a stylized assumption. If storage is costless, current stocks of the asset are infinite, and the spot market in the first period is perfectly competitive, the second-period spot market will also be perfectly competitive, so manipulation will not be possible. If any one of these assumptions are violated, however, spot markets at settlement will be imperfectly competitive and spot trades will have price impact, so the qualitative conclusions of the model will still hold.

In a model with costly storage, manipulation may happen in a more dynamic manner. A spot trader who intends to manipulate settlement prices might build up a large spot good position in advance of settlement, then use this as “ammunition” to move spot prices. Similarly, other spot traders may stockpile the spot good, in order to profit from market-making, anticipating that manipulators will trade non-fundamentally to move spot prices.\footnote{These are interesting directions to explore, but I leave these to future work.}

A related concern is that, if the contract market is very vulnerable to manipulation, market participants could hedge factor risk by holding spot goods directly. However, as subsection 2.1 discusses, market participants would generally prefer to hedge using derivatives, because holding spot goods involves higher logistical and capital costs. If manipulation makes derivative contracts sufficiently unattractive, market participants may hedge using spot goods instead, despite their higher costs; this is socially inefficient, because market participants could share risk at lower cost using derivatives in the first-best outcome.

\footnote{Note that storage arbitrageurs, who buy the spot good in the first period to sell it in the second period, cannot risklessly profit from manipulation: since manipulators’ contract positions are private information, arbitrageurs would not know, when they make their storage decisions, whether $p$ will be higher or lower than $\psi$ at contract settlement.}
I assume agents optimize independently of each other; however, in the model, spot traders have incentives to collude. A spot trader who increases settlement prices generates profits for all other traders with long contract positions; thus, colluding spot traders, who coordinate their spot market bids to maximize joint profits, would manipulate more per unit contract than independently optimizing traders. This is an interesting direction to explore, but I leave this to future work.

4 Equilibrium

Proposition 1 describes equilibrium values of spot traders’ bid curves, auction prices, contract purchases, and expected utility.

**Proposition 1.** For any \( \alpha, \sigma_\psi^2, \kappa, \sigma_\chi^2, n \), there is a unique equilibrium, in which traders’ spot market bids are:

\[
 z_{Bi}(p; c_i, \psi) = \frac{1}{n-1} c_i - \frac{n-2}{n-1} \kappa (p - \psi) \tag{7}
\]

Spot auction prices are:

\[
 p - \psi = \frac{\sum_{i=1}^{n} c_i}{n (n-2) \kappa} \sim N \left( 0, \frac{\sigma_c^2}{n (n-2)^2 \kappa^2} \right) \tag{8}
\]

Spot traders’ contract positions \( c_i(x_i) \) are linear in traders’ factor exposures \( x_i \), so contract positions are normally distributed with mean 0 and variance \( \sigma_c^2 \), where:

\[
 c_i(x_i) = -t x_i, \quad \sigma_c^2 = t^2 \sigma_\chi^2 \tag{9}
\]

where \( t \) satisfies:

\[
 t \equiv \left( 1 + \frac{\alpha \sigma_\psi^2 - \kappa}{(\alpha \sigma_\psi^2 \kappa) ((n^2 - 2n) \kappa + \alpha \sigma_\psi^2 \kappa)} \right)^{-1} \tag{10}
\]
and $\sigma^2_\eta$ is the unique positive value satisfying:

$$\sigma^2_\eta = \frac{\sigma^2_x}{n-1} \left( 1 + \frac{\alpha \sigma^2_x - \kappa}{\left( \alpha \sigma^2_x \kappa \right) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_\eta \right)} \right)^{-2}$$

$$\sigma^2_\eta > \frac{\kappa - \left( \alpha \sigma^2_\psi \right) \left( (n^2 - 2n) \kappa^2 \right)}{\alpha \left( 1 + \alpha \sigma^2_\psi \kappa \right)}$$

Spot traders’ expected utility, as a function of $t$, is:

$$-\frac{\sqrt{\frac{(n^2-2n)\kappa}{\alpha \left( \frac{\sigma^2_x}{n-1} \right) + (n^2-2n)\kappa}}}{1 - \alpha \sigma^2_x \left( \alpha \sigma^2_\psi (1-t)^2 + \frac{\alpha \left( \frac{\sigma^2_x}{n-1} \right) - \kappa}{\left( \alpha \sigma^2_\psi \kappa \right) \left( (n^2-2n) \kappa + \alpha \left( \frac{\sigma^2_x}{n-1} \right) \right)} \right) t^2}$$

### 4.1 Spot market distortions and manipulation-induced basis risk

Spot traders’ equilibrium bids, (7), differ from their competitive bids, (6), in two ways. The first difference, which is known in the double-auctions literature, is that traders shade their bids due to their price impact. The slope of (7) with respect to prices is lower than the slope of (6), by a factor $\frac{n-2}{n-1}$.

The second difference is that traders’ equilibrium bid curves depend on their contract positions $c_i$, even though these do not affect traders’ marginal value for spot goods. If a spot trader holds contracts whose payoffs depends on the auction price, and the trader can move the auction price by trading the spot good, she has an incentive to trade the spot good in a way that increases her profits from her contract position. I refer to this phenomenon as manipulation in the context of my model.

Expression (7) shows that increasing a trader’s contract position by 1 unit causes her to increase her spot good bid curve by $\frac{1}{n-1}$ units; as $n$ increases, contract positions affect bids less. Intuitively, when the spot auction is competitive, agents need to buy more of
the spot good to move prices a given amount; the cost of manipulation is higher, so spot traders manipulate less per unit contract that they hold. In the limit as \( n \) grows large, expression (7) converges towards (6), and traders’ contract positions have no effect on their bids.

Since auction prices depend on spot traders’ contract positions \( c_i \), which depend on traders’ random factor exposures \( x_i \), in equilibrium, the auction price \( p \) is a noisy signal of the risk factor \( \psi \). The difference, \( p - \psi \), is normally distributed, and expression (8) characterizes its variance. I call this variance manipulation-induced basis risk. Basis risk is higher when the variance in spot traders’ contract positions \( \sigma_c^2 \) is larger, when the number of traders \( n \) is smaller, and when traders’ spot good holding capacity \( \kappa \) is lower.

### 4.2 Spot trader contract positions

Expression (9) of proposition 1 shows that, in the unique equilibrium of the model, agents’ optimal contract purchases \( c_i \) are linear in their factor exposures \( x_i \). The coefficient of proportionality, defined as \( t \) in expression (10), describes how aggressively agents are hedging; that is, how many contracts agents purchase per unit of their factor exposures.

If all traders bid competitively in spot markets, traders would perfectly hedge, setting \( t = 1 \). In equilibrium, \( t \) can be greater or smaller than 1. On the one hand, spot traders anticipate that others will manipulate, making spot prices noisy signals of \( \psi \), decreasing their incentives to buy contracts to hedge factor risk. On the other hand, spot traders anticipate that they can profit on average from their contract positions, since they can move spot prices in their favor, increasing their incentives to buy contracts.

The second force may dominate the first, implying that \( t > 1 \) in equilibrium. In this case, spot traders will “over-hedge”, buying contract positions which are larger than their original factor risk exposures \( x_i \). In other words, agents may purchase contract positions so large that they actually increase their exposures to factor risk, because of anticipated profits from moving contract settlement prices in their favor. From expression (10), \( t \) is greater or smaller than 1 in equilibrium depending on whether:

\[
\kappa > \alpha \sigma_c^2 \eta
\]
where $\sigma_\nu^2$, from (11), is the expected variance of residual supply in the spot market. Intuitively, when residual supply has low variance, spot traders’ risk aversion $\alpha$ is low, and spot traders’ holding capacity for the spot good, $\kappa$, is high, the expected profits from manipulation are large relative to the costs, so $t$ will tend to be larger.

Increasing competition in spot markets decreases both basis risk, and the expected profits from manipulation. Formally, appendix B.2 shows that increasing $n$, holding other parameters fixed, causes $t$ to rapidly converge to its competitive value of 1.

### 4.3 Spot trader welfare

Expression (13) shows spot traders’ expected welfare, over all sources of uncertainty in the model. This allows us to assess whether spot traders are better or worse off in equilibrium, relative to the competitive-bidding benchmark. Expression (13) also allows us to calculate welfare for values of $t$ other than its equilibrium value. This corresponds to welfare in a limited social planner’s problem, in which the planner can force all spot traders to buy contracts according to some prespecified value of $t$, but cannot influence traders’ behavior in spot markets. This can be thought of as a reduced-form model of various actions contract market regulators can take to limit the size of traders’ contract positions, such as imposing contract position limits.

The three panels of figure 1 shows spot traders’ welfare as a function of $t$, alongside the equilibrium and competitive values of $t$ and spot trader welfare, for three different sets of input parameters. A given spot trader’s manipulation generates both positive and negative externalities on other spot traders, so a number of different outcomes are possible: spot traders may gain or lose on average from manipulation, the equilibrium $t$ may be greater or smaller than 1, and spot traders may hedge more or less than is optimal for spot traders’ welfare.

In the left panel, manipulation increases spot traders’ welfare, relative to the competi-

---

26I do not directly analyze position limits because my model requires agents’ contract positions to be Gaussian, so bounds on the size of agent’s contract positions would be intractable. In the context of the model, appendix B.4 shows that the planner could implement any positive value of $t$ in equilibrium by imposing quadratic taxes or subsidies on spot traders’ contract positions, charging spot traders $kc^2_t$ for buying $c_t$ contracts.
tive benchmark. Spot traders also over-hedge, choosing $t > 1$ in equilibrium. However, spot traders would do even better as a group if they could commit to holding smaller contract positions. Spot traders face a kind of prisoner’s dilemma: manipulation by one spot trader creates a negative externality on other spot traders by increasing basis risk, so all spot traders would prefer a lower value of $t$ than the equilibrium value. A lower value of $t$ would also benefit pure hedgers, since it would decrease basis risk, so a regulator could create a Pareto improvement by limiting the size of spot traders’ contract positions.

In the middle panel, like the left panel, spot traders would prefer for $t$ to be smaller than its equilibrium value. In addition, spot traders’ equilibrium welfare is below the competitive value of $-1$. As the following section shows, hedgers are always worse off in equilibrium relative to the competitive benchmark. Thus, in this example, manipulation is Pareto disimproving relative to the competitive benchmark, decreasing welfare for both spot traders and hedgers. Intuitively, this is because spot traders’ losses from manipulation-induced basis risk outweigh their gains from extracting a transfer from hedgers in expectation.

In the right panel, spot traders would actually prefer for $t$ to be higher than its equilibrium value. This is because manipulation by one spot trader actually creates a positive externality on other spot traders in spot markets: manipulators make non-fundamental trades in spot markets, so other spot traders profit as market makers, buying low and selling high in the spot auction. In this case, spot traders face a kind of coordination problem: all spot traders would be better off if each trader bought more contracts and manipulated more. This would, however, increase basis risk and decrease hedgers’ welfare, so an increase in $t$ can never be Pareto-improving.

Together, these examples show that manipulation can be Pareto-dominated relative to the competitive benchmark, and regulatory intervention to decrease spot traders’ hedging aggressiveness can be Pareto improving. Appendix B.1 shows additional comparative statics of the model, illustrating how equilibrium outcomes are affected by different model primitives.
Notes. Expected welfare of spot traders, \( (13) \), as a function of the hedging aggressiveness parameter \( t \). The green vertical and horizontal lines denote the competitive values of \( t \) and welfare, which are always equal to 1 and -1 respectively. The blue line denotes the unique equilibrium value of \( t \), and the orange line denotes the value of \( t \) which maximizes spot traders’ welfare. The parameters for the left panel are \( n = 3, \alpha = 1, \kappa = 0.5, \sigma_{\psi}^2 = 0.3, \sigma_{x}^2 = 0.5 \); for the middle panel, they are \( n = 5, \alpha = 1, \kappa = 0.05, \sigma_{\psi}^2 = 0.9, \sigma_{x}^2 = 1 \); for the right panel, they are \( n = 3, \alpha = 1, \kappa = 0.8, \sigma_{\psi}^2 = 1.5, \sigma_{x}^2 = 0.05 \).

5 Measuring hedgers’ manipulation-induced welfare losses

This section shows how to measure manipulation-induced welfare losses for hedgers. Subsection 5.1 derives two measures of the impact of manipulation on hedgers’ welfare in the baseline model. Subsection 5.2 solves a more general model of the spot market, relaxing many symmetry assumptions imposed in the baseline model. Subsection 5.3 derives expressions for both welfare metrics in the general model, and shows how they can be estimated using market data.

5.1 Hedger welfare in the baseline model

Proposition 2 characterizes pure hedgers’ equilibrium contract purchasing decisions, the variance of hedgers’ wealth, and the total expected wealth transfer from hedgers to spot traders, in the baseline model.
Proposition 2. Pure hedgers’ contract positions are linear in hedgers’ factor exposures $x_i$:

$$c_i(x_i) = -\frac{\sigma^2}{\sigma^2 + \text{Var}(p-\psi)} x_i = -\frac{\sigma^2}{\sigma^2 + \frac{\sigma^2}{n(n-2)\kappa^2}} x_i$$

(14)

Pure hedgers’ wealth has mean 0. The variance of hedgers’ wealth, over uncertainty in $\psi$ and all $x_i$’s, is:

$$\text{Var}(p-\psi) \left\{ \frac{\sigma^2}{\sigma^2 + \frac{\sigma^2}{n(n-2)\kappa^2}} \right\} \sigma^2$$

(15)

The total expected wealth transfer from hedgers to spot traders is:

$$\tau \equiv \mathbb{E} \left[ (p - \mu_\psi) \sum_{i=1}^{n} c_i \right] = \frac{\sigma^2_c}{(n-2)\kappa}$$

(16)

Proposition 2 shows that manipulation has relatively simple effects on hedgers’ behavior. Hedgers buy less contracts per unit risk exposure, because under manipulation, contracts are noisier hedges for factor risk. Manipulation creates two sources of welfare loss for hedgers: it creates nonfundamental basis risk, and also creates a net transfer from hedgers to spot traders.

Basis risk: Expression (15) shows that manipulation-induced basis risk increases the variance of hedgers’ wealth, by a factor which depends on price variance, the size of hedgers’ factor exposures, and the magnitude of factor risk. Based on (15), we can define a simple index of hedgers’ welfare losses from basis risk.

Definition 1. Define the basis risk coefficient $\rho$ as:

$$\rho \equiv \frac{\text{Var}(p-\psi)}{\text{Var}(p-\psi) + \sigma^2_\psi}$$

(17)

From (15), hedgers’ wealth variance can be written as:

$$\rho \sigma^2_\psi \sigma^2_x$$
\( \rho \) can thus be thought of as the fraction of total factor risk that hedgers are exposed to, assuming they purchase contracts optimally, given anticipated basis risk. When \( \rho = 0 \), hedgers can perfectly hedge and face no wealth uncertainty; when \( \rho = 1 \), hedgers buy no contracts, and are fully exposed to factor risk.

**Expected manipulation rents:** Expression (16) is the total expected transfer from spot traders to manipulators. I call this the *expected manipulation rent*, as it represents the rents that spot traders, as a group, extract from hedgers as a result of their ability to move prices in spot markets. This may be a useful number in regulatory proceedings, to quantify the total expected profits that spot traders as a group extract from pure hedgers.

Qualitatively, \( \rho \) and \( \tau \) respond similarly to changes in model parameters. Both are higher when spot traders’ contract positions, \( \sigma_c^2 \), are large; when spot traders’ holding capacities \( \kappa \) are low, so the price impact of spot trades is higher; and when \( n \) is lower, so auctions are less competitive.

In the context of the model, only basis risk matters for hedgers’ welfare. This is because we have made the simplifying assumption that there is a continuum of hedgers, so manipulation rents are infinitesimally diffused among hedgers. Basis risk, on the other hand, affects all hedgers and is not diffused away. It is an empirical question which metric matters more in any given practical setting. As I show in the following subsections, both are fairly simple to estimate and have similar data requirements, so both could be used in practice.

### 5.2 Hedger welfare in a generalized spot market model

The baseline model imposes a number of unrealistic symmetry assumptions on market participants. To bring the model to data, I study a more general model of the spot market, taking spot traders’ contract positions as exogeneous random variables. I do not study the full multi-stage game because it is analytically intractable in the general case.

Suppose that the wealth of spot market participant \( i \), if she buys \( z_i \) net units of the spot good at price \( p \), is:
\[ W_{\text{spot},i} (c_i, z_i, y_i, p) = \underbrace{pc_i}_{\text{Contract payoff}} + \underbrace{\psi z_i - \frac{(z_i + y_i)^2}{2\kappa_i}}_{\text{Spot good payoff}} - \underbrace{pz_i}_{\text{Spot good price}} \]  

(18)

I omit factor exposure and contract price terms from (18); these terms do not depend on \( z_i \), so they do not affect traders’ decisions in the spot markets.\(^{27}\)

Expression (18) generalizes (3) from the baseline model in a number of ways. First, I assume that contract positions \( c_i \) must have mean 0, full support, and finite variances and covariances with other \( c_j \), but otherwise can be arbitrarily distributed. In particular, \( c_i \) need not be identically distributed across agents.

Second, (18) assumes that agents enter the spot market with some existing inventory position, \( y_i \), in the spot good.\(^{28}\) I assume that each \( y_i \) has mean 0, full support, and finite variances and covariances with all \( y_j \) and \( c_j \), but otherwise \( y_i \) can be arbitrarily distributed. Inventory shocks imply that agents will trade the spot good even if they do not hold contract positions, so that not all trade volume in the spot market is caused by manipulation.

