Competition and Manipulation in Derivative Contract Markets*

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Abstract

This paper develops methods and metrics for quantifying manipulation risk in cash-settled derivative contract markets. I show how to estimate market participants’ manipulation incentives, and predict manipulation-induced market distortions, using commonly observed market data. I develop a simple manipulation index, which can be used as a diagnostic metric to detect potentially manipulable contract markets, similar to the Herfindahl-Hirschman index (HHI) in antitrust settings. I apply these methods to estimate manipulation risk in a number of contract markets.

Keywords: derivative contracts, manipulation, regulation

JEL classifications: D43, D44, D47, G18, K22, L40, L50

*The online appendix to the paper is available here. I appreciate comments from Mohammad Akbarpour, Yu An, Sam Antill, Anirudha Balasubramanian, Alex Bloedel, Jeremy Bulow, Shengmao Cao, Gabe Carroll, Juan Camilo Castillo, Scarlet Chen, Wanning Chen, Yiwei Chen, Yuxin (Joy) Chen, Cody Cook, Peter DeMarzo, Rob Donnelly, Tony Qiaofeng Fan, Winston Feng, Robin Han, Benjamin Hebert, Hugh J. Hoford, Emily Jiang, David Kang, Zi Yang Kang, Fuhito Kojima, Nadia Kotova, Pete Kyle, Eddie Lazear, Wenhao Li, Yucheng Liang, Jacklyn Liu, Jialing Lu, Carol Hengheng Lu, Danqi Luo, Hannu Lustig, Anthony Lynch, Suraj Malladi, Giorgio Martini, Negar Matoorian, Ellen Muir, Evan Munro, Mike Ostrovsky, Paul Oyer, Agathe Pernoud, Peter Reiss, Sharon Shiao, John Shim, Ryan Shyu, Andy Skrzypacz, Paulo Somaini, Takuo Sugaya, Cameron Taylor, Lulu Wang, Bob Wilson, Milena Wittwer, Alex Wu, David Yang, Jessica Yu, Ali Yurukoglu, Becky Zhang, Jeffrey Zhang, and Mingxi Zhu. I am very grateful to my advisors, Lanier Benkard and Paul Milgrom, as well as my committee members, Darrell Duffie, Glen Weyl and Brad Larsen, for their continuous guidance and support. I especially thank Darrell Duffie, who originally inspired this work.

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“Generally speaking… market power is a structural issue to be remedied, not by behavioral prohibitions, but by processes to identify and, where necessary, mitigate market power…”

– 114 FERC ¶61,165

1 Introduction

This paper studies manipulation in cash-settled derivative contract markets. An agent who holds a cash-settled derivative contract, at contract expiration, receives a payment linked to a price benchmark, determined based on trade prices of certain underlying assets. The volume of derivative contracts settled based on a given benchmark is often much larger than the volume of underlying trades used to calculate the benchmark; thus, agents holding large contract positions may have incentives to manipulate contract markets, trading relatively small amounts of underlying assets to move price benchmarks and influence payoffs on their larger contract positions. Manipulation has been a problem in contract markets since their inception, and fines for contract market manipulation have totalled billions of dollars in the past two decades alone.

Contract market manipulation is illegal in the US and many other jurisdictions, but it is notoriously difficult to define and prosecute. US authorities currently regulate manipulation using a primarily behavioral approach. Trading with the intent to move prices and influence contract payoffs is illegal per se, and regulators often bring cases against manipulators based on evidence from email or telephone communications between traders. This approach is difficult to apply in settings where regulators cannot easily access traders’ communications, and it is insensitive to the magnitude of traders’ economic incentives to manipulate contract markets.

Contract market manipulation could instead be regulated using a primarily structural approach, similar to antitrust regulation. Antitrust regulators do not punish firms for pricing above marginal cost; they instead aim to keep industries sufficiently competitive that market competition disciplines firms’ pricing behavior, without the need for case-by-case regulatory intervention. Regulators use concentration metrics such as the Herfindahl-Hirschman index (HHI), and methods such as demand system and markup estimation, to monitor the size of market power-induced distortions in oligopolistic industries, guiding
policy interventions, such as blocking mergers and forcing divestitures, for structurally controlling industry competitiveness.