Third, (18) allows spot market participants to have different holding capacities, \( \kappa_i \), for the spot good. Agents with larger \( \kappa_i \) have more elastic demand for the spot good; these may be agents who have more storage space for physical spot goods, or lower capital costs for trading financial spot goods.

The following proposition characterizes equilibrium bids and prices in the unique equilibrium of the general model.

\(^{27}\)More generally, (18) is an accurate model of spot market behavior under many different assumptions about how the contract market works: for example, as appendix B.3 discusses, spot traders may buy contracts to speculate rather than hedge, or as appendix B.4 discusses, spot traders’ contract purchases may have price impact. A spot trader who holds a long contract position profits from higher spot prices, regardless of why she entered into the contract in the first place. Thus, across many different models, spot traders’ contract positions are an observable sufficient statistic for spot traders’ incentives to manipulate spot markets.

\(^{28}\)Inventory shocks \( y_i \) can alternatively be interpreted as preference shocks for the spot good: expanding the quadratic term in (18), we have:

\[ -\frac{z^2}{2\kappa_i} - \frac{y_iz}{\kappa_i} - \frac{y_i^2}{2\kappa_i} \]

Ignoring the constant \( \frac{y_i^2}{2\kappa_i} \) term, \( y_i \) simply linearly shifts i’s marginal value of the spot good.
Proposition 3. There is a unique linear ex-post equilibrium in the general model, in which \( i \) submits the bid curve:

\[
 z_{Bi}(p; y_i, c_i) = -\frac{b_i}{\kappa_i} y_i + \frac{b_i}{\sum_{j\neq i} b_j} c_i - b_i (p - \psi) \quad (19)
\]

The spot auction price is:

\[
p - \psi = \frac{1}{\sum_{i=1}^{n} b_i \sum_{i=1}^{n} \left[ -\frac{b_i}{\kappa_i} y_i + \frac{b_i}{\sum_{j\neq i} b_j} c_i \right]} \quad (20)
\]

Bid slopes \( b_i \) satisfy:

\[
b_i = \frac{B + 2\kappa_i - \sqrt{B^2 + 4\kappa_i^2}}{2} \quad (21)
\]

and \( B = \sum_{i=1}^{n} b_i \) is the unique positive solution to the equation

\[
B = \sum_{i=1}^{n} \frac{2\kappa_i + B - \sqrt{B^2 + 4\kappa_i^2}}{2} \quad (22)
\]

5.2.1 The symmetric case

First, to illustrate how inventory shocks affect spot traders’ behavior, I specialize proposition 3 to the case in which agents are fully symmetric.

Corollary 1. Suppose that \( \kappa_i = \kappa \), \( \text{Var}(y_i) = \sigma_y^2 \), \( \text{Var}(c_i) = \sigma_c^2 \) for all \( i \), and all \( y_i \) and \( c_i \) are independent. Spot traders’ equilibrium bids are:

\[
z_{Bi}(p; c_i, y_i) = -\frac{n - 2}{n - 1} y_i + \frac{1}{n - 1} c_i - \frac{n - 2}{n - 1} \kappa (p - \psi) \quad (23)
\]

Price variance is:

\[
\text{Var}(p - \psi) = \frac{\sigma_y^2}{n\kappa^2} + \frac{\sigma_c^2}{n(n - 2)^2 \kappa^2} \quad (24)
\]

Expressions (23) and (24) are very similar to the expressions for bids and prices in the baseline model, in proposition 1; the only difference is the inclusion of \( y_i \) terms. In (23),
\( y_i \) and \( c_i \) have qualitatively similar effects, shifting agents’ bids vertically. However, as \( n \) increases, the \( y_i \) coefficient increases towards 1, whereas the \( c_i \) coefficient decreases towards 0. Intuitively, \( y_i \) reflects a fundamental, utility-driven component of bids, whereas \( c_i \) reflects a nonfundamental component, reflecting agents’ preferences over prices driven by their contract positions. In competitive markets, agents have less price impact, so their bids reflect fundamentals more and preferences over prices less.

Another implication of \((23)\) is that the coefficient on \( c_i \) tends to be much smaller than the coefficient on \( y_i \): an agent who receives a unit inventory shock changes her spot bid curve by approximately one unit, whereas an agent with a unit contract shock changes her spot bid by approximately \( \frac{1}{n} \) units. Thus, when \( n \) is large, most of the variation in agents’ bids (and thus settlement prices) will be driven by inventory shocks, even if the variance of \( c_i \) is much larger than the variance of \( y_i \). Quantitatively, from \((24)\), inventory positions and contract positions contribute equal amounts to settlement price variance when:

\[
\sigma_c = (n - 2) \sigma_y \tag{25}
\]

As I discuss in subsection 2.1, contract markets are often much larger than spot markets in practice. It is a puzzle why this liquidity mismatch can be sustained, without creating very large manipulation incentives for spot market participants. Expression \((25)\) provides a simple answer: contract positions can be much larger than inventory positions, as long as spot markets are sufficiently competitive.

Expression \((25)\) could also be used as a simple rule-of-thumb for evaluating whether a given market is vulnerable to manipulation: regulators need only check whether the total size of spot traders’ outstanding contract positions is more than \((n - 2)\) times larger than the total size of their inventory positions. This test only formally works under strong symmetry assumptions, but may be a reasonable first-pass when limited data are available.

5.2.2 Asymmetric agents

When agents are asymmetric, the coefficients on \( y_i \) and \( c_i \) differ across agents. From \((21)\) and \((19)\), agents with higher values of \( \kappa_i \) have lower \( y_i \) coefficients, and higher \( c_i \)
coefficients. Intuitively, agents with high $\kappa_i$ values are large relative to the market, so they shade their inventory shocks more, and also manipulate more per unit contract position that they hold.

Expression (20) shows that equilibrium prices depend on a weighted sum of $c_i$ and $y_i$; the weights on $c_i$ are higher for agents with larger $\kappa_i$ values. In words, this means that contract positions held by larger agents create larger distortions in benchmark prices, so manipulation risk in the general asymmetric case depends on the covariance between agents’ spot holding capacities, $\kappa_i$, and the variance of agents’ contract positions, $\text{Var}(c_i)$. The following subsection derives general expressions for basis risk and manipulation rents, which allow for arbitrary asymmetries between agents.

### 5.3 Measuring basis risk and manipulation rents in the general model

Expression (20) shows that auction prices are linear in agents’ inventory and contract positions. This implies that the moments of auction prices are simple functions of the moments of $y_i$ and $c_i$, allowing us to derive simple expressions for basis risk and manipulation rents. As in proposition 3, we define $B \equiv \sum_{i=1}^{n} b_i$ as the sum of agents’ bid slopes $b_i$. Define the coefficient vectors $k_c, k_y$ as:

$$
    k_c = \begin{pmatrix}
      \frac{b_1}{\sum_{j \neq 1} b_j} \\
      \vdots \\
      \frac{b_n}{\sum_{j \neq n} b_j}
    \end{pmatrix},
    k_y = \begin{pmatrix}
      \frac{b_1}{\kappa_1} \\
      \vdots \\
      \frac{b_n}{\kappa_n}
    \end{pmatrix}
$$

Define the covariance matrices of agents’ contract position, $\Sigma_{cc}$, agents’ inventory positions, $\Sigma_{yy}$, and the inventory-contract covariance matrix $\Sigma_{yc}$, respectively, as matrices with elements:

$$
    \Sigma_{cc} (i, j) = \text{Cov}(c_i, c_j),
    \Sigma_{yy} (i, j) = \text{Cov}(y_i, y_j),
    \Sigma_{yc} (i, j) = \text{Cov}(y_i, c_j)
$$

**Proposition 4.** In the general model of proposition 3, basis risk is:

$$
    \text{Var}(p - \psi) = \frac{1}{B^2} \left( k'_y \Sigma_{yy} k_y - 2k'_y \Sigma_{yc} k_c + k'_c \Sigma_{cc} k_c \right) \tag{26}
$$
Expected manipulation rents are:

$$\tau = \frac{1}{B} (-2k_y' \Sigma y c 1 + k_c' \Sigma c c 1)$$  \hspace{1cm} (27)$$

where \(1\) is the length-\(n\) unit vector.

Proposition 4 shows that two kinds of data are required to estimate basis risk and manipulation rents: data on agents’ bid slopes in spot markets, and data on the variances and covariances of agents’ inventory and contract positions.

**Bid slopes.** In order to estimate \(k_c\) and \(k_y\), the econometrician must estimate \(b_i\) for all agents, that is, the slope of each agent’s equilibrium bid curve with respect to prices. The coefficient vectors \(k_c\) and \(k_y\) are functions of both \(b_i\) and agents’ spot holding capacity \(\kappa_i\); however, \(\kappa_i\) can recovered from the vector of \(b_i\) values, and the aggregate bid slope \(B\), by inverting (21) of proposition 3.

For benchmarks which are set in auctions, bid slopes could be estimated directly from agents’ auctions bids. If bidding data are not available, an alternative approach would be to model or estimate agents’ spot holding capacities, \(\kappa_i\). \(\kappa_i\) is the inverse of the slope of \(i\)’s marginal utility for the spot good; there are a variety of econometric models and methods that can be used to estimate agents’ marginal utility functions in different settings. Given agents’ \(\kappa_i\) values, the econometrician could then calculate implied values of bid slopes \(b_i\) by solving for the unique equilibrium in proposition 3.

**Inventory-contract variance and covariances.** The econometrician must also estimate the variance-covariance matrices of \(c_i\) and \(y_i\). In many markets, regulators have access to detailed and high-frequency data on agents’ positions in both spot and derivative markets, so the variance and covariance matrices could be estimated using historical sample moments of \(c_i\) and \(y_i\).\(^{29}\) If \(c_i\) is not observable, the econometrician could derive bounds on \(\Sigma cc\) based on contract position limits which are imposed by exchanges or regulators. If \(y_i\) is not observable, under some distributional assumptions, the econometrician could estimate or bound \(\Sigma yy\) using the observed volume of trade, since more variable inventory shocks imply higher trade volumes in spot markets.

\(^{29}\)The covariance matrices are still high-dimensional, so finite-sample error may be a concern; this can be solved using a variety of econometric methods, such as low-dimensional factor models for agents’ contract and inventory covariances.
Summary. This section shows that the effects of manipulation on hedgers’ welfare depend on a fairly small number of parameters, which can conceivably be estimated by regulators in many markets. This is in contrast to the case for spot traders. The results of subsection 4.3 show that the effects of manipulation on spot traders’ welfare are complex even in the baseline model, and requires determining values of parameters, such as spot traders’ factor exposures and risk aversions, which are likely difficult to estimate from data. I thus focus on estimating hedgers’ welfare, firstly because it is easier to estimate, and secondly because in many (but not all) cases hedgers’ total contract positions are much larger than spot traders’ total positions.

An important question is whether the metrics I propose can be estimated precisely enough to be practically useful for contract market regulators. In the following section, I address this question by estimating both metrics using data on the LBMA gold auctions.

6 The LBMA Gold Price

In this section, I address the following question: how large would basis risk and manipulation rents be, if COMEX gold futures contracts were cash-settled using the London Bullion Market Association (LBMA) gold price benchmark? There are many sources of uncertainty in the LBMA gold auctions that may affect manipulation risk, such as the number of auction participants, the absolute and relative sizes of their bid slopes, and correlations between their contract positions and inventory shocks. I show that, across all of these sources of uncertainty, I can estimate informative bounds on basis risk and manipulation rents, which imply that both are small.

6.1 Background

The LBMA gold price is a benchmark price for gold, set twice each business day at 10:30AM and 3:00PM London time. While the popular COMEX gold futures contract (ticker symbol GC) is physically settled, the LBMA gold price is used to settle some exchange-traded gold futures and options, as well as some OTC gold derivatives.

2014). The LBMA gold price is also used by other market participants, such as miners, central banks, and jewellers, for purposes such as inventory valuation (Aspris, Foley and O’Neill 2020).

Prior to 2015, the price was set in a private teleconference auction between five members; this took between 10-15 minutes to conclude, and data on auction bidding was not made public. In 2014, the five banks involved in setting the LBMA gold price were accused of manipulation (Reuters Staff, 2014); the UK Financial Conduct Authority fined Barclays $43.9 million for bidding strategically to move benchmark prices, in order to avoid paying USD $3.9m to a customer who held a benchmark-linked option contract FCA (2014). In 2015, the ICE Benchmark Administration (IBA) took over the administration of the LBMA gold price; IBA moved to an electronic auction system, allowing more participants to enter, and began publishing detailed information about bids in intermediate rounds of the auction.

The LBMA gold price is now determined in a static auction which uses a dynamic price setting mechanism. During each round, IBA publishes a round price, and participants have 30 seconds to enter how much gold they would be willing to buy or sell at the announced price. If the difference between buy and sell volume is within an imbalance threshold – during the time period that my data covers, usually 10,000oz – the auction concludes and agents buy and sell the amounts that they bid. Otherwise, IBA adjusts the price in the direction of the difference between total buy and sell volume, and a new round begins. Thus, all trade occurs in the final round, but I observe agents’ bids for early rounds as IBA searches for the market-clearing price, and I can use these bids to estimate the slope of auction demand.

6.2 Data

The primary data source I use is the IBA Gold Auction Historical Transparency Reports. The data covers daily morning and afternoon auctions; the full dataset covers 1,650 auctions over the period 2015-03-20 to 2018-06-29. For each round of each auction, I observe the number of participants, the round price, and the total volume of gold that

31 Note that, throughout this section, I will use oz as shorthand for troy ounces.
32 I accessed the data at the ICE website
auction participants wish to buy and sell during the round.

To my knowledge, the formal rules used by ICE for updating prices between auction rounds are not public; however, the data suggest that ICE essentially runs a Walrasian auction. The left panel of figure 2 plots, for non-terminal auction rounds, the volume imbalance in round $r$ (that is, the difference between buy and sell volume) against the change in prices from round $r$ to round $r + 1$. Price changes correlate with volume imbalances: if there is excess demand, the round price increases, and if there is excess supply, the round price decreases.

The right panel of figure 2 shows that auction participants appear to respond to round price changes in the expected direction: if prices go up, the volume imbalance decreases, and vice versa. For each auction round $r > 1$, the x-axis shows the change in prices from round $r - 1$ to $r$, and the y-axis shows the change in aggregate volume imbalance from round $r - 1$ to $r$. The relationship between price changes and volume imbalance changes is negative: if the auction price increases from round $r - 1$ to $r$, the difference between buy and sell volume tends to decrease, and vice versa.

This relationship is statistically quite robust: round volume imbalance differences have opposite signs to round price changes in 88.4% of all auction rounds. The OLS coefficient from regressing volume differences on price differences is $-49,888 \text{ oz}^2/\text{USD}$, implying that each dollar change in round price is associated with a change in buy-sell imbalance of 49,888 oz, on average across auctions.

### 6.3 Auction demand slope measurement

The first step in the empirical analysis is to map quantities in the data to $B$, the slope of aggregate auction demand in the model. I will essentially identify $B$ based on the pattern observed in the right panel of figure 2 by regressing volume imbalances on prices. This faces a number of challenges.

First, the LBMA gold auction uses a dynamic price setting process, and it is unclear that the static bidding game I analyzed in previous sections is an appropriate model.\(^{33}\)

\(^{33}\)Note that the LBMA gold auction uses a dynamic process to set prices, but it is a static auction, since all trade occurs in a single period at a single price. It is thus much simpler to analyze than dynamic auctions.
Figure 2: Price and bid behavior in LBMA gold auctions

Notes. The left panel shows the round \( r \) volume imbalance (buy minus sell volume) on the x-axis, and the price change from round \( r \) to round \( r+1 \) on the y-axis. The right panel shows the round \( r \) to round \( r+1 \) price change on the x-axis, and the round \( r \) to round \( r+1 \) change in volume imbalance on the y-axis. Each data point is one auction round. The sample of auctions used is the set of 1,331 auctions which last for more than a single round.

In appendix D.1 I study a stylized model of the ICE dynamic price-setting mechanism, and show that it admits an equilibrium which is isomorphic to the unique static bidding equilibrium: if the round price is \( p \), agents simply bid their unique equilibrium bids \( z_{B_{p}}(p; y_i, c_i) \) from proposition 3. The intuition is that, if all agents are bidding non-dynamically, according to their static equilibrium strategies, no agent can do better by deviating.

This shows that the static bid submission game is at least an internally consistent model of behavior in the dynamic LBMA gold auctions. I have not proven that the equilibrium is unique, so other equilibria may exist; I must assume that agents are playing the static equilibrium to bring the model to the data. Empirically, however, dynamic behavior does not appear to affect bid slope estimates substantially: figure 5 in the appendix estimates demand slopes using only bids in later rounds of auctions, and shows that these estimates are statistically very similar to my baseline slope estimates.

Second, I must choose how flexibly to allow demand slopes to vary across auctions, with multiple trading periods, as papers such as Sannikov (2016) and Du and Zhu (2017) study.
Under the assumption of linear equilibrium bid functions, the slope of aggregate demand $B$ is identified using data from any two rounds within an auction: $B$ is simply the ratio of volume imbalance differences to price differences for any two rounds. I thus estimate $B$ separately for each auction $a$ in the dataset; I will use $B_a$ to refer to the estimated demand slope for auction $a$. In any auction with more than two rounds, the slope of demand is overidentified, so I estimate demand slopes by regressing volume imbalances on prices within auctions.

I describe my demand slope measurement procedure more formally in appendices D.2 and D.3. To test whether the estimated demand slopes indeed behave like a measure of gold dealers’ cost of liquidity provision, in appendix D.3 I regress estimated demand slopes on the VIX, realized gold volatility, and implied volatility from gold ETF options; all coefficients are negative and significant. In appendix D.4 I show that the linear bid model fits the data fairly well, explaining around 80% of the variance in volume imbalances across rounds within auctions.

Third, I must choose a sample of auctions to use for estimation. In the main text, I limit the sample to auctions with a fixed number of participants, and with at least 3 rounds. This restricts the sample to 502 of the 1,331 auctions used in figure 2. This restriction helps in interpreting the results, because my model does not formally apply to auctions with a varying number of participants over time, and because auction demand slopes are estimated more precisely for auctions with more rounds. However, this restriction is not required for my results: in appendix D.8 I repeat the analysis using the full sample of all 1,331 auctions which last more than one round, and find that all results are quantitatively very similar.

The left panel of figure 3 shows the distribution of estimated auction demand slopes, $B_a$, for each auction $a$. The modal demand slope is around 50,000 oz$^2$/USD, but some auctions have slopes as high as 200,000 oz$^2$/USD or as low as around 6,000 oz$^2$/USD. Auction demand slopes are somewhat higher for auctions with more participants.

I also use total auction trade volumes to identify the variance of traders’ inventory shocks. The right panel of figure 3 shows the distribution of the total volume of gold traded in auctions. Modal volume is around 150,000 oz, but volumes can be as high as 500,000 oz. Volumes are also somewhat higher for auctions with more participants.
Table 1: Gold auctions descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>P10</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds</td>
<td>4.17</td>
<td>1.5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Participants</td>
<td>7.69</td>
<td>1.21</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Price (USD / oz)</td>
<td>1223.98</td>
<td>75.24</td>
<td>1118.16</td>
<td>1327.47</td>
</tr>
<tr>
<td>Price range (USD / oz)</td>
<td>1.05</td>
<td>0.79</td>
<td>0.45</td>
<td>1.9</td>
</tr>
<tr>
<td>Volume (1000 oz)</td>
<td>167.58</td>
<td>85.26</td>
<td>90.08</td>
<td>244.24</td>
</tr>
</tbody>
</table>

N | 502

Notes. Each observation is an auction, indexed by a. “Rounds” is the number of rounds the auction took to complete, R_a. “Participants” is the number of auction participants, n_a. “Price” is the auction clearing price, p_a. “Price range” is the difference between the highest and lowest round prices. “Volume” is v_a, the sum of buy volume and sell volume in the final round of the auction.

Figure 3: Gold auctions demand slopes and volumes

Notes. The left panel shows the distribution of estimated aggregate demand slopes, B_a, for different numbers of auction participants n_a, estimated by regressing round volume imbalances on prices within auctions, as described in appendices D.2 and D.3. The right panel shows the distributions of volumes v_a for different n_a. The dataset used is the primary estimation sample of 502 auctions.

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6.4 Estimation

The next step in the estimation is to map the measured aggregate demand slopes $B_a$, participant counts $n_a$, and trade volumes $v_a$ for each auction $a$ into bidder-level primitives: spot holding capacities $\kappa_i$ and inventory variances $\sigma^2_{y_i}$ for individual bidders. Then, based on these primitives, and assumptions about the counterfactual variances and covariances of contract positions $\sigma^2_{e_t}$, I can calculate implied basis risk and manipulation rents. In the process, I must address three sources of model uncertainty. First, I observe only aggregate demand slopes and volumes, not $\kappa_i$ and $\sigma^2_{y_i}$. Second, these bid slopes and volumes vary across auctions. Third, I must take a stance on the covariances between contract positions and inventory shocks to calculate basis risk and manipulation rents.

6.4.1 Uncertainty in $\kappa_i$ and $\sigma^2_{y_i}$ given $B$ and $v$

To deal with uncertainty in individual parameters within auctions, I use a bounds approach. For each set of target moments $n, B, v$, I randomly sample 10,000 possible vectors of individual bidders’ parameters, $\kappa_i$ and $\sigma^2_{y_i}$, which are consistent with the target moments. I then calculate and report the maximum and minimum values of basis risk and manipulation rents over all draws of $\kappa_i$ and $\sigma^2_{y_i}$. This tells us how much knowing the aggregate quantities $B$ and $v$ bounds basis risk and manipulation rents, over any possible combination of individual bidders’ parameters $\kappa_i$ and $\sigma^2_{y_i}$ which are consistent with $B$ and $v$.

While I can estimate bid slopes $b_i$ and thus spot holding capacities $\kappa_i$ without imposing any distributional assumptions, I assume inventory shocks $y_i$ are independent normal random variables, in order to match $\sigma^2_{y_i}$ to observed volumes. Further details of the random sampling process are described in appendix D.5.

As tables 2 and 3 below show, these random sampling-based bounds are surprisingly tight: for any given choice of $n, B, v$, the max and min values for basis risk and manipulation risk differ by a factor of around 2. Intuitively, holding fixed a given auction demand slope $B$, increasing one agent’s holding capacity $\kappa_i$ implies that some other agent’s $\kappa_j$ must decrease, and the effects of these changes on outcome metrics approximately offset each other.
6.4.2 Uncertainty in $B_a$ and $v_a$ across auctions

As figure 3 shows, there is substantial variation in demand slopes and auction volume across auctions. According to my model, auctions with higher slopes $B_a$ and lower volumes $v_a$ have lower manipulation risk, since prices are harder to move, and lower trade volumes imply lower inventory shock-driven basis risk. Calibrating the model separately to each auction is very computationally intensive; instead, I calibrate the model to match three sets of target moments for each $n$: a low manipulation risk, setting $B$ to its 80th percentile and $v$ to its 20th percentile across auctions; medium, setting both $B$ and $v$ to their median values; and high, setting $B$ to its 80th percentile and $v$ to its 20th percentile. Appendix table 5 shows the values of target moments, that is, the p20, p50, and p80 values of $B_a$ and $v_a$ across auctions for each $n_a$.

6.4.3 Assumptions on contract and inventory covariances

As subsection 5.2 discusses, both basis risk and manipulation rents depend on the variances and covariances between $c_{ai}$ and $y_{ai}$. I set the standard deviations of agents’ contract positions to 1,500 contracts, worth 150,000oz of gold exposure, which is half the CME position limit of 300,000oz, implying that agents’ contract positions will be below position limits with approximately 95% probability. I then consider three possible cases for covariances, ranked in order of increasing size of manipulation-induced distortions.

First, I assume contracts $c_{ai}$ are independent of inventory shocks $y_{ai}$, as well as all other agents’ $c_{aj}$, $y_{aj}$. Second, I assume $c_{ai}$ is perfectly negatively correlated with $y_{ai}$, but independent of $c_{aj}$, $y_{aj}$; this maximizes basis risk and manipulation rents under the constraint that $c_{ai}$, $y_{ai}$ are uncorrelated across agents. Third, I assume $c_{ai}$ is independent of $y_{ai}$, but is perfectly correlated with all other agents’ contract positions $c_{aj}$; this maximizes basis risk and manipulation rents under the constraint that $c_{ai}$, $y_{ai}$ are uncorrelated. Since I require inventory shocks to be independent and Gaussian to match the auction trade volume moment, for internal consistency, I do not allow correlations between inventory shocks $y_{ai}$ across agents.

Appendix D.7 derives expressions for basis risk and expected manipulation rents under each of these three assumptions about the covariance structure of $c_{ai}$, $y_{ai}$.
6.5 Results

Tables 2 and 3 show my estimates of basis risk and manipulation rents across all sources of uncertainty: estimates are shown separately for different numbers of participants, for the low, medium and high choices of target slope and volume moments, and for different assumptions about contract and inventory covariances. Each cell reports the median value of basis risk or manipulation rents over all draws of $\kappa_i$ and $\sigma_{yi}$, with the minimum and maximum values in parentheses. Column 3 assumes contract positions are independent across agents; column 4 assumes contract positions and inventory shocks are perfectly negatively correlated for each agent, but independent across agents; and column 5 assumes contract positions are perfectly correlated across agents, but are independent of inventory shocks.

All four sources of uncertainty – $N$, target moments, $\kappa_i$ and $\sigma_{yi}$, and contract covariances – affect both outputs, but the effects of each source of uncertainty are relatively small. Increasing $N$ has a non-monotone effect on both outcomes; basis risk and rents are lowest for auctions with 8-9 participants, and increase somewhat for 10. The inflection at 10 is because, as appendix table 5 shows, aggregate demand slopes are actually somewhat lower for auctions with 10 participants, so price impact is cheaper in these auctions.

As we move from slope and volume target moments with low to high manipulation risk, (the “B+v type” column), both risk metrics increase by a factor of around 2. Similarly, the max and min of both outcomes, over different draws of $\kappa_i$ and $\sigma_{yi}$, differ by a factor of around 2-3, depending on other parameters. Finally, the different assumptions about contract covariances change basis risk by a factor of around 2-3, and manipulation rents by a factor of around 10. Collectively, this implies that the max and min values in tables 2 and 3, over all sources of uncertainty, are not extremely far apart: basis risk is between 0.795 to 9.916 USD/oz. The range of possible manipulation rents is somewhat larger, varying from $348,272 to $12,954,987 USD.

Position limit-sized contract positions are fairly large: a 150,000oz gold contract position is around a $225 million USD exposure to gold prices. However, this is a very small fraction of the gold futures market as a whole: on average from 2015-2016, the most liquid COMEX gold contract month has around 300,000 outstanding contracts, equal to a
total gold exposure of around $45 billion USD.\footnote{34} Dividing total manipulation rents by the total value of outstanding contracts, hedgers lose only between 0.077 to 2.88 basis points of their notional gold exposure to manipulation rents; while these bounds are fairly wide, even the upper bound is very low.

Basis risk, on the other hand, can create quantitatively large transfers from longs to shorts. Multiplying basis risk per ounce by the volume of outstanding gold exposure, basis risk creates net transfers from longs to shorts with a standard deviation between $23.8 million to $297.5 million USD; this is large in absolute terms, although it is a transfer mostly from hedgers to other hedgers. To evaluate how much this basis risk increases hedgers’ wealth variance, I calculate the basis risk coefficient \(\rho\). Since gold futures contracts expire once per month, I estimate factor risk, \(\sigma_{\psi}^2\), as the variance of monthly gold futures prices, over the period 2015-2018; the standard deviation of these differences is $67.94 USD/oz. Plugging price variance and the upper and lower bounds for factor risk into expression (17), I find that the basis risk coefficient \(\rho\) is between 0.000137 and 0.0209.

In words, even under the unrealistic assumption that all spot traders hold position-limit-sized contract positions, which are perfectly correlated across agents, manipulation-induced basis risk only exposes hedgers to 2.09% of total monthly variance in gold prices. Under more realistic assumptions, basis risk exposes hedgers to well under 1% of total monthly factor risk. Together, these results suggest that, under current contract position limits, the LBMA gold could be used to settle COMEX gold futures contracts, with very small welfare losses to hedgers from manipulation.

### 6.6 Discussion of assumptions

The analysis omits a number of factors. First, participants in the LBMA gold price can submit bids on behalf of their clients\footnote{35} if dealers simply perfectly pass through their clients’ bids to the auction, the actual number of effective auction participants may be...

\footnote{34}These calculations effectively assume all 300,000 outstanding contracts are held to expiration. In practice, contract holders will rarely hold their contracts to expiration, as many will roll into the next month close to expiration, and manipulation would only directly affect agents holding to expiration, so the numbers here are likely to overestimate manipulation-induced distortions.

\footnote{35}See, for example, [London Gold Fix](#).
Table 2: Gold auctions basis risk estimates

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>1.76 (1.36-2.89)</td>
<td>2.43 (2.05-3.15)</td>
<td>3.31 (3.03-4.30)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>2.55 (1.97-4.19)</td>
<td>3.96 (3.38-5.16)</td>
<td>4.80 (4.39-6.23)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>4.06 (3.13-6.67)</td>
<td>4.48 (3.68-6.75)</td>
<td>7.64 (6.99-9.92)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>1.38 (1.04-2.28)</td>
<td>1.91 (1.64-2.55)</td>
<td>2.77 (2.54-3.52)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>2.36 (1.78-3.90)</td>
<td>3.72 (3.19-4.69)</td>
<td>4.74 (4.35-6.02)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>3.22 (2.43-5.34)</td>
<td>3.60 (2.96-5.48)</td>
<td>6.47 (5.94-8.22)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>1.30 (0.98-2.19)</td>
<td>1.88 (1.61-2.48)</td>
<td>2.78 (2.57-3.48)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>1.68 (1.27-2.83)</td>
<td>2.81 (2.38-3.55)</td>
<td>3.59 (3.32-4.50)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>2.54 (1.92-4.25)</td>
<td>2.89 (2.36-4.37)</td>
<td>5.41 (5.00-6.78)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>1.13 (0.85-1.95)</td>
<td>1.75 (1.52-2.30)</td>
<td>2.53 (2.36-3.15)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>1.45 (1.09-2.50)</td>
<td>2.56 (2.23-3.27)</td>
<td>3.25 (3.02-4.03)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>2.03 (1.53-3.51)</td>
<td>2.37 (1.97-3.66)</td>
<td>4.55 (4.24-5.66)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>1.05 (0.79-1.80)</td>
<td>1.63 (1.43-2.13)</td>
<td>2.48 (2.32-3.05)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>1.46 (1.10-2.50)</td>
<td>2.63 (2.34-3.29)</td>
<td>3.44 (3.22-4.23)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>2.46 (1.86-4.21)</td>
<td>2.78 (2.26-4.32)</td>
<td>5.80 (5.43-7.13)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>1.23 (0.94-2.16)</td>
<td>2.15 (1.90-2.69)</td>
<td>3.02 (2.85-3.61)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>1.68 (1.28-2.95)</td>
<td>3.35 (2.98-4.10)</td>
<td>4.13 (3.89-4.94)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>2.95 (2.25-5.16)</td>
<td>3.24 (2.62-5.30)</td>
<td>7.23 (6.81-8.64)</td>
</tr>
</tbody>
</table>

Notes. Estimated standard deviations of basis risk, in units of prices (USD/oz). Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 5. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
## Table 3: Gold auctions manipulation rent estimates

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>0.497 (0.454-0.645)</td>
<td>0.762 (0.675-0.865)</td>
<td>2.484 (2.272-3.223)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>0.720 (0.659-0.935)</td>
<td>1.261 (1.095-1.412)</td>
<td>3.601 (3.295-4.674)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>1.146 (1.049-1.487)</td>
<td>1.329 (1.218-1.619)</td>
<td>5.730 (5.243-7.437)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>0.415 (0.381-0.528)</td>
<td>0.647 (0.584-0.725)</td>
<td>2.491 (2.285-3.165)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>0.710 (0.652-0.903)</td>
<td>1.284 (1.147-1.415)</td>
<td>4.262 (3.911-5.416)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>0.971 (0.891-1.233)</td>
<td>1.153 (1.062-1.386)</td>
<td>5.824 (5.344-7.401)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.417 (0.385-0.522)</td>
<td>0.687 (0.618-0.750)</td>
<td>2.916 (2.698-3.657)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>0.539 (0.498-0.675)</td>
<td>1.041 (0.909-1.136)</td>
<td>3.770 (3.489-4.727)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>0.811 (0.750-1.017)</td>
<td>0.994 (0.928-1.164)</td>
<td>5.676 (5.252-7.117)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.380 (0.354-0.473)</td>
<td>0.685 (0.619-0.755)</td>
<td>3.041 (2.832-3.780)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>0.487 (0.453-0.605)</td>
<td>1.014 (0.904-1.099)</td>
<td>3.894 (3.626-4.841)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0.683 (0.636-0.849)</td>
<td>0.866 (0.817-1.013)</td>
<td>5.466 (5.090-6.795)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.372 (0.348-0.457)</td>
<td>0.672 (0.623-0.734)</td>
<td>3.348 (3.134-4.114)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>0.516 (0.484-0.635)</td>
<td>1.100 (1.009-1.193)</td>
<td>4.648 (4.352-5.713)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>0.870 (0.815-1.070)</td>
<td>1.053 (0.994-1.227)</td>
<td>7.834 (7.335-9.627)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.454 (0.427-0.542)</td>
<td>0.941 (0.867-1.015)</td>
<td>4.535 (4.275-5.419)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>0.620 (0.584-0.740)</td>
<td>1.477 (1.348-1.593)</td>
<td>6.197 (5.841-7.403)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>1.084 (1.022-1.295)</td>
<td>1.267 (1.207-1.467)</td>
<td>10.843 (10.220-12.955)</td>
</tr>
</tbody>
</table>

Notes. Estimated expected manipulation rents, in units of millions of US dollars. Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1 “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2 Target moment values are in table 5. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
much larger than the 5-10 dealers who formally bid in the auction. Increasing the number of auction participants while holding fixed the aggregate demand slope has two offsetting effects on manipulation risk. When \( n \) is higher, auctions are more competitive, so each agent has lower manipulation incentives; however, since contract position limits apply to each independent agent, the total number of contracts held by all auction participants can increase.

In appendix D.9 I analyze the effect of increasing \( n \), while holding fixed agents’ contract position variances \( \sigma_c^2 \) and the aggregate bid slope \( B \), on basis risk and manipulation rents. Basis risk decreases linearly in \( n \), while total manipulation rents are asymptotically constant. Thus, my results will tend to overestimate basis risk, and would be within a constant factor of manipulation rents, if the number of effective participants is larger than the observed \( n_a \) values.

I estimate the slope of demand using relatively small price movements – table I shows that, within a single auction, the range of prices tends to be around $0.5 to $1.9 USD/oz. One concern is that the slope of market demand for large price movements could be different from this local slope. On the one hand, if a manipulator were able to “corner” the gold market, buying enough gold to exhaust the supplies of all other auction participants, gold supply could become much more inelastic, and the manipulator could increase prices essentially without bound. On the other hand, large price movements might cause auction participants who are normally inactive to enter the market, so the slope of market supply could also be larger for large price movements.