Contract market regulators have a variety of tools for controlling market structure: regulators set limits on the size of agents’ contract positions, and set rules for how price benchmarks are calculated. However, regulators do not know how to tell whether a given contract market is vulnerable to manipulation. Contract markets are often much larger than underlying markets; we do not know how to tell when a contract market is too large. In order to effectively apply structural policy tools to combat contract market manipulation, we need a theory-based way to measure the vulnerability of contract markets to manipulation.

This paper develops metrics and methods to quantify manipulation risk in derivative contract markets. I assume that agents hold contract positions tied to a price benchmark, which is determined in a uniform-price auction for an underlying asset. If an agent holds a large positive contract position, she has incentives to manipulate the market, buying the underlying asset to increase the benchmark price and thus her contract payoffs. However, buying the underlying asset to move prices is costly to the agent because it decreases her marginal utility for the underlying asset. I show that the size of an agent’s manipulation incentives depend on how large the slope of residual supply facing the agent is, relative to the slope of the agent’s own demand for the underlying asset.

In equilibrium, manipulation incentives are a function of all agents’ demand slopes. Define an agent’s capacity share as the agent’s share of the market’s aggregate demand slope, that is, the agent’s demand slope divided by the slope of aggregate market demand. In equilibrium, an agent with capacity share \( s_i \) trades approximately \( s_i \) additional units of the underlying asset per unit contract that she holds. Thus, in competitive markets, manipulation incentives are quantitatively small, and the volume of outstanding derivative contracts can be much larger than trade volume in the underlying asset without creating high manipulation risk. The relationship between demand slopes and manipulation incentives is quantitatively precise, so we can measure agents’ manipulation incentives in any setting where we can estimate agents’ demand slopes.

I propose two ways to measure manipulation-induced benchmark distortions. If we observe sufficiently rich data to estimate all agents’ demand slopes, I show how to estimate the fraction of intertemporal benchmark price variance which is attributable
to manipulation. In settings with more limited data, I propose a simpler manipulation index, which can be calculated using only aggregate data on total contract volume and underlying trade volume, along with the largest market participant’s capacity share. Under some assumptions, the square of the manipulation index is an approximate upper bound for the fraction of total benchmark noise which is attributable to manipulation. I propose a simple rule-of-thumb for detecting manipulable contract markets: if the manipulation index for a given market is higher than 0.7, the market is potentially vulnerable to manipulation.

I apply my results to analyze whether the London Bullion Market Association (LBMA) gold price would be vulnerable to manipulation if it were used for cash settlement of COMEX gold futures contracts. I estimate agents’ demand slopes using auction bidding data, allowing me to predict manipulation-induced benchmark variance for any conjectured size of contract positions. I find that, even though the calibrated size of contract positions is 4-10 times larger than the volume of gold traded in the auction, manipulation-induced variance would only constitute 0.14%-0.46% of monthly variation in the LBMA gold price. Thus, my model predicts that the LBMA gold price benchmark could be used to settle COMEX gold futures with negligible manipulation risk.

I then evaluate my rule-of-thumb test by calculating the manipulation index for three other contract markets: ICE Brent crude futures, CME feeder cattle futures, and ICE HSC basis futures. I find that the manipulation index is lower than the rule-of-thumb cutoff for the Brent crude and feeder cattle futures contracts, but is much higher than the cutoff for ICE HSC basis futures, suggesting that the HSC basis contract may be vulnerable to manipulation. Consistent with this prediction, the HSC basis contract was the subject of a high-profile manipulation case in 2009, whereas authorities have not brought successful manipulation cases in the Brent crude or feeder cattle contract markets. I then use the rule-of-thumb to argue that, under current market conditions, position limits for SOFR-linked interest rate derivatives could be on the order of $700 billion in notional exposure for each market participant, without creating high levels of manipulation risk.
1.1 Related literature

To my knowledge, this is the first paper to attempt to develop an empirical measure of manipulation risk in derivative contract markets. There are a number of related strands of literature. There is a small but growing literature theoretically and empirically analyzing market and benchmark manipulation. Abrantes-Metz et al. (2012), Gandhi et al. (2015), and Bonaldi (2018) analyze LIBOR manipulation, Griffin and Shams (2018) analyzes VIX manipulation, and Birge et al. (2018) studies manipulation in electricity markets. A number of theoretical papers analyze the question of optimal benchmark design, such as Duffie, Dworczak and Zhu (2017), Duffie and Dworczak (2018), Eisl, Jankowitsch and Subrahmanyan (2017), Coulter, Shapiro and Zimmerman (2018) and Baldauf, Frei and Mollner (2018). Duffie and Dworczak (2018) and Baldauf, Frei and Mollner (2018) propose using volume-weighted average price schemes, whereas Coulter, Shapiro and Zimmerman (2018) proposes an incentivized announcement scheme which bears some resemblance to an auction. In contrast to these papers, I do not attempt to find an optimal mechanism for benchmark determination in this paper; instead, I adopt a reduced-form model of benchmark setting in order to quantify agents’ manipulation incentives.