The data I have do not allow me to distinguish between these two possibilities. I can only state that, within the range of demand slopes I observe in my sample, basis risk and manipulation rents are fairly low. As figure 3 shows, these slope estimates vary by a factor of 3-4 across auctions. So long as demand slopes for large price movements are not substantially lower than the lowest demand slopes observed in my sample, my conclusions that manipulation risk is low should be robust for larger price movements.

The LBMA gold auctions are currently run twice daily, while COMEX gold futures contracts settle only once per month. If a single auction each month were used to settle COMEX gold futures, this auction would likely attract more participants than standard daily auctions. Entry would decrease manipulation risk, by increasing competition and
the slope of aggregate demand: expression (8) of proposition [1] shows that basis risk decreases at rate $\frac{1}{n^3}$, and expression (16) shows that manipulation rents decrease at rate $\frac{1}{n^3}$.

Finally, my analysis estimates manipulation risk assuming the standard deviation of agents’ contract positions is half the current position limit of 3,000 contracts. The model could also be used to measure basis risk under alternative position limits by plugging in different values for $\sigma^2_c$. A regulator could thus use this analysis to determine how large position limits could be, while ensuring that basis risk and manipulation rents stay below certain target values.

7 Discussion

7.1 Implications for contract market regulation

Contract market regulators use two main structural policy tools to regulate contract market manipulation: position limits and benchmark regulation. My results have implications for how regulators should apply both tools.

7.1.1 Position limits

Under the Commodity Exchange Act, the CFTC has the authority to impose speculative position limits on contract market participants, to limit the risk of large price movements. These limits apply not only to individual market participants, but also to groups of participants who are coordinating their trades. Position limit violations are a very

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36 As I discuss in subsection 3.2, the costs of entry vary widely across different kinds of spot markets. Some assets, such as the S&P 500 options underlying the VIX, or the stocks underlying S&P 500 futures, are traded on public-facing exchanges, so there is essentially no entry cost. For goods such as oil, gas and electricity, entry is effectively impossible over short time horizons, since spot market participants require physical infrastructure – pipelines and storage facilities – which are costly and take time to build. For other goods, such as gold, FX, ISDAFIX and repo loans, benchmarks are set using inter-dealer trade prices, often on trade platforms with restricted participation, so there are institutional and regulatory constraints to entry in spot markets.

37 See Section 4a(1) of the Commodity Exchange Act, 7 U.S.C. § 6a(1).
common class of enforcement actions. In a famous 1979 case,\footnote{Commodity Futures Trading Com’n v. Hunt, 591 F.2d 1211 (7th Cir. 1979)} seven Hunt family members and an associated corporation coordinated to purchase soybean futures positions totalling over 23 million bushels of soybeans, well over the position limit of 3 million bushels. More recent cases involving wheat,\footnote{CFTC Press Release 7955-19} soybean,\footnote{CFTC Press Release 8021-19} cattle,\footnote{CFTC Press Release 8002-19} and other markets are documented on the CFTC’s press release website.

While the primary stated purpose of position limits is to prevent excessive futures price volatility,\footnote{The CFTC’s stated purpose for speculative position limits is to “protect futures markets from excessive speculation that can cause unreasonable or unwarranted price fluctuations”; hence position limits do not exist solely to combat manipulation, although according to my theory they can be an effective tool for doing so.} the results of this paper suggest that position limits can also be effective as a tool for reducing manipulation risk. Interestingly, the CFTC currently applies position limits more harshly for pure hedgers than spot traders.\footnote{Contract position limits do not apply for market participants who have bona fide commercial risks to hedge; see the CFTC’s website on Speculative Limits} This is justifiable if the goal is to limit the risk of default, or margin calls and large price movements resulting from large unhedged positions: a spot trader with offsetting contract and spot positions has less total exposure to factor risk than a pure hedger with only a contract position.

This makes less sense if the goal is to limit manipulation risk, however, since spot traders with offsetting spot and contract factor risk exposures still have large incentives to trade spot goods to move the basis, \( p - \psi \). In contrast, pure hedgers who are unable to trade in spot markets do not create manipulation risk, regardless of the size of their contract positions. Thus, my results imply that, if manipulation risk is a first-order concern, regulators may wish to reverse their current policies: position limits should actually be applied more harshly to spot market participants than pure hedgers.

Quantitatively, my metrics can be used to calculate, based on spot market structure, how large position limits can be, to ensure that basis risk and manipulation rents remain below a certain desired level. This appears to be novel to the literature: from my conversations with market participants, it appears that there is currently no systematic, theory-based approach to determine appropriate contract position limits in any given
market.

7.1.2 Benchmark regulation

Benchmark regulation is a comparatively new form of structural market intervention. Following the LIBOR manipulation scandal, there has been a concerted effort across jurisdictions to move hundreds of trillions of dollars of interest rate derivatives towards new, less manipulable interest rate benchmarks. Since then, principles for financial benchmarks have been released by the International Organization of Securities Commissions (IOSCO (2013)), the FCA began regulating a number of benchmarks, and then in 2018 EU law was revised to include benchmark regulation, along similar principles to the IOSCO report.

The IOSCO (2013) report outlines many details of benchmark governance, calculation methodology, transparency, and accountability; it does not discuss features of spot market structure which influence the robustness of benchmarks. This paper shows that, even if benchmarks are based on correctly reported prices of actual trades, if spot traders have market power and price impact, they have incentives to distort price benchmarks to profit from their derivative positions. Changes in governance and benchmark calculation methodology have limited effectiveness in markets which are structurally vulnerable to manipulation. In such settings, regulators should instead target structural features of contract and spot markets directly, by limiting the size of spot traders’ contract positions, encouraging entry to make spot markets more competitive, and monitoring market participants’ spot trading costs to ensure spot markets are sufficiently liquid.

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44 See, for example, Duffie and Stein (2015).
45 FCA to regulate seven additional financial benchmarks.
46 FCA (2016).
47 A number of manipulation cases in practice appear to involve problems with market structure, rather than governance or reporting. Manipulation in the FX markets, and manipulation of the ISDAFIX benchmark, both involved actual and correctly reported trades made by market participants in auction or limit-order-book markets. See, respectively, Levine (2014) for FX and Leising (2017) for ISDAFIX manipulation.
7.1.3 Optimal regulation?

In the baseline model of the paper, if the regulator wishes to maximize spot traders’ welfare, assuming that the regulator cannot influence traders’ behavior in spot markets, subsection 4.3 shows that there is some quadratic subsidy or tax scheme which can implement the optimal linear equilibrium for spot traders. If the regulator wishes to maximize pure hedgers’ welfare, without regard for spot traders, she can simply ban spot traders from holding derivative contract positions. Spot traders will then bid identically in the spot auction, and the spot price will be identically equal to $\psi$, so hedgers will be able to perfectly share factor risk.

In practice, regulators likely have positive welfare weights on both spot traders and hedgers. Since hedgers’ welfare is strictly decreasing in $t$, the regulator will want to choose some value of $t$ which is lower than the spot-trader optimum. However, there appears to be no simple theoretical way to determine relative welfare weights, that is, to trade off spot trader and hedger welfare, in any given contract market.

Rather than attempting to solve for optimal policy, a simple alternative approach is to set maximum allowable levels for basis risk and manipulation rents, and to intervene in markets when either or both metrics rise above these upper bounds. This is similar to the approach used in antitrust regulation: the FTC and DOJ’s Horizontal Merger Guidelines classify markets as moderately or highly concentrated based on the level of the Herfindahl-Hirschman index, although various other industry-specific concerns also factor into regulators’ decisions.

7.2 Mechanisms for benchmark setting

This paper uses uniform-price double auctions as a reduced-form model of price benchmarks. Some benchmarks, such as VIX and the LBMA gold price discussed in this paper, are determined using actual auctions. Derivative contracts for many equity indices are also settled based on exchange opening or closing auction prices.

Some benchmark-setting mechanisms may produce outcomes similar to uniform-price
double auctions. The WM/Reuters FX fixing\textsuperscript{49} and the ISDAFIX interest rate swap benchmark\textsuperscript{50} (now the ICE swap rate) are set using exchange prices within a few minutes; if agents submitted fixed bid curves at the start of the fixing period and do not adjust bids through the course of the auction, outcomes will coincide with static uniform-price double auction outcomes.\textsuperscript{51} Some benchmarks for commodities such as oil and gas are set using volume-weighted average prices in specific geographical locations, over relatively short time spans; if the underlying goods are relatively homogeneous, market outcomes can be approximated by supply function competition between dealers (Klemperer and Meyer, 1989), which is equivalent to a uniform-price double auction.

Other benchmarks are less well approximated by auctions. Some benchmarks are based on trades of underlying assets in markets with large search or transportation frictions. For example, the CME Feeder Cattle Index is based on US-wide cattle trade prices; the price of cattle traded in New York on any given day may differ substantially from the price of cattle traded in California. Other markets are organized as core-periphery networks, with central dealers trading with peripheral counterparties (Wang, 2017; Duffie and Wang, 2016). In these markets, agents’ manipulation incentives may depend on their physical location, or their position in the dealer network, in addition to their holding capacity for spot goods.

Some benchmarks are not based on prices of verifiable trades, but rely on market participants to self-report trades or potential trades. For example, LIBOR is based on banks’ announcements of their borrowing costs\textsuperscript{52} and some natural gas benchmarks are based on market participants’ reports of their trades\textsuperscript{53} In these settings, market participants can manipulate benchmarks simply by falsely reporting trades, so manipulation is potentially much easier than in the auction model studied in this paper.

A normative interpretation of my results is that auctions are a relatively robust way

\textsuperscript{49}WM/Reuters FX Benchmarks
\textsuperscript{50}ICE Swap Rate
\textsuperscript{51}Since the WM/Reuters fixing period is only a few minutes long, there is likely little arrival of information during the fixing, and thus no fundamental reason for participants to change their bids over time. However, there may be strategic reasons to adjust bids dynamically; Du and Zhu (2017) argue that the frequency of auctions does in fact affect market outcomes even in the absence of information arrival.
\textsuperscript{52}ICE LIBOR
\textsuperscript{53}CFTC Press Release 5409-07
to set price benchmarks, so regulators could encourage broader use of auctions for benchmark setting. Auctions have a number of benefits besides those discussed in this paper. They are anonymous, so any offer to buy or sell can be taken up by any other agent; thus, agents cannot distort benchmarks by trading at artificially high or low prices with favored counterparties. They are also straightforward to run; unlike average-price benchmarks, they do not require costly infrastructure to track all trades, and thus cannot be manipulated by falsifying trades.\footnote{As an example, the CFTC brought charges against many companies for false reporting of natural gas trades to price index compilers; see \textit{CFTC Press Release 5300-07}.}

### 7.3 Settlement by physical delivery

Many derivative contracts are settled, not by cash payments, but by physical delivery of the underlying asset. Under normal market conditions, physical delivery and cash-settled contracts function similarly, because most holders of physical delivery contracts close out their positions for cash payments prior to delivery. \footnote{Markham (2014, pg. 3). Note that “corners” and “squeezes” are defined slightly differently by different authors.} Kyle (2007) derives a set of conditions under which cash-settled contracts and physical delivery contracts are economically equivalent; one important condition is that agents can trade an arbitrary quantity of the underlying asset at the cash settlement price.

While this paper focuses on cash-settled derivatives, the measures I propose may also be applicable to physical delivery contract markets in some cases. In one common manipulation strategy, called a “squeeze”, a long manipulator buys the underlying asset to create a shortage of the underlying asset, raising prices of the underlying asset and increasing the prices at which shorts can close out their contracts. In these settings, shorts can choose to make delivery, but they would have to pay elevated prices for the spot asset. This appears to function very similarly to cash-settled contract manipulation, and my proposed risk metrics could potentially apply to these settings.

Another form of manipulation, which is sometimes called a “corner”, works as follows. A long manipulator aims to buy up enough of the underlying asset that it is essentially impossible for shorts to fulfill their delivery requirements. The long manipulator then uses the threat of default to extract large payments from shorts to close out contracts.\footnote{Markham (2014, pg. 3). Note that “corners” and “squeezes” are defined slightly differently by different authors.}
Corners thus require that long contract holders can threaten to take delivery, and that delivery for shorts is logistically impossible at any cost.

The assumptions of the \[\text{Kyle (2007)}\] equivalence result are violated for corners, because shorts cannot purchase arbitrary quantities of underlying assets at settlement prices; that is, if shorts owe more of the underlying asset than the total deliverable market supply, settlement prices are essentially undefined. Thus, corners seem to be a qualitatively different form of manipulation than squeezes, which are closely linked to the ability of long contract holders to demand delivery; my metrics likely do not fully capture the risk of this form of manipulation.

8 Conclusion

The regulation of contract market manipulation is a contentious topic in both academic and policy circles. Illegal manipulation is essentially defined as trading with the intent to move prices. This definition is vague, and it is unclear that regulators and courts currently apply it in a way that is optimal for improving market quality and social welfare.

This paper makes two main contributions. First, I develop a theoretical basis for structurally regulating manipulation; I construct a model in which manipulation represents a market failure, which can cause equilibrium outcomes to be Pareto dominated. Regulatory intervention can alleviate this market failure, potentially improving the welfare of all market participants. Second, I develop empirical methods for predicting the size of manipulation-induced distortions, using market data that is often observed by regulators.

The methods developed in this paper can be used by regulators to regulate manipulation in a primarily structural manner. If predicted manipulation risk is high in a given market, regulators can use policy tools such as position limits to decrease spot traders’ contract positions, and thus their incentives to manipulate. If predicted risk is low, regulators can largely leave markets to their own devices. Such an approach could both reduce regulatory uncertainty for market participants, and also direct scarce regulatory attention towards settings in which manipulation is likely to be most harmful for social welfare.

\[\text{authors};\] for example, the definitions in \[\text{Kyle (1983)}\] are slightly different.
References


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Internet Appendix

A Proof of proposition 1

Repeating (3), spot traders’ total wealth can be written as:

\[ W_{\text{spot}}(x_i, c_i, z_i, p, \psi) = \psi x_i - \mu \psi x_i - \mu \psi c_i + p c_i + z_i \psi - \frac{1}{2\kappa} z_i^2 - p z_i \]  

\[ (28) \]

A.1 Spot market bidding

To solve the spot market auction, I adopt the standard solution concept of equilibrium in ex-post optimal bid curves. A bid curve is ex-post optimal if it is optimal for any realization of other agents’ bid curves which occurs in equilibrium.

From the perspective of agent \( i \), the spot auction defines a residual supply curve, \( z_{RSi}(p) \), specifying the number of units of the underlying asset that \( i \) is able to trade at price \( p \). This is the negative of the sum of all other agents’ bid curves:

\[ z_{RSi}(p) = - \sum_{j \neq i} z_{ Bj}(p; c_j, \psi) \]  

\[ (29) \]

In equilibrium, given my assumptions on agents’ utility functions, residual supply functions will be affine with a fixed slope:

\[ z_{RSi}(p) = \frac{d}{\psi} (p - \psi) + \eta_i \]  

\[ (30) \]

Where the random intercept \( \eta_i \) depends on uncertainty in other traders’ bid curves, resulting from uncertainty in their contract positions \( c_i \). I solve the spot auction model using the standard Kyle (1989) trick: I assume agents can choose the quantity they want to purchase for every possible realization of \( \eta_i \), then show that these choices can be
implemented by an affine demand schedule.

**Claim 1.** In the spot market, given \(d\), agents’ optimal bid curves are:

\[
z_{Bi} (p; c_i, \psi) = \frac{\kappa}{d + \kappa} c_i - \frac{\kappa d}{d + \kappa} (p - \psi)
\]

**Proof.** Spot trader \(i\)’s wealth is given by (28). Trader \(i\) chooses her bid curves after \(x_i\) and \(\psi\) are realized, so I analyze agents’ choices conditional on \(x_i\) and \(\psi\). Assume the agent faces a residual supply curve as described in (30), and rearrange to get the inverse residual supply function:

\[
p_{RS} (z_i; \eta_i, \psi) = \psi + \frac{z_i - \eta_i}{d}
\]

Suppose \(i\) can condition her purchase decision on \(\eta_i\). We can write (28) as:

\[
C = \psi x_i - \psi \mu x + p_{RS} (z_i; \eta_i, \psi) c_i - \mu x c_i + z_i \psi - \frac{1}{2 \kappa} z_i^2 - z_i p_{RS} (z_i; \eta_i, \psi)
\]

All components of (33) are known to \(i\), so \(i\) simply chooses her purchase quantity \(z_i\) to maximize (33). Differentiate with respect to \(z_i\):

\[
p'_{RS} (z_i; \eta_i, \psi) c_i + \psi - z_i p'_{RS} (z_i; \eta_i, \psi) - p (z_i; \eta_i, \psi) - \frac{z_i}{\kappa} = 0
\]

From (32), we have \(p'_{RS} (z_i; \eta_i) = \frac{1}{d}\), hence (34) becomes:

\[
\frac{c_i}{d} + \psi - \frac{z_i}{d} - p_{RS} (z_i; \eta_i, \psi) - \frac{z_i}{\kappa} = 0
\]

Expression (35) implicitly defines the optimal choice of \(z_i\) given \(\eta_i\). Expression (35) defines an affine bid curve; solving for \(z_i\), we attain expression (31). Since (31) passes through exactly all pairs \((z_i, p)\) which are \(i\)’s optimal choices for some realization of \(\eta_i\), \(i\) can do no better than submitting bid curve (31).