1.2 Outline

The remainder of the paper proceeds as follows. Section 2 discusses some history and institutional details of contract market manipulation. Section 3 introduces the model and derives the main theoretical results. Section 4 shows how the model can be used
to empirically measure manipulation risk. Section 5 studies the LBMA gold price, and section 6 presents case studies of a number of other contract markets. Section 7 discusses implications of my findings, and section 8 concludes. Proofs, derivations and other supplementary material are presented in the appendix, and additional supplementary material is presented in an online appendix.

2 Institutional background

Derivative contracts are financial instruments – futures, options, swaps, and others – whose prices are linked to some underlying asset. This paper focuses on cash-settled derivative contracts, which entitle contract holders to cash payments linked to price benchmarks, which are set based on trade prices of the underlying asset. For example, the CME Feeder Cattle Index is a benchmark which measures the average sale price per pound of cattle in the US over the past seven days, based on cattle sale prices published by the USDA. An agent who holds a single cash-settled CME Feeder Cattle futures contract, at contract expiration, is entitled to a cash payment totalling 50,000 times the price benchmark; one long contract thus represents 50,000 pounds of exposure to cattle prices.

Cash-settled derivative contracts are used in a variety of settings. The first cash-settled contract was the Eurodollar futures contract, introduced on the Chicago Mercantile Exchange in 1981. Futures and options contracts for livestock and many dairy products are financially settled based on USDA-published average transaction prices for underlying commodities. Many derivative contracts for energy products, such as oil, gas, and electricity are settled using price benchmarks calculated based on government or industry sources. Interest rate benchmarks such as LIBOR and SOFR are used to determine payments on many floating-rate loans and related products, such as mortgages, student
loans, bonds, and interest rate swaps. FX benchmarks such as the WM/Reuters rates are widely used to value portfolios and evaluate execution quality, and a variety of exchange-traded FX futures and options are also financially settled based on these and other benchmarks. There are also a variety of contracts written based on indices derived from aggregated prices of many underlying securities: aggregated indices and associated derivative contracts exist for equities, commodities, FX, volatility, and many others.

Derivative contract activity tends to be concentrated in a small number of contracts for any given underlying asset. For example, as of December 2016, over half of wheat futures open interest recorded by the CFTC was based on the Chicago SRW contract over 3/4 of all open interest in crude oil futures was in contracts based on WTI and Brent crude, and over half of gas futures open interest was in contracts based on gas traded at Henry Hub, Louisiana. Derivative contracts also tend to be highly leveraged, so investors can purchase large volumes of exposure to underlying assets at low upfront capital cost; for example, an investor buying ICE Brent Crude futures can purchase exposure to 1,000 barrels of oil, worth approximately $70,000 at the time of writing, by depositing at ICE a margin payment of approximately $3,500.

Due to their concentration and leverage, the volume of derivative contracts settled using a given price benchmark is often much larger than the volume of underlying trade used to construct the benchmark. For example, the Platts Inside FERC Houston Ship Channel natural gas price benchmark is based on weekly trade volume totalling around 1.4 million MMBtus of natural gas open interest in the ICE HSC basis future, which

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8 Financial Stability Board (2018)
9 Financial Stability Board (2014)
10 CME Korean Won futures are settled based on a benchmark price reported by Seoul Money Brokerage Service Limited, which represents activity in the Korean Won spot market, and the CME Chinese Renminbi futures are settled based on a similar benchmark reported by the Treasury Markets Association, Hong Kong.
11 FTSE 100 Index
12 Bloomberg Commodity Index
13 U.S. Dollar Index
14 VIX
15 My calculations based on the CFTC Commitments of Traders report for December 27, 2016. In a CME report titled “Understanding Wheat Futures Convergence”, Fred Seamon writes that “Often, because of its liquidity, the CBOT Wheat futures contract is used as a benchmark for world wheat prices”.
16 My calculations based on the CFTC Commitments of Traders report for December 27, 2016.
17 Brent Crude Futures margin rates.
is financially settled based on the Platts benchmark, is more than 75 million MMBtus for many delivery months. The ICE Brent Crude index is based on around 75 million barrels of physical oil traded in the North Sea each month; open interest in ICE Brent Crude futures is over 200 million barrels of oil for some delivery months. The CME Feeder Cattle Index is based on 10-20 million pounds of cattle trades; open interest in CME feeder cattle futures for many delivery months is over 500 million pounds of cattle.