Now, note that from (29), the slope of residual supply \(d\) facing any given agent is equal to the sum of all \(n - 1\) other agents’ bid slopes. Thus, in equilibrium, we must have:

\[
d = (n - 1) \frac{\kappa d}{d + \kappa}
\]
Solving for $d$, we get:

$$d = (n - 2) \kappa$$  \hspace{1cm} (36)

Plugging this into (31), we get (7) of proposition 1. This implies that agents’ optimal bids are:

$$z_{Bi} (p; c_i, \psi) = \frac{1}{n-1} c_i - \frac{n-2}{n-1} \kappa (p - \psi)$$

proving (7).

To get prices, sum bids and add to 0:

$$\sum_{i=1}^{n} \frac{1}{n-1} c_i - \frac{n-2}{n-1} \kappa (p - \psi) = 0$$

Solving for $p - \psi$, and using that agents’ contract positions are normally distributed with variance $\sigma_c^2$, we get (8).

A.2 Spot trader welfare conditional on $x_i$

Now, I analyze how much utility $i$ achieves in expectation, if she has factor exposure $x_i$. Suppose an agent with factor exposure $x_i$ is bidding against a residual supply curve of the form (30).

Claim 2. Agent $i$’s expected utility given $\alpha, \sigma_\psi^2, \kappa, x_i, c_i, \sigma_n^2, d$ is:

$$\sqrt{\frac{d^2 + 2kd}{\alpha \kappa \sigma_n^2 + d^2 + 2kd}} \left( - \exp \left( -\frac{\alpha}{2} \left( -\alpha \sigma_\psi^2 (c_i + x_i)^2 - \frac{\alpha \sigma_n^2 - \kappa}{\alpha \kappa \sigma_n^2 + d^2 + 2kd c_i^2} \right) \right) \right)$$  \hspace{1cm} (37)

Proof. To calculate expected utility over uncertainty in $\eta_i$ and $\psi$, we first write expected utility from the auction as a function of $\eta_i$, fixing $c_i$. Rearranging residual supply from (30), we have:

$$p = \psi + \frac{z_i + \eta_i}{d}$$  \hspace{1cm} (38)

Wealth is:

$$W = \psi x_i - \mu_p x_i - \mu_\psi c_i + pc_i + z_i \psi - \frac{1}{2\kappa} z_i^2 - z_i p$$

Plugging in (38) for prices and rearranging, we have:
\[ W = \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{z_i (\eta_i; c_i) c_i}{d} - \frac{(z_i (\eta_i; c_i))^2}{2\kappa} - \frac{(z_i (\eta_i; c_i))^2}{d} - \frac{\eta_i z_i (\eta_i; c_i)}{d} \]  \tag{39}

Now, to find an expression for \( z_i (\eta_i; c_i) \), we eliminate prices from expression (31) for optimal bid curves and expression (30) for residual supply, to get \( z_i \) as a function of \( \eta_i \):

\[ z_i (\eta_i; c_i) = \frac{\kappa}{d + 2\kappa} (c_i - \eta_i) \]  \tag{40}

Plugging (40) into expression (39) for wealth, and simplifying, we have that wealth is:

\[ \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{(c_i - \eta_i)^2 \kappa}{2d^2 + 4\kappa d} \]

Given our assumption of CARA utility, agents' utility is:

\[ -\exp \left( -\alpha \left( \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{(c_i - \eta_i)^2 \kappa}{2d^2 + 4\kappa d} \right) \right) \]  \tag{41}

We first integrate (41) over uncertainty in \( \eta_i \), assuming that \( \eta_i \) is normally distributed with mean 0 and variance \( \sigma_{\eta_i}^2 \), to get:

\[ -\sqrt{\frac{d^2 + 2\kappa d}{\alpha \kappa \sigma_{\eta_i}^2 + d^2 + 2\kappa d}} \exp \left[ \frac{-\alpha}{\kappa} \left( \psi x_i - \mu \psi x_i + \psi c_i - \mu \psi c_i \right) + \frac{\alpha}{2} \left( \frac{\alpha \sigma_{\eta_i}^2 - \kappa}{\alpha \kappa \sigma_{\eta_i}^2 + d^2 + 2\kappa d} \right) c_i^2 \right] \]  \tag{42}

This gives expected utility over uncertainty in \( \eta_i \), conditional on \( \psi \). Now, we integrate (42) over uncertainty in \( \psi \), which is normally distributed with mean \( \mu \psi \) and variance \( \sigma_{\psi}^2 \), to get:

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\[
\sqrt{\frac{d^2 + 2\kappa d}{\alpha \kappa \sigma^2_n + d^2 + 2\kappa d}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\alpha \sigma^2_{\psi} (c_i + x_i)^2 - \frac{\alpha \sigma^2_n - \kappa}{\alpha \kappa \sigma^2_n + d^2 + 2\kappa d} c_i^2 \right) \right) \right)
\]

as desired. \( \square \)

### A.3 Optimal hedging

Using claim 2, we can find spot traders’ optimal choice of \( c_i \).

**Claim 3.** If:

\[
1 + \frac{\alpha \sigma^2_n - \kappa}{\alpha \sigma^2_{\psi} \left( \alpha \kappa \sigma^2_n + d^2 + 2\kappa d \right)} > 0 \tag{43}
\]

then spot traders’ objective function is strictly concave in \( c_i \), and there is a unique optimal choice of \( c_i \), which satisfies:

\[
c_i \left( 1 + \frac{\alpha \sigma^2_n - \kappa}{\alpha \sigma^2_{\psi} \left( \alpha \kappa \sigma^2_n + d^2 + 2\kappa d \right)} \right) = -x_i \tag{44}
\]

**Proof.** Take conditional expected utility from (37). Since only the exponent depends on \( c \), and the function \( -\exp \left( -\frac{\alpha}{2} x \right) \) is increasing in \( x \), we choose \( c_i \) to maximize:

\[
-\alpha \sigma^2_{\psi} (c_i + x_i)^2 - \frac{\alpha \sigma^2_n - \kappa}{\alpha \kappa \sigma^2_n + d^2 + 2\kappa d} c_i^2 \tag{45}
\]

Taking the second derivative, note that the problem is only concave if:

\[
1 + \frac{\alpha \sigma^2_n - \kappa}{\alpha \sigma^2_{\psi} \left( d^2 + 2\kappa d + \alpha \kappa \sigma^2_n \right)} > 0
\]

proving (43). Assuming (43) holds, differentiate (45) with respect to \( c_i \) and rearrange to get (44). Claim 3, plugging in \( d = (n - 2) \kappa \) from (36), gives expressions (9) and (10) of proposition 1. \( \square \)
A.4 Equilibrium \( \sigma^2_{\eta} \)

From (9) of proposition 1, traders’ optimal contract purchases are linear in their factor exposures \( x_i \), and traders’ factor exposures \( x_i \) are mean-0 normal, so traders’ equilibrium contract positions are also mean-0 normally distributed. Thus, the residual supply intercept term, \( \eta_i \), is also mean-0 normally distributed. We can then solve the model by requiring agents’ optimal behavior given \( \sigma^2_{\eta} \), the variance of the residual supply intercept \( \eta_i \), to generate residual supply curves with variance \( \sigma^2_{\eta} \).

**Claim 4.** For any \( \alpha, \sigma^2_{\psi}, \kappa, \sigma^2_{\chi}, n \) there is a unique equilibrium value of \( \sigma^2_{\eta} \), satisfying:

\[
\sigma^2_{\eta} = \frac{\sigma^2_{\chi}}{n-1} \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{(\alpha \sigma^2_{\psi} \kappa) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_{\eta} \right)} \right)^{-2} \tag{46}
\]

\[
\sigma^2_{\eta} > \frac{\kappa - (\alpha \sigma^2_{\psi}) \left( n^2 - 2n \right) \kappa^2}{\alpha \left( 1 + \alpha \sigma^2_{\psi} \kappa \right)} \tag{47}
\]

**Proof.** Since (44) implies that contract positions \( c_i \) are linear in exposures \( x_i \), agents’ contract positions are also normally distributed, with mean 0 and variance:

\[
\sigma^2_c = \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{(\alpha \sigma^2_{\psi}) \left( d^2 + 2d\kappa + \alpha\kappa \sigma^2_{\eta} \right)} \right)^{-2} \sigma^2_{\chi} \tag{48}
\]

Now, note that residual supply facing \( i \), summing over other agents’ bids using (7), is:

\[
-\sum_{j \neq i} z_{Bi}(p; c_i) = -\sum_{j \neq i} \left( \frac{1}{n-1} c_i - \frac{n-2}{n-1} \kappa (p - \psi) \right)
\]

hence, using representation (30) of residual supply, we have:

\[
\eta_i = -\sum_{j \neq i} \frac{1}{n-1} c_i
\]

and all factor exposures \( x_i \) are independent. Hence, the variance of the residual supply
intercept, $\eta_i$, is:

$$\sigma^2_{\eta_i} = \frac{\sigma^2_c}{n-1} \quad (49)$$

Combining this with (48), and plugging in $d = (n - 2) \kappa$ from (36), we obtain (46). Note also that, using the definition of $t$ in (9), we can write (49) as:

$$\sigma^2_{\eta} = \frac{t^2\sigma^2_x}{n-1} \quad (50)$$

By claim 3, in order for (11) to solve agents’ optimal contract purchasing problem, the concavity condition (43) must also hold; plugging in $d = (n - 2) \kappa$ to (43), we get:

$$1 + \frac{\alpha\sigma^2_{\eta} - \kappa}{(\alpha\sigma^2_{\psi}\kappa) ((n^2 - 2n) \kappa + \alpha\sigma^2_{\eta})} > 0$$

Solving for $\sigma^2_{\eta}$, we get (47).

To show that there is a unique value of $\sigma^2_{\eta}$ satisfying (46) and (47), rearrange (46) to:

$$(n-1) \sigma^2_{\eta} = -\left(1 + \frac{\alpha\sigma^2_{\eta} - \kappa}{(\alpha\sigma^2_{\psi}\kappa) ((n^2 - 2n) \kappa + \alpha\sigma^2_{\eta})}\right)^{-2} \sigma^2_x \quad (51)$$

The LHS of (51) is increasing in $\sigma^2_{\eta}$ from 0 to $\infty$. As $\sigma^2_{\eta}$ varies from the lower bound (47) to 0, the RHS decreases from $\infty$ towards the finite quantity:

$$-\left(1 + \frac{1}{\alpha\kappa\sigma^2_{\psi}}\right)^{-2} \sigma^2_x$$

Hence, there is exactly one positive value of $\sigma^2_{\eta}$ greater than the lower bound (47) which equates the LHS and RHS of (51), proving that there is a unique equilibrium value of $\sigma^2_{\eta}$.

This proves claim 4, and thus (11) and (12) of proposition 1.
A.5 Expected welfare over uncertainty in factor exposures

Plugging in $c_i = tx_i$, $d = (n - 2)\kappa$ to (37) of claim 2, we get expected utility conditional on $x_i$, for any linear contract purchasing rule:

$$
\sqrt{\frac{(n^2 - 2n)\kappa}{\alpha\sigma^2 + (n^2 - 2n)\kappa}} \left( -\exp\left( -\frac{\alpha}{2} \left( -\alpha\sigma^2 (1 - t)^2 - \frac{\alpha\sigma^2 - \kappa}{(\alpha\sigma^2, \kappa)} \left( \frac{(n^2 - 2n)\kappa + \alpha\sigma^2}{} \right) t^2 \right) \right) \right) x_i^2
$$

Integrating this against uncertainty in $x_i$, with mean 0 and variance $\sigma_x^2$, we get:

$$
- \sqrt{\frac{(n^2 - 2n)\kappa}{\alpha\sigma^2 + (n^2 - 2n)\kappa}} \sqrt{1 - \alpha\sigma^2 (1 - t)^2 + \left( \frac{\alpha\sigma^2 - \kappa}{(\alpha\sigma^2, \kappa)} \left( \frac{(n^2 - 2n)\kappa + \alpha\sigma^2}{} \right) t^2 \right)}
$$

(52)

Substituting for $\sigma_{\eta}^2$ using (50), we get (13).

B Supplementary material for section 4

B.1 Comparative statics

Figure 4 illustrates the effects of varying input parameters on equilibrium outcomes. When spot traders’ risk aversion $\alpha$ is low, traders are more willing to bear factor risk in order to attain manipulation profits, so the equilibrium and spot-trader-optimal values of $t$ increase, causing price variance to increase. When the spot good holding capacity $\kappa$ decreases, price variance increases. The welfare-maximizing value of $t$ for spot traders tends to be lower than the equilibrium $t$ when $\kappa$ is low, because the negative basis risk externalities from manipulation are larger.

Decreasing the variance of the risk factor, $\sigma_{\psi}^2$, makes spot traders more willing to buy large contract positions and manipulate, which increases $t$ in equilibrium and increases basis risk. Increasing agents’ factor exposure variance, $\sigma_x^2$, increases price variance, but actually decreases the equilibrium and socially optimal values of $t$, as it becomes more...
Figure 4: Comparative statics

Notes. Comparative statics of the equilibrium and spot trader welfare-maximizing values of \( t \), spot traders’ equilibrium welfare gain (minus the competitive equilibrium value of -1), and equilibrium price variance, \( \text{Var} (p - \psi) \), as input parameters vary. The baseline values that parameters are varied around are \( n = 3, \alpha = 1, \kappa = 0.8, \sigma_{\psi}^2 = 1.5, \sigma_{x}^2 = 0.05 \).

costly for agents to deviate from full hedging. Larger factor exposures also imply that spot traders’ welfare losses are larger, and because negative basis risk externalities are larger, the spot-trader welfare maximizing \( t \) tends to fall below the equilibrium \( t \) when \( \sigma_{x}^2 \) is large. Finally, increasing \( n \) causes all parameters to converge rapidly to their competitive values: the equilibrium \( t \) converges to 1, and price variance and net spot trader welfare losses from manipulation converge to 0.
B.2 Convergence rate of equilibrium $t$

From (11) and (10), we have:

$$
\sigma^2_{\eta} = \frac{\sigma^2_{\chi}}{n-1} \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \sigma^2_{\psi} \kappa \left( (n^2 - 2n) \kappa + \alpha \sigma^2_{\eta} \right)} \right)^{-2}
$$

and:

$$
t = \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \sigma^2_{\psi} \kappa \left( (n^2 - 2n) \kappa + \alpha \sigma^2_{\eta} \right)} \right)^{-2}
$$

The RHS of (54) is decreasing in $\sigma^2_{\eta}$. Hence an upper bound for the equilibrium $t$ comes from setting $\sigma^2_{\eta}$ to 0:

$$
t \leq \left( 1 - \frac{\kappa}{\alpha \sigma^2_{\psi} \kappa \left( (n^2 - 2n) \kappa \right)} \right)^{-1}
$$

Now, we can get an upper bound for $\sigma^2_{\eta}$ by plugging the upper bound on $t$, (55) into (53):

$$
\sigma^2_{\eta} \leq \frac{\sigma^2_{\chi}}{n-1} \left( 1 - \frac{\kappa}{\alpha \sigma^2_{\psi} \kappa \left( (n^2 - 2n) \kappa \right)} \right)^{-2}
$$

The RHS is decreasing in $n$, hence, we can set $n = 3$ in (56) to get:

$$
\sigma^2_{\eta} \leq M \equiv \frac{\sigma^2_{\chi}}{2} \left( 1 - \frac{\kappa}{\alpha \sigma^2_{\psi} \kappa \left( 3\kappa \right)} \right)^{-2}
$$

where $M$ does not depend on $n$. Now, a lower bound for $t$ comes from plugging the upper bound $M$ into (54).

$$
t \geq \left( 1 - \frac{\alpha M - \kappa}{\alpha \sigma^2_{\psi} \kappa \left( (n^2 - 2n) \kappa + \alpha M \right)} \right)^{-1}
$$
Together, (55) and (57) bound the equilibrium value of $t$. Now, the function $1 - \frac{\kappa}{\alpha \sigma^2_{\psi}} \left( n^2 - 2 \kappa \right)$ is differentiable at $x = 1$ with derivative equal to 1. Hence, (55) converges to 1 at the same rate that $1 - \frac{\kappa}{\alpha \sigma^2_{\psi}} \left( n^2 - 2 \kappa \right)$ converges to 1, which is $\frac{1}{n^2}$; (57) is analogous. Thus, the equilibrium value of $t - 1$ converges to 0 at rate $\frac{1}{n^2}$.

**B.3 Heterogeneous beliefs**

In this appendix, I show that contract purchases can also be generated by dispersion in agents’ beliefs about $\psi$. Suppose an agent believes the mean of $\psi$ is $\beta_{\psi i}$, and its variance is $\sigma^2_{\psi i}$. The agent can purchase contracts at a fixed price $\mu_{\psi i}$, and has no factor exposure $x_i$. From (42), the agent’s utility as a function of $\psi$, over uncertainty in the spot market, is:

$$\sqrt{\frac{d^2 + 2 \kappa d}{\alpha \kappa \sigma^2_{\eta} + d^2 + 2 \kappa d}} \left( -\exp \left( -\alpha \left( \psi c_i - \mu_{\psi} c_i \right) - \frac{\alpha}{2} \left( \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \kappa \sigma^2_{\eta} + d^2 + 2 \kappa d} \right) c_i^2 \right) \right)$$

To calculate the agent’s expected utility (under her beliefs), we integrate assuming $\psi$ has mean $\beta_{\psi i}$ and variance $\sigma^2_{\psi i}$. This gives:

$$= \sqrt{\frac{d^2 + 2 \kappa d}{\alpha \kappa \sigma^2_{\eta} + d^2 + 2 \kappa d}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\alpha \sigma^2_{\psi i} c_i^2 + 2 \beta_{\psi i} c_i - 2 \mu_{\psi} c_i - \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \kappa \sigma^2_{\eta} + d^2 + 2 \kappa d} c_i^2 \right) \right) \right)$$

Only the exponent depends on $c_i$, so we maximize:

$$-\alpha \sigma^2_{\psi i} c_i^2 + 2 \left( \beta_{\psi i} - \mu_{\psi} \right) c_i - \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \kappa \sigma^2_{\eta} + d^2 + 2 \kappa d} c_i^2$$

Differentiating and solving for $c_i$, we have:

$$c_i \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \sigma^2_{\psi} \left( \alpha \sigma^2_{\psi} \left( d^2 + 2 \kappa \alpha \kappa \sigma^2_{\eta} \right) \right) \right) = \frac{\beta_{\psi i} - \mu_{\psi}}{\alpha \sigma^2_{\psi}} \quad (58)$$
Comparing (58) to (9) and (10), a belief shock $\beta_{\psi i}$ is isomorphic to a factor exposure $x_i$ of size:

$$x_i = -\frac{\beta_{\psi i} - \mu_{\psi}}{\alpha \sigma_{\psi}^2}$$

in the sense that it generates the same contract purchasing decisions and spot market behavior. Hence, contract purchases can be motivated either by disagreement or risk-sharing. If we do not observe factor exposures directly, based only on agents’ contract purchases and spot market behavior, we cannot separately identify factor exposures from heterogeneous beliefs, complicating welfare analysis for spot traders.