The Secured Overnight Financing Rate (SOFR), designed to replace USD LIBOR as an interest rate benchmark, is based on average daily volumes of approximately $1 trillion in overnight treasury-backed repo loans; as of 2014, the total notional volume of contracts linked to USD LIBOR was estimated to be greater than $160 trillion.

Since contract markets are much larger than underlying markets, agents holding large contract positions may attempt to manipulate contract markets, trading relatively small volumes of the underlying asset to move price benchmarks and generate large profits in contract markets. Suppose that a gas trader is able to acquire a long position in Houston Ship Channel (HSC) basis futures totalling 10 million MMBtu of natural gas. If gas is currently trading at $3 USD/MMBtu at HSC, the gas trader can then buy a relatively large amount of physical gas at HSC – for example, 3 million MMBtus – thus raising the Platts Inside FERC price at HSC to, for example, $5 USD/MMBtu. The trader makes a loss in the underlying market, as she purchases physical gas at an elevated price, and accumulates a large amount of physical gas that she has no commercial use for; however, by raising the gas benchmark price, she increases her total contract payoff by $20 million.

Symmetrically, the trader could accumulate a large short contract position and then sell volume during “bid week,” the five business days during which index prices are set, was between 50 to 280 thousand MMBtu per day; multiplying by 5, we get an upper bound of 1.4 million. Volume in other years is much larger; in 2008-2010, many delivery months have total trade volume over 10 million MMBtus.

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19 [ICE Report Center](#) End of Day reports for HSC basis futures, as of October 24th, 2018. Open interest is above 30,000 contracts for many delivery months, and the contract multiplier is 2,500 MMBtus.

20 [ICE Brent FAQ](#)

21 [ICE Report Center](#) End of Day reports for Brent Crude futures, as of October 24th, 2018.

22 See “Feeder Cattle Daily Index Data” at the [CME website](#).

23 As of 2018-09-29, [open interest](#) in the Nov 18 and Jan 19 contracts were over 20,000 and 15,000 respectively, and the contract multiplier is 50,000 pounds.

24 [NY Fed’s Secured Overnight Financing Rate Data](#)

25 [Financial Stability Board (2018)](#). Note that the $160 trillion number measures the total volume of outstanding contracts across expiration dates; the volume of interest rate derivatives expiring on any given settlement date will be substantially smaller.
large quantities of gas in order to lower benchmark prices.

The result of this kind of manipulation is that the trader makes losses in the underlying market, from buying or selling large undesired quantities of gas at unfavorable prices, but profits in the derivative contract market by moving the HSC index price and thus increasing her derivative contract payoffs. Manipulation is socially harmful for two reasons. First, the HSC benchmark price becomes a noisier signal of true supply and demand conditions at HSC for natural gas, so derivative contracts based on the benchmark become noisier hedges against shifts in fundamentals. Second, allocations of gas are distorted, as manipulative trades made to move benchmark prices may create shortages or surpluses of gas at HSC.

Contract market manipulation has led to billions of dollars of fines in the past two decades alone. But manipulation is notoriously hard to define and prosecute. In most recent cases, authorities have adopted a primarily behavioral approach to regulation: charges are brought against alleged manipulators largely based on “smoking gun” evidence of manipulative intent, and regulators search through traders’ email and telephone communications attempting to find evidence that traders acted with the intent to move benchmark prices. But role that traders play in contract markets is precisely to manage and optimize the price impact of their trades. Trades move prices regardless of whether traders intend to move prices, so it is impractical to prohibit traders’ acknowledgement of price impact per se, but it has been difficult to define clear rules determining when the intentional exercise of price impact is manipulative and illegal. The behavioral

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26 The CFTC and the FERC have fined energy traders millions of dollars for manipulating oil and gas futures markets; see, for example, [CFTC Press Release 6041-11](https://www.cftc.gov/法制/pressrelease/6041-11) and [128 FERC 61,269](https://www.ferc.gov/legal/staff-reports/orders/128-ferc-61-269.pdf). Banks have been fined over $10 billion for FX manipulation (Levine, 2015), over $8 billion USD for manipulation of LIBOR and other interest rate benchmarks (Ridley and Freifeld, 2015), and over $500 million for manipulation of the ISDAFIX interest rate swap benchmark (Leising, 2017).