However, spot traders’ optimal behavior in spot markets only depends on the size of their contract positions and the structure of spot markets, not the motivations of spot traders for purchasing contracts. Hence, the results of section 5, analyzing the effects of manipulation on hedgers’ welfare, hold even if spot traders’ contract positions are driven by differences in beliefs.

### B.4 Price impact in the contract market, quadratic taxes/subsidies

In this appendix, I show that, if spot traders’ contract purchases have price impact, we can still solve for linear optimal contract purchasing rules. Moreover, price impact is isomorphic to subsidies or taxes to spot traders which are quadratic in the size of traders’ contract positions. Thus, a regulator can implement any desired choice of hedging aggressiveness $t$ in equilibrium using some quadratic subsidy or tax scheme.

Suppose that spot traders’ contract purchases move the spot price linearly: if a spot trader purchases $c_i$ contracts, the price per contract is:

$$\mu_{\psi} + kc_i$$

The total cost of buying $c_i$ contracts is then:

$$\mu_{\psi}c_i + kc_i^2 \quad (59)$$

Combining this cost with (3) and integrating, a spot trader’s conditional expected utility
if she has factor exposure $x_i$ and contract position $c_i$ is:

$$-\frac{1}{\sqrt{2\pi\sigma^2_\psi}} \int \sqrt{\frac{d^2 + 2kd}{\alpha\kappa\sigma^2_\eta + d^2 + 2kd}} \exp \left[ -\alpha \left( \psi x_i - \mu_\psi x_i + \psi c_i - \mu_\psi c_i - kc_i^2 \right) - \frac{\alpha}{2} \left( \frac{\alpha^2_\eta - \kappa}{\alpha\kappa\sigma^2_\eta + d^2 + 2kd} \right) c_i^2 \right] \exp \left( -\frac{(\psi - \mu_\psi)^2}{2\sigma^2_\psi} \right) \, d\psi$$

$$= \sqrt{\frac{d^2 + 2kd}{\alpha\kappa\sigma^2_\eta + d^2 + 2kd}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\alpha\sigma^2_\psi(c_i + x_i)^2 - 2kc_i^2 - \frac{\alpha^2_\eta - \kappa}{\alpha\kappa\sigma^2_\eta + d^2 + 2kd} c_i^2 \right) \right) \right)$$

Hence, agents choose $c_i$ to maximize:

$$-\alpha\sigma^2_\psi(c_i + x_i)^2 - 2kc_i^2 - \frac{\alpha^2_\eta - \kappa}{\alpha\kappa\sigma^2_\eta + d^2 + 2kd} c_i^2$$

The optimal choice of $c_i$ thus satisfies:

$$c_i = -\left( 1 + \frac{\alpha\sigma^2_\eta - \kappa}{\alpha\sigma^2_\psi} \left( d^2 + 2d\kappa + \alpha\kappa\sigma^2_\eta \right) \right)^{-1} x_i \quad (60)$$

Substituting for $d$ using the spot market equilibrium bid curves, we have:

$$c_i = -\left( 1 + \frac{\alpha\sigma^2_\eta - \kappa}{\alpha\sigma^2_\psi} \left( (n^2 - 2n)\kappa + \alpha\sigma^2_\eta \right) \right)^{-1} x_i \quad (61)$$

Expression (61) shows that spot traders' optimal contract positions are still linear in their factor exposures $x_i$, with coefficient:

$$t \equiv \left( 1 + \frac{\alpha\sigma^2_\eta - \kappa}{\alpha\sigma^2_\psi} \left( (n^2 - 2n)\kappa + \alpha\sigma^2_\eta \right) \right)^{-1} \quad (62)$$

Comparing (62) to (10) of proposition 1, price impact in the contract market decreases $t$, causing spot traders to hedge less per unit of their factor exposures. Given this $t$, the
equilibrium \( \sigma^2_n \) is the unique value that satisfies:

\[
\sigma^2_n = \frac{\sigma^2_x}{n-1} \left( 1 + \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_{\psi} \kappa) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_n \right)} + \frac{4k}{\alpha \sigma^2_{\psi}} \right)^{-2}
\]

and outcomes in the spot market are described by (7) and (8). Qualitatively, price impact in contract markets simply causes spot traders to buy less contracts per unit of their factor exposures.

Expressions (60) and (62) also imply that regulators can implement any desired choice of \( t \) as a unique equilibrium, by imposing quadratic taxes or subsidies on agents’ contract positions. To see this, suppose that a regulator can charge all spot traders some net amount \( \tau c^2_i \) for buying \( c_i \) contracts, where \( \tau \) can be positive or negative. A spot trader’s total cost for buying \( c_i \) contracts is thus:

\[
\mu \psi c_i + kc_i^2 + \tau c_i^2
\]

Analogous to (62), spot traders’ optimal hedging decisions are linear, satisfying:

\[
t \equiv \left( 1 + \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_{\psi}) \left( d^2 + 2d\kappa + \alpha \kappa \sigma^2_n \right)} + \frac{4k}{\alpha \sigma^2_{\psi}} + \frac{4\tau}{\alpha \sigma^2_{\psi}} \right)^{-1}
\]

(63)

Now, suppose the regulator wishes to implement some positive level of hedging aggressiveness, \( t^* > 0 \), in equilibrium. From (50) of appendix A.4, we have:

\[
\sigma^2_n = \frac{(t^*)^2 \sigma^2_x}{n-1}
\]

Hence, in order to implement \( t^* \), we must choose \( \tau \) such that:

\[
t^* = \left( 1 + \frac{\alpha \left( \frac{(t^*)^2 \sigma^2_x}{n-1} \right) - \kappa}{(\alpha \sigma^2_{\psi}) \left( d^2 + 2d\kappa + \alpha \kappa \left( \frac{(t^*)^2 \sigma^2_x}{n-1} \right) \right)} + \frac{4k}{\alpha \sigma^2_{\psi}} + \frac{4\tau}{\alpha \sigma^2_{\psi}} \right)^{-1}
\]
By changing $\tau$, the RHS can be varied from 0 to $\infty$, so for any $t^*$, and any values of other primitives, there is a unique value of $\tau$ which implements $t^*$ as an equilibrium outcome.

Throughout this appendix, we have taken price impact as exogeneous. In a more realistic model, price impact would result endogeneously from the contract market, in which spot traders and pure hedgers bid for contracts in a double auction. This is analytically difficult to solve, because spot traders’ order flow would be informative about their spot market traders, and thus contract settlement prices.

The $k\psi_i^2$ term in (59) could also be used to model holding costs for contracts, which may arise from margin capital requirements or related factors. The results of this appendix then imply that quadratic holding costs, like price impact, decrease spot traders’ hedging aggressiveness in equilibrium.

Once again, spot traders’ optimal behavior in spot markets depends only on the size of their contract positions and spot market structure. Thus, price impact in contract markets would not affect the results of section 5.

C Supplementary material for section 5

C.1 Proof of proposition 2

From (4), hedgers’ wealth is:

$$W_{hedger}(x_i, c_i, p, \psi) = \psi x_i - \mu_\psi x_i - \mu_\psi c_i + pc_i$$

Since the expectation of $p$, over uncertainty in $\psi$ and all $x_i$’s, is $\mu_\psi$, the expectation of hedgers’ wealth is always equal to 0. To find the variance, adding and subtracting $\psi c_i$, we can write this as:

$$= \psi x_i - \mu_\psi x_i - \mu_\psi c_i + \psi c_i + (p - \psi) c_i$$
Since the mean of hedgers’ wealth is independent of \(c_i\), hedgers simply purchase contracts to minimize the variance of wealth, which is:

\[
= \sigma^2_{\psi} (x_i + c_i)^2 + c_i^2 \text{Var} (p - \psi)^2
\]  

(64)

Minimizing with respect to \(c_i\), we have (14). Plugging (14) in to (64), and taking expectations with respect to \(x_i\), we get (15).

To calculate the total expected wealth transfer from hedgers to spot traders, since we have assumed that \(\psi\) is independent of factor exposures \(x_i\), which are linearly related to contract positions \(c_i\), we have:

\[
E \left[ \psi \sum_{i=1}^{n} c_i \right] = E \left[ \psi \right] E \left[ \sum_{i=1}^{n} c_i \right] = E \left[ \mu_{\psi} \sum_{i=1}^{n} c_i \right]
\]

Hence,

\[
E \left[ (p - \mu_{\psi}) \sum_{i=1}^{n} c_i \right] = E \left[ (p - \psi) \sum_{i=1}^{n} c_i \right]
\]

Now, plugging in for \((p - \psi)\) using (8) of proposition 1, this is:

\[
E \left( \left( \sum_{i=1}^{n} c_i \right) \left( \frac{\sum_{i=1}^{n} c_i}{n(n-2)} \right) \right)
\]

(65)

Since we have assumed agents’ factor exposures \(x_i\) are independent, agents’ contract positions \(c_i\) are also independent, so (65) becomes (16).

**C.2 Proof of proposition 3**

To prove proposition 3, claim 5 first characterizes agents’ best responses, given the slope of residual supply, and subsection C.2.2 proves proposition 3 using claim 5.
C.2.1 Best responses

Claim 5. If agent $i$ has inventory position $y_i$ and contract position $c_i$, and the slope of residual supply is $d_i$, then agent $i$’s unique ex-post optimal bid curve is:

$$z_{Bi}(p; y_i, c_i) = -\frac{d_i}{\kappa_i + d_i}y_i + \frac{\kappa_i}{\kappa_i + d_i}c_i - \frac{\kappa_i d_i}{\kappa_i + d_i} (p - \psi)$$  \hspace{1cm} (66)

Proof. Analogously to claim 1, assume that residual supply takes the form:

$$z_{RSi}(p, \eta_i) = d_i (p - \psi) + \eta_i$$

We will optimize pointwise in $\eta_i$. Define $p^*(\eta_i)$ as the optimal choice of $p$ for any given $\eta_i$, that is:

$$p^*(\eta_i) \equiv \arg \max_p W(z_{RSi}(p, \eta_i), p; y_i, c_i)$$

$$= \arg \max_p \psi z_{RSi}(p, \eta_i) - \frac{y_i^2}{2\kappa_i} - \frac{y_i z_{RSi}(p, \eta_i)}{\kappa_i} - \frac{z_{RSi}(p, \eta_i)^2}{2\kappa_i} + c_i p - z_{RSi}(p, \eta_i) p$$

Since $z_{RSi}(p, \eta_i)$ is affine and increasing in $p$, the objective function concave in $p$, thus the first-order condition is necessary and sufficient for $p^*(\eta_i)$ to be optimal. Differentiating with respect to $p$ and setting to 0, and using that $z'_{RSi}(p, \eta_i) = d_i$, we have:

$$-\frac{d_i}{\kappa_i}y_i - \frac{z_{RS}(p^*(\eta_i), \eta_i)}{\kappa_i} d_i + c_i - z_{RS}(p^*(\eta_i), \eta_i) - (p^*(\eta_i) - \pi) d_i = 0$$  \hspace{1cm} (67)

Hence, any pair $(p^*(\eta_i), z_{RS}(p^*(\eta_i), \eta_i))$ – that is, any point $(p, z)$ which is the agent’s optimal choice for some $\eta_i$ – satisfies (67). Hence, the unique bid curve which passes through the set of all ex-post optimal points is the curve implicitly defined by (67). Solving (67) for $z_{RS}(p^*(\eta_i), \eta_i)$, we have (66). \hfill \Box

C.2.2 Equilibrium

This proof is based on Appendix A.4 of Du and Zhu (2012), with notational modifications to suit the context of this paper. We seek a vector of demand and residual supply slopes
b_i which satisfy, for all i:

\[ d_i = \sum_{j \neq i} b_i = B - b_i \]  

(68)

\[ b_i = \frac{d_i \kappa_i}{\kappa_i + d_i} \]  

(69)

Rearranging, we have:

\[ d_i = \frac{b_i \kappa_i}{\kappa_i - b_i} \]  

(70)

Combining (68) and (70), we have:

\[ \sum_j b_j - b_i = \frac{b_i \kappa_i}{\kappa_i - b_i} \]

Defining \( B \equiv \sum_j b_j \), we have

\[ (\kappa_i - b_i) (B - b_i) = b_i \kappa_i \]

This has two solutions. In order for \( B > b_i \), we must pick:

\[ b_i = \frac{2\kappa_i + B - \sqrt{B^2 + 4\kappa_i^2}}{2} \]  

(71)

This is (21) of proposition 3. B must satisfy:

\[ B = \sum_j b_j = \sum_{i=1}^n \frac{2\kappa_i + B - \sqrt{B^2 + 4\kappa_i^2}}{2} \]  

(72)

This is (22) in the main text. By multiplying the top and bottom of the RHS by \( 2\kappa_i + B + \sqrt{B^2 + 4\kappa_i^2} \) and simplifying, this becomes:

\[ B = \sum_{i=1}^n \frac{2\kappa_i B}{2\kappa_i + B + \sqrt{B^2 + 4\kappa_i^2}} \]
Or,
\[ B \left( -1 + \sum_{i=1}^{n} \frac{2\kappa_i}{2\kappa_i + B + \sqrt{B^2 + 4\kappa_i^2}} \right) = 0 \]  \hspace{1cm} (73)

Now, define
\[ f(B) = -1 + \sum_{i=1}^{n} \frac{2\kappa_i}{2\kappa_i + B + \sqrt{B^2 + 4\kappa_i^2}} \]

In order for \( B \) to solve (73) when \( B > 0 \), we need \( f(B) = 0 \). Now, \( f(0) > 0 \), \( f(B) \to -1 \) as \( B \to \infty \), and \( f'(B) < 0 \) for \( B > 0 \). Hence, \( f(B) = 0 \) at some unique \( B \), hence there is a unique value of \( B \) which solves (73), and thus there is a unique linear equilibrium for any demand slopes \( \kappa_1 \ldots \kappa_n \).

Substituting \( b_i = \frac{\kappa_i d_i}{\kappa_i + d_i} \) into agents’ best-response bid curves from claim 5, we have agents’ equilibrium bids in expression (19). To find prices, sum agents’ demand curves and equate to 0:
\[ \sum_{i=1}^{n} \left[ -y_i b_i \frac{b_i}{\kappa_i} + c_i \frac{b_i}{\sum_{j \neq i} b_j} - (p - \psi) b_i \right] = 0 \]

Solving for \( p \), we have (20).

C.3 Proof of corollary 1

Conjecture that:
\[ b_i = \frac{n - 2}{n - 1} \kappa \]

Plugging this into (19) and (20), we get (23) and (24).

C.4 Proof of proposition 4

For basis risk, we simply take the variance of expression (20). Prices can be written as:
\[ p - \psi = \frac{1}{B} (k'_y y + k'_e c) \]
Since we have assumed that contract positions and inventory shocks have mean 0, price variance can be written as:

\[
E \left[ (p - \psi)^2 \right] = E \left[ \frac{1}{B^2} \left( k'_y y + k'_c c \right) \left( k'_y y + k'_c c \right)' \right] = E \left[ \frac{1}{B^2} \left( k'_y y y' y + 2k'_y y c' k_c + k'_c c c' k_c \right) \right]
\]

(74)

Now, since we assumed each element of \( y \) and \( c \) has mean 0, we have:

\[
E [yy'] = \Sigma_{yy}, \ E [yc'] = \Sigma_{yc}, \ E [cc'] = \Sigma_{cc}
\]

(75)

Substituting these into (74), we get (26). For (27), we have:

\[
E \left[ (p - \psi) \left( \sum_{i=1}^{n} c \right) \right] = E \left[ \left( \frac{1}{B} \left( k'_y y + k'_c c \right) \right) (c'1) \right] = E \left[ \frac{1}{B} \left( k'_y y c'1 + k'_c c c'1 \right) \right]
\]

Again, substituting using (75), we get (27).

C.5 Proof of proposition 2

Expression (23) satisfies proposition 3, with \( b_i = \frac{n-2}{n-1} \). To calculate price variance, set the sum of all agents’ bids to 0 and solve for price, to get:

\[
p - \psi = \frac{1}{n\kappa} \sum_{i=1}^{n} \left[ -y_i + \frac{1}{n-2} c_i \right]
\]

Taking the variance, and using that we get (24).

D Supplementary material for section 6

D.1 A dynamic price determination game

In this appendix, I model the LBMA gold auction as a continuous-time dynamic price determination game: agents announce their demand quantities in response to prices, and the game concludes when markets clear. I prove that, while we cannot rule out the
existence of other equilibria, the dynamic game always admits an equilibrium equivalent to the unique static auction equilibrium. The intuition is very simple: if all agents besides i are behaving according to their equilibrium strategies in a static bid submission game, then i can do no better than to bid her static equilibrium strategy.