27 Levine (2014) quotes a number of trader chat messages used in FX manipulation lawsuits; some other examples of evidence for manipulation drawn from traders’ communications can be found in the CFTC’s lawsuits brought against [Parnon Energy, Inc.](https://www.cftc.gov/法制/pressrelease/6041-11) and others for crude oil manipulation, [Energy Transfer Partners, L.P.](https://www.cftc.gov/法制/pressrelease/6041-11) and others for natural gas manipulation, and [Barclays](https://www.cftc.gov/法制/pressrelease/6041-11) for ISDAFIX manipulation.

28 As an example of the difficulties involved in defining manipulation, the CFTC dismissed a [1977 cotton futures manipulation case against the Hohenberg Brothers Company](https://www.cftc.gov/法制/pressrelease/6041-11), writing: “Even though respondents’ activities may have involved a “profit motive,” absent a finding of manipulative intent, trading with the purpose of obtaining the best price for one’s cotton does not constitute, in itself, a violation of the Commodity Exchange Act.” The legal literature has proposed a number of definitions for manipulation, but has not reached consensus. For example, Perdue (1987) proposes defining manipulation as “conduct that would be uneconomic and irrational, absent an effect on market price.” Fischel and Ross (1991) argue
approach to regulation is also insensitive to the magnitude of agents’ economic incentives to manipulate markets. In cases where regulators are able to obtain sufficient evidence to prosecute, massive punitive fines are levied which may not be proportionate to agents’ *ex ante* incentives to manipulate; on the other hand, many economically harmful cases of manipulation may be unprosecutable in settings where regulators cannot easily prove manipulative intent.

An approach more grounded in economic theory would be to regulate markets structurally, limiting the size of contract markets so that market participants do not have large economic incentives to manipulate. Regulators have a variety of tools to intervene in contract markets: regulators impose limits on the size of agents’ contract positions and impose rules determining how underlying trades can be used to construct benchmarks. But it is difficult to apply these policy tools to effectively combat manipulation, because we do not know how to measure manipulation risk. Contract markets are generally much larger than underlying markets, so it is impractical to limit contract markets to the size of underlying markets, but we do not know how large contract markets can be without producing overly large incentives for manipulation. Thus, in order to structurally regulate contract market manipulation, regulators need theory-based metrics and methods to measure manipulation risk in derivative contract markets.

3 Model

3.1 Utility functions

I assume that a price benchmark is set in a uniform-price double auction for the underlying good. Agents both hold derivative contract positions linked to the benchmark, and have some fundamental demand to buy or sell the underlying asset. Since agents’ trades affect benchmark prices, agents holding derivative contracts will have incentives to modify their trading decisions, in order to move the price benchmark and increase their derivative

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29 For example, see the CFTC’s website on Speculative Limits.
30 For example, see the Financial Conduct Authority website on EU Benchmarks Regulation, and the IOSCO report on Principles for Financial Benchmarks.
contract payoffs.

Formally, there are $n \geq 3$ agents indexed by $i$. Agent $i$’s utility for trading $z$ units of the underlying asset, and paying net monetary transfer $t$, is:

$$U_i(z, t) = \pi z + \frac{y_{Di} z}{\kappa_i} - \frac{z^2}{2\kappa_i} - t$$  \hspace{1cm} (1)

The $\frac{z^2}{2\kappa_i}$ term implies that agents have linearly declining marginal utility for the underlying asset. $\kappa_i$ represents the slope of agent $i$’s demand for the underlying asset. Declining marginal utility can be thought of as driven by, for example, agents’ capital costs for purchasing the underlying asset, or their physical infrastructure costs for storing the underlying asset once it is purchased. While I do not explicitly consider agents’ decisions to trade the asset over time, the $\frac{z^2}{2\kappa_i}$ term could also be thought of as representing curvature of the agent’s value function in $z$ in a richer dynamic model.

The term $y_{Di}$ is a demand shock, privately observed by $i$, which vertically shifts $i$’s marginal utility for the underlying asset. $y_{Di}$ is divided by $\kappa_i$ in expression $1$; this is just a normalization which implies that an agent who receives a unit increase in $y_{Di}$ optimally purchases an additional unit of the asset at any given price, regardless of $\kappa_i$. I assume that the joint distribution of $y_{Di}$ across agents has full support, but $y_{Di}$ can be arbitrarily correlated across agents. For analytical convenience, I assume that the means of agents’ demand shocks sum to 0, that is:

$$\sum_{i=1}^{n} E[y_{Di}] = 0$$  \hspace{1cm} (2)

This is not a substantive assumption, as we can always define the constant term $\pi$ in agents’ utility functions such that $2$ holds.

The baseline model assumes agents have private values: $i$’s utility per unit $z$ is not affected by $j$’s private information. This is arguably a reasonable model for many of the underlying markets for benchmark setting, such as currencies, interest rates, oil and gas; information about the value of the underlying asset is likely to be close to symmetric.

\footnote{The demand shock $y_{Di}$ is isomorphic, up to an additive constant in $U_i(z, t)$, to inventory shocks in the linear-quadratic double auctions literature.}
among large market participants, and trading is likely to be driven mostly by idiosyncratic preference or inventory shocks. I analyze an extension of the model in which agents have interdependent values, so adverse selection is a concern, in section 5 of the online appendix. The baseline model also restricts agents’ utility functions to be quadratic in $z$. This is standard in the literature, as multi-unit double auctions are difficult to solve for general utility functions; however, section 1.4 of the online appendix analyzes an extension of the model to the general nonlinear case.

3.2 Derivative contract positions

I model derivative contracts held by agent $i$ as a random, exogenously determined contract position, $y_{Ci}$. $y_{Ci}$ can be positive or negative, and $i$ receives net monetary payment $p_B y_{Ci}$ if the benchmark price is $p_B$. Thus, the total utility of agent $i$ with demand shock $y_{Di}$ and contract position $y_{Ci}$, if she buys $z$ units of the underlying asset and the benchmark price is $p_B$, is:

$$U_i (z, p_B; y_{Di}, y_{Ci}) = \pi z + \frac{y_{Di} z}{\kappa_i} - \frac{z^2}{2\kappa_i} - p_B z + p_B y_{Ci}$$

That is, the agent pays $p_B z$ to purchase $z$ units of the underlying asset, and receives $p_B y_{Ci}$ in contract payoffs.

I assume that the joint distribution of demand shocks and contract positions $y_{Di}, y_{Ci}$ across agents has full support, but contract positions and demand shocks can be arbitrarily correlated across agents. Unlike demand shocks, I allow contract positions to be either privately or commonly observed. Since derivative contracts are zero-sum bets on prices, contract positions must sum to 0 across agents in the broader economy; however, I do not require contract positions to sum to 0 across the $n$ agents in my model. We can think of other contracts being held by unmodelled agents who do not participate in benchmark setting, perhaps because they have very high costs of trading the underlying asset.

My model makes two important simplifying assumptions about derivative contract positions. First, I assume that contract positions are exogeneous. In practice, market participants may hold derivative contracts for many reasons: to hedge existing risk, to speculate on prices of the underlying asset, or even to profit from attempted manipulation. However, at the point of contract settlement, agents holding a given contract position have
the same manipulation incentives regardless of their original reasons for entering into their contract positions. Moreover, regulators can often directly observe agents’ contract positions at settlement. For the purpose of analyzing agents’ manipulation incentives given their observed contract positions, it is not necessary to take a stance on why agents originally chose to enter these positions.

Second, I do not take an explicit stance on the welfare consequences of manipulation in the main text. Once again, this is because we can measure manipulation incentives and manipulation-induced market distortions without taking a stance on the welfare effects of manipulation. We can, however, think of manipulation as causing two main sources of welfare loss. First, manipulation causes price benchmarks to be noisier signals for the common component $\pi$ of agents’ values. Agents who hold derivative contracts to hedge against movements in $\pi$ are thus exposed to additional basis risk. Second, manipulation distorts allocative efficiency, as manipulators’ trading decisions are influenced by their contract positions, which do not directly affect their marginal utility for the underlying asset.

I construct a simple model which endogenizes agents’ contract positions, and formally demonstrates both sources of welfare losses, in section 4 of the online appendix. For the remainder of the main text, I maintain the assumption of exogeneous contract positions.