As in the baseline model of section 3, i ∈ {1...n} agents have types κ1...κn, which are common knowledge, and inventory positions y1...yn and contract positions c1...cn, which are private information. As before, the utility of agent i for purchasing z units of the asset when the price is p is:

\[ \psi_z = \left( z + y_i \right)^2 / 2\kappa - pz + pc_i \]

Agents play a continuous-time auction, which I model as a simple differential game. At time t = 0, the auction begins at some deterministic price p(0), which is known to all participants. Participants simultaneously announce initial demands z_{Bi}(p(0)). Thereafter, at any given time t, the price evolves according to the differential equation:

\[ \frac{dp}{dt} = \begin{cases} k & z_{Bi}(p(t)) > 0 \\ -k & z_{Bi}(p(t)) < 0 \end{cases} \]

This stylized price-setting process matches the stylized fact shown in the left panel of figure 2 that prices decrease when aggregate demand is below 0, and increase when aggregate demand is above 0. Agents can update their demand functions z_{Bi}(p(t)) as the price p(t) changes; agents’ demand functions are required to be decreasing in price. The game ends at the first time T when aggregate demand is exactly equal to 0,

\[ \sum_{i=1}^{n} z_{Bi}(p(T)) = 0 \]

at which point each agent purchases z_{Bi}(p(T)) units of the good for p(T) per unit, and is paid p(T) per unit contract that she holds.

---

\[ ^{56} \text{IBA's published auctions specification documents do not describe how the auction starting price is chosen. However, the choice of starting price does not significantly affect agents' optimal strategies, so I assume the price is non-random and commonly known for simplicity.} \]
I assume that agents choose the rate at which demand changes with price, \( z'_B \left( p\left( t \right) \right) = \frac{dz_B}{dp}, \) rather than the level of demand; this ensures that resultant demand functions are continuous. Since \( \frac{dp}{dt} \) is constant throughout any instance of the game, choosing \( \frac{dz_B}{dt} \) is equivalent to choosing \( \frac{dz_B}{dp} \). I require agents’ reported demand slopes to be finite and bounded away from 0:

\[
-M \leq z'_B \left( p\left( t \right) \right) \leq -\epsilon < 0 \tag{76}
\]

This guarantees that the game will end in finite time. I will show that the dynamic differential auction game admits an equilibrium which coincides with the equilibrium of the static auction in section 3, since agents’ bid curves in the static game have slopes which are negative, finite and bounded away from 0, for any \( \kappa_1 \ldots \kappa_n, \epsilon \) can always be chosen small enough in magnitude, and \( M \) large enough, that the bounds in (76) are not binding in equilibrium.

\[ \text{Claim 6. The dynamic auction game always ends in finite time.} \]

\[ \text{Proof. Suppose that } \sum_{i=1}^{n} z_{B_i} \left( p\left( 0 \right) \right) > 0. \text{ Then, we have:} \]

\[
\frac{dz_B}{dt} = \sum_{i=1}^{n} \frac{dz_{B_i}}{dt} = \sum_{i=1}^{n} \frac{dz_{B_i}}{dp} \frac{dp}{dt} = k \sum_{i=1}^{n} \frac{dz_{B_i}}{dp} \in \left[ -kM, -k\epsilon \right]
\]

Thus,

\[
z_{B_i} \left( p\left( t \right) \right) \in \left[ z_{B_i} \left( p\left( 0 \right) \right) - kMt, z_{B_i} \left( p\left( 0 \right) \right) - k\epsilon t \right]
\]

Hence, \( z_{B_i} \left( p\left( t \right) \right) = 0 \) for some \( t \in \left[ \frac{z_{B_i}(0)}{kM}, \frac{z_{B_i}(0)}{k\epsilon} \right] \), hence the game ends in finite time. The proof of the case where \( \sum_{i=1}^{n} z_{B_i} \left( p\left( 0 \right) \right) < 0 \) is analogous. \[ \Box \]

The setup of this game fails to capture two features of the ICE gold auction in practice. First, in practice the price does not evolve smoothly, but jumps in increments; second, the auction does not end when supply and demand are exactly equal, but allows for some volume gap. These features are difficult to model tractably; modelling the first would require taking a stance on the price updating rule, which to my knowledge is not publicly documented by ICE, and the second induces noise in prices which implies that outcomes in the dynamic game not to correspond one-to-one to outcomes in the static bid submission game. Thus, for analytical tractability, I adopt the simpler monotone
differential game model as an approximation to the ICE auction.

A history is a sequence of observed demand slopes \( z'_B (p(t)) \) of all agents on the interval \( t \in [0, T] \). It is equivalent to assume we observe agents’ demand functions; thus a history at time \( t \) can be described as:

\[
h_T = \{(z_{D1}(p(t)) \ldots z_{Dn}(p(t)), t \in [0, T]\}
\]

A strategy is a decision \( z_{Bi}(p(0)) \) about what demand to announce at the starting price \( p(0) \), and then a choice of \( z'_B (p(t)) \) for every possible history. Both decisions may also depend on the realization of agents’ inventory \( y_i \) and contract position \( c_i \). Thus, in full generality, strategies can be very complex. I will restrict attention to a class of naive linear strategies, which I define below.

**Definition 2.** A naive linear strategy for agent \( i \) is a strategy in which:

\[
z_{Bi}(p(0)) = f_i(y_i, c_i), \quad z'_B (p(t)) = b_i
\]

that is, agent \( i \)’s demand function has an intercept which may depend on \( y_i \) and \( c_i \), and constant slope.

Agents playing naive strategies cannot condition their slopes on the observed behavior of other agents, or their own contract positions and inventory positions – they must commit to a constant slope, independent of the history of the game. Note that (76) implies that we must have

\[
b_i \in [-M, -\epsilon]
\]

Naive linear strategies are isomorphic to affine bid curves. For any naive linear strategies described by \( f_i(y_i, c_i), b_i \), we can construct the equivalent bid curve as:

\[
z_{Bi}(p; f_i(y_i, c_i), b_i) = f_i(y_i, c_i) + \int_{p_0}^{p} b_i dp = f_i(y_i, c_i) + b_i(p - p_0)
\]

**Proposition 5.** Naive linear strategies \( f_i(y_i, c_i), b_i \) corresponding to equilibrium bidding strategies \( z_{Bi}(p; y_i, c_i) \) in the static auction game in proposition 3 also constitute an equilibrium in the dynamic auction game.
I prove proposition 5 in two steps. Claim 7 shows that, if all agents are restricted to naive linear strategies, the dynamic auction game produces exactly the same outcomes as the static auction game. Claim 8 shows that, if all agents other than \( i \) are playing naive linear strategies, agent \( i \) can do no better than to play a naive linear strategy. Thus, equilibria in naive linear strategies are equilibria in the broader game. This proves proposition 5 as equilibrium strategies in proposition 3 for the static auction game can be implemented as naive linear strategies in the dynamic auction game.

**Claim 7.** If all agents play naive linear strategies, the outcomes of the dynamic auction game are exactly the outcomes of a static auction game in which agents submit the bid functions specified by (77).

**Proof.** Suppose agents are playing the naive linear strategies \( f_i(y_i, c_i), b_i \). Construct the equivalent strictly decreasing bid curves \( z_{B_i}(p; f_i(y_i, c_i), b_i) \). Fix some realization of \( y_i, c_i \) across agents, and consider the aggregate demand function,

\[
\sum_{i=1}^{n} z_{B_i}(p; f_i(y_i, c_i), b_i)
\]

Claim 6 states that the dynamic auction game always ends in finite time; thus, the dynamic auction must end at some time \( T \), at some price \( p(T) \), at which

\[
\sum_{i} z_{B_i}(p(T); f_i(y_i, c_i), b_i) = 0
\]

Then each agent purchases

\[
z_{B_i}(p; f_i(y_i, c_i), g_i(y_i, c_i))
\]

units of the good at price \( p(T) \), and is paid \( p(T) \) per unit contract she holds. This is exactly the same outcome as agents receive in a static auction game in which agents submit bid curves \( z_{B_i}(p; f_i(y_i, c_i), b_i) \).

The strategies in the dynamic auction game with naive linear strategies are a subset of their strategies in the static auction, since they are not allowed to condition the slopes of

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their demand on \( y_i, c_i \). However, all equilibria in the static auction game correspond to naive linear strategies, because agents’ bid slopes in equilibrium do not depend on \( y_i, c_i \), as shown by proposition 3 in the baseline model of section 3. Thus, if agents are restricted to playing naive linear strategies, equilibria in the dynamic auction game correspond to those of the static auction game. The following Claim shows that these strategies also constitute equilibria without the restriction to naive linear strategies:

**Claim 8.** If all agents other than \( i \) are playing naive linear strategies, it is weakly optimal for agent \( i \) to play a naive linear strategy.

**Proof.** Suppose all other agents are playing naive linear strategies, described by

\[
 f_j (y_j, c_j), b_j
\]

From the perspective of agent \( i \), there is a random residual supply curve:

\[
 z_{RSi} (p, y_{-i}, c_{-i}) \equiv - \sum_{j \neq i} z_{Bj} (p, y_j, c_j) = - \sum_{j \neq i} [f_j (y_j, c_j) + b_j (p - \psi)]
\]

Importantly, the slope of the residual supply curve is constant at \( d = \sum_{j \neq i} b_j \). The dynamic auction game concludes when \( \sum_{i=1}^{n} z_{Bi} (p) = 0 \), that is, when

\[
 z_{Bi} (p) = z_{RSi} (p, y_{-i}, c_{-i})
\]

Hence, the set of all attainable combinations of quantity and price available to the agent, for any realization of \((y_{-i}, c_{-i})\), are described by \( z_{RSi} (p, y_{-i}, c_{-i}) \). The agent can do no better, using arbitrarily complex strategies, than choosing the optimal point on \( z_{RSi} (p, y_{-i}, c_{-i}) \) point for every realization of \((y_{-i}, c_{-i})\). Proposition 3 shows that this can be accomplished in the static auction game by submitting the bid curve:

\[
 z_{Bi} (p, y_i, c_i) = - \frac{d}{\kappa + d} y_i + \frac{\kappa}{\kappa + d} c_i - \frac{\kappa d}{\kappa + d} (p - \psi)
\]

Since the slope of \( z_{Bi} \) does not depend on \( y_i \) or \( c_i \), agent \( i \) can implement this bid curve
by playing the naive linear strategy:

\[ f_i(y_i, c_i) = -\frac{\kappa}{\kappa + d} y_i + \frac{\kappa}{\kappa + d} c_i + \frac{\kappa d}{\kappa + d} \psi, \quad b_i = -\frac{\kappa d}{\kappa + d} \]

Hence, agent \( i \) can do no better than playing a naive linear strategy.

Claim 8 shows that there exists an equilibrium in the game in which agents play naive linear strategies corresponding to equilibrium strategies in the static auction game, as in proposition 3. However, this does not imply that naive strategies are the only equilibrium strategies in this game; the strategy space is rich, and it is possible that many collusive equilibria of the kind documented by, for example, Wilson (1979) and others may exist.

One concern is that, if agents adopt dynamic bidding strategies, the estimated slopes of auction demand may depend on which auction rounds we use to estimate slopes. To gauge how much this would affect my results, in figure 5, I estimate auction demand slopes using only data from rounds 3 and above, for auctions which lasted 4 or more rounds. If, for example, agents shade bids in early rounds and bid more aggressively in late rounds, demand slopes estimated using late-round bidding data should be higher than demand slopes using the full sample. However, figure 5 shows that late-round demand slopes are statistically almost indistinguishable from demand slopes estimated using the full dataset, suggesting that differential bid shading across rounds does not have large effects on estimated demand slopes.

D.2 Data cleaning

Let auctions be indexed by \( a \in \{1 \ldots A\} \), and suppose that auction \( a \) lasts for \( R_a \) rounds, indexed by \( r \). For each round \( r \) of each auction \( a \), I observe the number of participants, \( n_{ar} \), the round price, \( p_{ar} \), and the total volume of gold that auction participants wish to buy and sell, respectively \( b_{ar} \) and \( s_{ar} \). For estimating my model, I filter to auctions with at least 3 rounds, \( R_a \geq 3 \), as I will estimate slopes of demand by regressing round buy and sell volume on round prices. I also filter to auctions in which the number of participants \( n_{ar} \) is constant over the course of the auction, and to auctions with 5-10 participants, as there are too few auctions with \( n = 4 \) and \( n = 11 \) to reliably estimate the distributions of

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Figure 5: Gold auction demand slope, all rounds vs late rounds only

Notes. Auction demand slopes by auction participant number \( n_a \) in primary estimation sample, estimated using data from all auction rounds (blue) and only data from auction rounds 3 and above (red). Dotted lines denote 80th and 20th percentile values.

volumes and demand slopes. This reduces the estimation sample to 502 auctions.

Table 1 shows features of my sample. Most auctions have 6-9 participants and conclude in 3-6 rounds. The range of prices between rounds for any given auction is small, relative to variation in gold prices between auctions: the difference between the highest and lowest round prices in a given auction is around $1 USD/oz on average. An average of 167,577oz of gold is traded on average in each auction in my estimation sample.

Define the volume imbalance in round \( r \) of auction \( a \) as the difference between buy and sell volume, that is:

\[
i_{ar} \equiv b_{ar} - s_{ar}
\]

The auction clearing price, buy and sell volume, and volume imbalance at the final round \( R_a \) of auction \( a \) are:

\[
p_{aR_a}, b_{aR_a}, s_{aR_a}, i_{aR_a}
\]

Define the total trade volume at the end of the auction as the sum of buy and sell volume:

\[
v_{aR_a} = b_{aR_a} + s_{aR_a}
\]

For simplicity, when discussing final-round quantities, I will omit the round subscript
R_a, thus, I will write p_a, v_a, i_a to mean the final auction price, trade volume, and volume imbalance in auction a. Let n_a represent the number of participants in auction a; this is uniquely defined within my estimation subsample, since I filter to auctions in which participation is constant.

To calculate gold futures open interest, I use CME group end-of-day data on gold futures from 2015-02-02 to 2016-12-30, provided by the Fama-Miller Center. COMEX gold futures position limits are publicly available on the CME group’s website. To calculate the monthly volatility of gold prices, I use a time series of front-month COMEX gold futures prices from November 2008 to September 2018, downloaded from Factset, and calculate the standard deviation of monthly differences in gold prices, calculated using the first trading day of each month.

D.3 Details on demand slope measurement

Suppose an auction has n_a participants, with holding capacities κ_a1 \ldots κ_an_a, and the risk factor is ψ_a. In a static auction, agent i would bid:

\[
z_{Bai}(p_a, c_{ai}, y_{ai}) = -\frac{b_{ai}}{κ_{ai}} y_{ai} + \frac{b_{ai}}{\sum_{j \neq i} b_{aj}} c_{ai} - b_{ai} (p_a - ψ_a)
\]  

(78)

where the bid slopes (b_{a1} \ldots b_{an_a}) satisfy expressions (21) and (22) of proposition 3. I assume that agents bid their static equilibrium bids in each round of the dynamic price-setting mechanism: if the round prices is p_{ar}, i bids to buy net quantity z_{Bi}(p_{ar}, c_{ai}, y_{ai}). Appendix [D.1] shows that this behavior is an equilibrium in a stylized model of the dynamic price-setting mechanism. Under this assumption, in round r of auction a, the aggregate volume imbalance will be:

\[
i_{ar} = \sum_{i=1}^{n} z_{Bai}(p_{ar}, y_{ai}) = \sum_{i=1}^{n} \left[ -\frac{b_{ai}}{κ_{i}} y_{ai} - (p_{ar} - ψ_a) b_{ai} \right]
\]  

(79)

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Thus, using any two rounds \( r \) and \( \tilde{r} \) of the same auction, we can measure the auction demand slope:

\[
\frac{i_{ar} - i_{a\tilde{r}}}{p_{ar} - p_{a\tilde{r}}} = -\sum_{i=1}^{n} b_{ai} = -B_a
\]

In any auction with more than two rounds, \( B_a \) is overidentified. I estimate \( \hat{B}_a \) by regressing volume imbalance on prices using all rounds of auction \( a \):

\[
\hat{B}_a \equiv -\frac{\sum_{r=1}^{R_a} (i_{ar} - \bar{i}_a) (p_{ar} - \bar{p}_a)}{\sum_{r=1}^{R_a} (p_{ar} - \bar{p}_a)} \tag{80}
\]

where \( \bar{i}_a \) and \( \bar{p}_a \) are the averages of \( i_{ar} \) and \( p_{ar} \), respectively, within auction \( a \).

If \( \hat{B}_a \) truly reflects auction bid slopes, it should be correlated with measures of dealers’ cost of liquidity provision. To test this, in table 4, I regress \( \hat{B}_a \) on three predictor variables: the CBOE VIX; the CBOE gold ETF volatility index (GVZ), which is calculated using prices of options on the SPDR gold ETF; and the 30-day average realized volatility of the LBMA gold price. The VIX reflects market-wide volatility conditions, which should correlate with the cost of liquidity provision for many different assets. The forward- and backward-looking measures of gold volatility should also correlate with gold dealers’ willingness to make markets in spot gold, since spot gold positions are more risky in periods where gold price volatility is high.

All coefficients in table 4 are significant, and have the expected signs: when VIX is high, or when forward- or backward-looking gold volatility is high, the auction slope \( \hat{B}_a \) tends to be lower. The \( R^2 \) is relatively low for the VIX, and substantially higher for both gold volatility measures; this is intuitive, as the gold-specific volatility measures should be better measures of gold market liquidity in particular. Together, these results suggest that my demand slope measurements indeed capture gold dealers’ costs of liquidity provision.
Table 4: Determinants of estimated demand slopes

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>−0.822***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.212)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GVZ</td>
<td>−2.773***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.287)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-day gold vol</td>
<td></td>
<td>−0.428***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.048)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,180</td>
<td>1,180</td>
<td>1,180</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.012</td>
<td>0.072</td>
<td>0.063</td>
</tr>
</tbody>
</table>

Notes. Regressions of estimated demand slope $\hat{B}_a$, in units of 1000 oz$^2$/USD, against the CBOE VIX, the CBOE gold ETF volatility index (GVZ), and the 30-day rolling realized volatility of the LBMA AM gold price. The sample is the full dataset of 1,331 auctions, on all days where FRED data on the CBOE VIX is available.
D.4 Descriptive evidence on model fit

Define the de-meaned volume imbalances and prices as:

\[
\tilde{i}_{ar} \equiv i_{ar} - \bar{i}_a, \quad \tilde{p}_{ar} \equiv p_{ar} - \bar{p}_a
\]

We can evaluate the fit of the model by estimating the following regression:

\[
\tilde{i}_{ar} = B_a \tilde{p}_{ar} + \epsilon_{ar} \tag{81}
\]

this evaluates how well volume imbalances can be fitted by linear functions of prices, separately for each auction. Estimating specification (81) for the primary estimation sample using OLS, we get an adjusted $R^2$ of 0.8295, and an F-statistic of 21.29, with a p-value below double precision, $2.2 \times 10^{-16}$. The estimated standard deviation of $\epsilon_{ar}$ is 11,080oz. This implies that, as the left panel of figure 3 suggests, volume imbalances are significantly associated with prices, and simple linear functions of prices explain around 83% of variation in volume imbalances within auctions. Results of estimating (81) using the full dataset are very similar: we get a slightly lower $R^2$ of 0.7923, and we estimate the standard error of $\epsilon_{ar}$ to be 13,587oz.

D.5 Random sampling of $\kappa_i$ and $\sigma_{y_i}$

By inspection, (21) and (22) have constant returns to scale in $\kappa_i$: scaling all $\kappa_i$’s up by some factor increases the unique equilibrium $b_i$ and $B$ by the same factor. Hence, we can sample a vector $(\tilde{\kappa}_1 \ldots \tilde{\kappa}_n)$ consistent with some observed aggregate demand slope $B$, by sampling a random vector of $\tilde{\kappa}_i$’s and then scaling them to match the desired $B$. Formally, I first draw some random vector:

\[
\tilde{\kappa}_1 \ldots \tilde{\kappa}_n
\]

where each $\tilde{\kappa}_i$ is an i.i.d. random sample; to generate relatively high levels of concentration in my samples, I draw these from an exponential distribution with mean 100. I then
numerically solve (21) and (22) to find the unique equilibrium \( \tilde{b}_i \)'s and \( \tilde{B} \). I then set:

\[
(\kappa_1 \ldots \kappa_n) = \frac{B}{\tilde{B}} (\tilde{\kappa}_1 \ldots \tilde{\kappa}_n)
\]

Now, fix a given draw for holding capacities \( \kappa_i \). If we assume that agents’ inventory shocks are independent normal random variables, appendix D.5.1 below shows that volume is a linear function of \( \sigma_{yi} \); thus, to simulate draws of \( \sigma_{yi} \) which match some target value for volume, we draw some random vector \( \tilde{\sigma}_{yi} \) and then scale it up to match the volume target.

### D.5.1 Expected volume with independent normal inventory shocks

Setting the sum of agents’ bid curves in (78) to 0 and solving for \( p_a \), the auction clearing price is:

\[
p_a - \psi_a = \frac{1}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{i=1}^{n} - \frac{b_{ai}}{\kappa_i} y_{ai} + \frac{b_{ai}}{\sum_{j \neq i} b_{aj}} c_{ai} \right]
\]

Since I assume agents do not hold contract positions when estimating the model, set \( c_{ai} = 0 \). Demand for each agent at the auction clearing price is:

\[
z_{Bi}(p_a) = - \frac{b_{ai}}{\kappa_i} y_{ai} - b_{ai} (p_a - \psi_a)
\]

\[
= - \left( 1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \right) \left( \frac{b_{ai}}{\kappa_i} y_{ai} \right) + \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{j \neq i} \frac{b_{aj}}{\kappa_j} y_{aj} \right]
\]

Assuming \( y_{aj} \) are independent mean-0 normal random variables, \( z_{Bi}(p_a) \) is also normal, with mean 0 and variance:

\[
\text{Var} (z_{Bi}(p_a)) = \left( 1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \right)^2 \left( \frac{b_{ai}}{\kappa_i} \right)^2 \sigma_{yai}^2 + \left( \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \right)^2 \sum_{j \neq i} \left( \frac{b_{aj}}{\kappa_j} \right)^2 \sigma_{yaj}^2
\]

Thus, the expected total trade volume for participant \( i \) in equilibrium is:

\[
E [z_{Bi}(p_a)] = \sqrt{\frac{2\text{Var} (z_{Bi}(p_a))}{\pi}}
\]
Expected volume for all participants the sum of $E \left[ \sum_{i} z_{Bai} (p_{ai}) \right]$ across all participants in auction $a$; this scales linearly with $\sigma_{yai}$.

### D.6 Target moments

Table $\text{[5]}$ shows the 20th, 50th, and 80th percentile estimates of auction demand slopes and auction volumes, for auctions with different numbers of participants.

### D.7 Basis risk and manipulation rent expressions

I set the standard deviation of contract positions, $\sigma_{ci}$, to 150,000oz of gold per agent, and I set the standard deviation of inventory shocks, $\sigma_{yi}$, to match volume, as I describe in appendix D.5 above. Assuming $c_{ai}$ is independent of $y_{ai}$ and $c_{aj}, y_{aj}$, basis risk and manipulation rents are respectively:

$$
\text{Var}(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right)^2 \sum_{i=1}^{n} \left( \frac{b_i}{d_i} \sigma_{ci} \right)^2
$$

$$
\mathbb{E} \left( \sum_{i} c_i \right) (p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right) \sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}^2
$$

If we assume $c_{i}$ can be correlated with $y_{ai}$, but is independent of $c_{aj}, y_{aj}$, basis risk and manipulation rents are maximized if $c_{ai}$ and $y_{ai}$ are perfectly negatively correlated, in which case we have:

$$
\text{Var}(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right)^2 \sum_{i=1}^{n} \left( \frac{b_i}{d_i} \sigma_{ci} + \frac{b_i}{k_i} \sigma_{yi} \right)^2
$$

$$
\mathbb{E} \left( \sum_{i} c_i \right) (p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right) \left( \sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}^2 + \frac{b_i}{k_i} \sigma_{ci} \sigma_{yi} \right)
$$

If we assume $c_{i}$ is independent of $y_{i}$, but contract positions can be correlated across agents, basis risk and rents are maximized if all agents’ contract positions are perfectly
Table 5: Target moments

<table>
<thead>
<tr>
<th>n</th>
<th>Demand slope</th>
<th>Volume</th>
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<tbody>
<tr>
<td></td>
<td>p20</td>
<td>p50</td>
</tr>
<tr>
<td>5</td>
<td>26.84</td>
<td>42.70</td>
</tr>
<tr>
<td>6</td>
<td>30.35</td>
<td>41.46</td>
</tr>
<tr>
<td>7</td>
<td>35.12</td>
<td>52.87</td>
</tr>
<tr>
<td>8</td>
<td>40.65</td>
<td>57.06</td>
</tr>
<tr>
<td>9</td>
<td>31.25</td>
<td>52.67</td>
</tr>
<tr>
<td>10</td>
<td>24.68</td>
<td>43.19</td>
</tr>
</tbody>
</table>

Notes. Estimated 20th, 50th and 80th percentile values for auction demand slopes and auction volumes, in the primary estimation sample of 502 auctions. Volume is in units of 1,000oz, and demand slopes are in units of 1,000 (oz^2/USD).
correlated, in which case we have:

\[
\text{Var}(p - \psi) = \left(\frac{1}{\sum_{i=1}^{n} b_i}\right)^2 \left(\sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}\right)^2
\]

\[
E\left(\sum_{i} c_i\right)(p - \psi) = \left(\frac{1}{\sum_{i=1}^{n} b_i}\right) \left(\sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}\right) \left(\sum_{i=1}^{n} \sigma_{ci}\right)
\]

## D.8 Full sample results

To ensure that my results are robust to the sample of auctions used, I repeat the analysis of the main text, using all 1,331 auctions which lasted more than one round. I cannot use auctions which lasted a single round for the estimation, because auction demand slopes are not identified. Since participation may change through the course of these auctions, I classify auctions by the maximum number of participants observed through the auction. Table 6 shows the estimated percentiles of demand slopes and volumes, which are the target moments used for estimation from the full sample, and tables 7 and 8 show estimated basis risk and manipulation rents. Results are quantitatively quite similar; in fact, auction demand slopes are actually somewhat higher in the full dataset than my primary estimation sample, implying that basis risk and manipulation rents estimated using the full sample are somewhat lower than those from the main estimation sample.

## D.9 Increasing participation holding fixed \(B\)

Suppose all agents have symmetric spot holding capacities \(\kappa\), with independent contract positions \(c_i\) with variance \(\sigma_c^2\), and no inventory shocks. From proposition 1, price variance is:

\[
\frac{\sigma_c^2}{n (n - 2)^2 \kappa^2}
\]

and the slope of aggregate auction demand is:

\[
B = n \frac{n - 2}{n - 1} \kappa
\]
Table 6: Target moments, full sample

<table>
<thead>
<tr>
<th>n</th>
<th>Demand slope</th>
<th></th>
<th>Volume</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p20 p50 p80</td>
<td></td>
<td>p20 p50 p80</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>24.69 39.34 65.96</td>
<td>79.76</td>
<td>118.79</td>
<td>171.27</td>
</tr>
<tr>
<td>6</td>
<td>30.16 50.00 81.48</td>
<td>77.81</td>
<td>122.00</td>
<td>181.88</td>
</tr>
<tr>
<td>7</td>
<td>46.27 65.05 93.31</td>
<td>86.55</td>
<td>126.45</td>
<td>179.56</td>
</tr>
<tr>
<td>8</td>
<td>44.85 68.73 96.89</td>
<td>105.74</td>
<td>146.21</td>
<td>200.51</td>
</tr>
<tr>
<td>9</td>
<td>36.12 62.75 98.38</td>
<td>121.31</td>
<td>166.99</td>
<td>244.60</td>
</tr>
<tr>
<td>10</td>
<td>34.81 54.92 84.14</td>
<td>144.26</td>
<td>191.07</td>
<td>260.52</td>
</tr>
</tbody>
</table>

Notes. Estimated 20th, 50th and 80th percentile values for auction demand slopes and auction volumes, in the full sample of 1,316 auctions. Volume is in units of 1,000 oz, and demand slopes are in units of 1,000 (oz²/USD).
Table 7: Gold auctions basis risk estimates, full sample

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>1.65 (1.27-2.63)</td>
<td>2.20 (1.86-2.87)</td>
<td>3.11 (2.84-3.95)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>2.77 (2.14-4.41)</td>
<td>4.20 (3.59-5.48)</td>
<td>5.22 (4.77-6.63)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>4.41 (3.40-7.03)</td>
<td>4.84 (3.96-7.12)</td>
<td>8.31 (7.60-10.56)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>1.20 (0.911-1.94)</td>
<td>1.59 (1.35-2.12)</td>
<td>2.41 (2.21-3.03)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>1.96 (1.48-3.17)</td>
<td>3.01 (2.54-3.83)</td>
<td>3.93 (3.61-4.93)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>3.24 (2.46-5.25)</td>
<td>3.63 (2.98-5.40)</td>
<td>6.51 (5.98-8.17)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.953 (0.713-1.58)</td>
<td>1.31 (1.15-1.75)</td>
<td>2.03 (1.88-2.53)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>1.37 (1.02-2.26)</td>
<td>2.15 (1.89-2.71)</td>
<td>2.92 (2.69-3.63)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>1.92 (1.44-3.18)</td>
<td>2.29 (1.92-3.32)</td>
<td>4.10 (3.79-5.11)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.854 (0.65-1.48)</td>
<td>1.26 (1.10-1.68)</td>
<td>1.91 (1.78-2.41)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>1.20 (0.916-2.09)</td>
<td>2.02 (1.76-2.53)</td>
<td>2.70 (2.51-3.40)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>1.84 (1.40-3.21)</td>
<td>2.18 (1.81-3.36)</td>
<td>4.13 (3.85-5.21)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.785 (0.588-1.32)</td>
<td>1.22 (1.08-1.57)</td>
<td>1.84 (1.72-2.20)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>1.23 (0.922-2.07)</td>
<td>2.21 (1.98-2.77)</td>
<td>2.89 (2.70-3.44)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>2.14 (1.60-3.60)</td>
<td>2.45 (2.01-3.71)</td>
<td>5.02 (4.70-5.98)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.862 (0.648-1.45)</td>
<td>1.45 (1.29-1.86)</td>
<td>2.12 (1.99-2.55)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>1.32 (0.993-2.23)</td>
<td>2.55 (2.27-3.17)</td>
<td>3.25 (3.05-3.91)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>2.08 (1.57-3.52)</td>
<td>2.38 (1.96-3.68)</td>
<td>5.12 (4.82-6.17)</td>
</tr>
</tbody>
</table>

Notes. Estimated standard deviations of basis risk for full sample of 1,316 auctions, in units of prices (USD/oz). Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 6. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
Table 8: Gold auctions manipulation rent estimates, full sample

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>0.467 (0.427-0.593)</td>
<td>0.688 (0.611-0.772)</td>
<td>2.333 (2.133-2.966)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.782 (0.715-0.995)</td>
<td>1.336 (1.167-1.503)</td>
<td>3.912 (3.577-4.974)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>1.247 (1.140-1.585)</td>
<td>1.429 (1.309-1.718)</td>
<td>6.233 (5.698-7.924)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>0.362 (0.332-0.454)</td>
<td>0.536 (0.491-0.599)</td>
<td>2.170 (1.993-2.723)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.589 (0.541-0.739)</td>
<td>1.036 (0.932-1.147)</td>
<td>3.536 (3.248-4.437)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>0.977 (0.897-1.226)</td>
<td>1.160 (1.078-1.372)</td>
<td>5.863 (5.385-7.355)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.305 (0.282-0.380)</td>
<td>0.475 (0.437-0.528)</td>
<td>2.136 (1.972-2.659)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.438 (0.404-0.545)</td>
<td>0.794 (0.719-0.870)</td>
<td>3.064 (2.828-3.814)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>0.615 (0.568-0.766)</td>
<td>0.798 (0.746-0.923)</td>
<td>4.308 (3.976-5.362)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.287 (0.267-0.362)</td>
<td>0.487 (0.448-0.537)</td>
<td>2.295 (2.140-2.893)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.404 (0.377-0.510)</td>
<td>0.794 (0.722-0.862)</td>
<td>3.235 (3.017-4.079)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0.620 (0.578-0.781)</td>
<td>0.802 (0.750-0.942)</td>
<td>4.957 (4.623-6.250)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.277 (0.259-0.330)</td>
<td>0.503 (0.469-0.546)</td>
<td>2.490 (2.328-2.966)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.434 (0.406-0.517)</td>
<td>0.921 (0.848-0.986)</td>
<td>3.904 (3.650-4.649)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>0.754 (0.704-0.897)</td>
<td>0.936 (0.888-1.068)</td>
<td>6.783 (6.340-8.077)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.318 (0.299-0.383)</td>
<td>0.632 (0.587-0.681)</td>
<td>3.180 (2.988-3.829)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.487 (0.458-0.587)</td>
<td>1.125 (1.033-1.201)</td>
<td>4.871 (4.578-5.866)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>0.769 (0.722-0.926)</td>
<td>0.951 (0.909-1.094)</td>
<td>7.686 (7.224-9.256)</td>
</tr>
</tbody>
</table>

Notes. Estimated expected manipulation rents for full sample of 1,316 auctions, in units of millions of US dollars. Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in Table 6. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
Suppose we fix the slope of aggregate auction demand at some value $B$, and the variance of individual spot traders’ contract positions $\sigma^2_c$, and increase the number of auction participants $n$. For any $n$, there is a unique value of $\kappa$ which rationalizes any $B$:

$$
\kappa(B, n) = B \frac{n - 1}{(n - 2) n}
$$

(84)

Plugging (84) into (83) and rearranging, we have price variance as a function of $\sigma^2_c$, $B$, and $n$:

$$
\frac{\sigma^2_c}{B^2} \frac{n}{(n - 1)^2}
$$

This is decreasing in $n$ for $n \geq 3$. Thus, if spot traders are symmetric, increasing the number of spot traders, fixing $B$ and $\sigma^2_c$, decreases basis risk.

Similarly, plugging (84) into expression (16) for manipulation rents, we get:

$$
\tau = \frac{\sigma^2_c}{B} \frac{n}{(n - 1)}
$$

so manipulation rents are also decreasing in $n$, converging to the constant $\frac{\sigma^2_c}{B}$.

Intuitively, as we increase $n$, there are a number of counteracting effects: competition increases, so agents’ manipulation incentives decrease; however, since each agent holds a contract position with variance $\sigma^2_c$, the variance of contract positions summed across all agents increases. Basis risk is decreasing in $n$, because the competition effect dominates; for manipulation rents, the effects asymptotically offset each other exactly. Note also that the variance of total contract positions increases linearly in $n$, so manipulation rents per agent – or per contract – decrease towards 0 rapidly as $n$ increases.