### 3.3 Benchmark prices and contract settlement

I assume that the benchmark price $p_B$ is set in a uniform-price double auction. After observing demand shocks $y_{Di}$ and contract positions $y_{Ci}$, agents simultaneously submit affine and strictly decreasing bid curves $z_{Bi}(p; y_{Di}, y_{Ci})$, representing the net amount of the underlying asset they are willing to purchase at price $p$. The auction clearing price...

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32A natural question is why agents would not build up extremely large contract positions just before settlement, then manipulate the underlying market to increase contract payoffs. There are a number of reasons why this would be difficult in practice. First, manipulation is illegal in the US, and entering large contract positions close to settlement dates and then trading aggressively in underlying markets would be strong evidence of manipulative intent. Second, in practice there is an imperfectly elastic market supply curve for contracts; so agents entering into large positive contract positions will face progressively worse prices, even in the absence of any manipulative ability or intent. Third, every contract has a counterparty, and if a large agent blatantly attempted to purchase large contract positions for manipulative purposes, at some point she would be unable to find willing counterparties at reasonable prices.
sets the sum of agents’ bids to 0:

\[ p_B = \left\{ p : \sum_{i=1}^{n} z_{Bi}(p; y_{Di}, y_{Ci}) = 0 \right\} \]

Each agent then pays \( p_B z_{Bi}(p; y_{Di}, y_{Ci}) \), and purchases net quantity \( z_{Bi}(p; y_{Di}, y_{Ci}) \) of the underlying asset. \( p_B \) is then used for contract settlement, so agent \( i \) receives a net transfer of \( p_B y_{Ci} \).

I use auctions as a reduced-form model for benchmark setting. Some benchmarks, such as the CBOE VIX and the LBMA gold price, are in fact set using uniform-price auctions. Other benchmark setting mechanisms may be reasonably approximated by auctions: for example, benchmarks for oil and gas, the WM/Reuters FX benchmark and the and the ISDAFIX interest rate swap benchmark are based on trades of relatively homogeneous commodities over relatively short periods of time. Auctions are a less appropriate model for benchmarks based on underlying markets with large distortions other than market power, and the analysis of this paper may not be suitable for these settings. Some assets are traded in decentralized markets with large search frictions; the assumption that all trades happen at the same price is not appropriate. In other markets, such as the interbank loan market which the LIBOR interest rate benchmark is based on, trades are difficult to verify, so false reporting is an important concern. I briefly discuss the extent to which my model applies to different benchmark-setting mechanisms in subsection 7.2.

### 3.4 Best responses

From the perspective of agent \( i \), the auction defines a residual supply curve, \( z_{RSi}(p) \), specifying the number of units of the underlying asset that \( i \) is able to trade at price \( p \). This is the negative of the sum of all other agents’ bid curves:

\[ z_{RSi}(p) = - \sum_{j \neq i} z_{Bj}(p; y_{Dj}, y_{Cj}) \]
In equilibrium, given my assumptions on agents’ utility functions, residual supply functions will be affine with a fixed slope:

\[ z_{RS_i}(p, \eta_i) = d_i (p - \pi) + \eta_i \]  

(4)

where \( \eta_i \) is a random term which captures uncertainty in other agents’ submitted bid curves. I adopt the standard solution concept of \textit{ex-post optimal bidding}, with respect to \( \eta_i \). If agent \( i \) submits bid curve \( z_{Bi}(p; y_{Di}, y_{Ci}) \) and residual supply is \( z_{RS_i}(p) \), define the price which equates residual supply to \( i \)’s bid as:

\[ p(z_{Bi}(p; y_{Di}, y_{Ci}), z_{RS_i}(p, \eta)) \equiv \{ p: z_{Bi}(p; y_{Di}, y_{Ci}) = z_{RS_i}(p, \eta) \} \]  

(5)

**Definition 1.** Bid curve \( z_{Bi}(p; y_{Di}, y_{Ci}) \) is \textit{ex-post optimal} with respect to the residual supply function (4) if \( z_{Bi}(p; y_{Di}, y_{Ci}) \) implements the optimal point on the residual supply \( z_{RS_i}(p, \eta_i) \) for every realization of \( \eta_i \); that is,

\[ p(z_{Bi}(p; y_{Di}, y_{Ci}), z_{RS_i}(p, \eta)) = \arg \max_p U_i(z_{RS_i}(p, \eta_i), p; y_{Di}, y_{Ci}) \quad \forall \eta_i \]

As is known in the literature on multi-unit double auctions with quadratic utility (Klemperer and Meyer, 1989; Vayanos, 1999; Rostek and Weretka, 2015; Du and Zhu, 2012), the unique ex-post optimal bid curve of agent \( i \) is linear, and depends only on the slope of residual supply \( d_i \), independent of the distribution of \( \eta_i \).

**Proposition 1.** If agent \( i \) has demand shock \( y_{Di} \) and contract position \( y_{Ci} \), and the slope of residual supply is \( d_i \), then agent \( i \)’s unique ex-post optimal bid curve is:

\[ z_{Bi}(p; y_{Di}, y_{Ci}) = \frac{d_i}{\kappa_i + d_i} y_{Di} + \frac{\kappa_i}{\kappa_i + d_i} y_{Ci} - \frac{\kappa_i d_i}{\kappa_i + d_i} (p - \pi) \]  

(6)

Proposition 1 shows that \( i \)’s best-response bid curve is increasing in both \( y_{Di} \) and \( y_{Ci} \). When \( y_{Di} \) is high, \( i \)’s fundamental utility for the underlying asset is higher, and when \( y_{Ci} \) is higher, \( i \) wants the benchmark price \( p_B \) to be higher so that her contract payoffs \( p_B y_{Ci} \) increase; both forces cause \( i \) to increase her bid curve to purchase more of the underlying asset. However, the coefficients on \( y_{Di} \) and \( y_{Ci} \) differ: when the slope of residual supply \( d_i \) grows large, the coefficient on \( y_{Di} \) approaches 1, whereas the
coefficient on $y_{Ci}$ approaches 0. Intuitively, this is because bidding to move prices is only effective for $i$ when the slope of residual supply $d_i$ is small relative to $i$’s demand slope $\kappa_i$; that is, when $i$’s marginal purchases of the underlying asset increase benchmark prices faster than they decrease $i$’s marginal utility for the underlying asset. In the limit as $d_i$ approaches infinity, $i$’s trades do not move prices, so $i$ cannot affect her contract payoffs, and thus $y_{Ci}$ has no effect on $i$’s optimal bid curve.

Expression (6) implies that, in the context of the model, fundamental-driven bidding and manipulation-driven bidding are conceptually distinguishable: demand shocks $y_{Di}$ and contract positions $y_{Ci}$ show up as separate terms in agents’ optimal bid curves. One could thus define manipulation, in the context of the model, as the effect that contract positions $y_{Ci}$ have on agents’ bidding decisions. This definition suggests a potential behavioral restriction to combat manipulation. Regulators could require that agents’ trading decisions are driven only by fundamental shocks and not by contract positions: formally, agents would be required to set the coefficient on $y_{Ci}$ in expression (6) to 0, instead of submitting the profit-maximizing bid curve of expression (6).

While conceptually sound, such an approach is difficult to implement empirically. If the regulator only observes agents’ bid curves and contract positions, without further restrictions on primitives, it is impossible to empirically determine the coefficient on $y_{Ci}$ in expression (6). This is because contract positions and demand shocks have, up to scale, identical effects on agents’ bidding behavior, vertically shifting agents’ optimal bid curves. If a regulator argues that a given trader is manipulating, based on the fact that $y_{Ci}$ is large and the agent is buying large quantities of the underlying asset, the trader could always argue that her purchases were driven not by her contract positions, but by a large fundamental demand shock $y_{Di}$ for the underlying asset. Without directly observing $y_{Di}$, it is impossible to distinguish between these two explanations of the trader’s behavior.\footnote{This example somewhat resembles the cotton manipulation case referenced in footnote 28: the alleged cotton manipulator essentially claimed that she was not attempting to manipulate, but simply had a fundamental commercial reason to buy a large quantity of cotton.}

Expression (6) implies, however, that it is possible to estimate any given agent’s incentives to manipulate, under the assumption of profit-maximizing trading behavior. A natural measure of agent $i$’s manipulation incentives is the coefficient $\frac{\kappa_i}{\kappa_i + d_i}$ on $y_{Ci}$; I will call this $i$’s \textit{manipulation coefficient}. This coefficient measures the pass-through of contract
positions into i’s optimal bid curve: how many additional units of the underlying asset i bids for when her contract position is increased by one unit. Agent i’s manipulation coefficient is a function of her demand slope, κ_i, and the slope of residual supply that she faces, d_i, both of which can in principle be estimated by regulators. These estimates can be used to structurally regulate market manipulation, taking actions to influence market structure so that manipulation incentives and manipulation-induced distortions are small. I will show how to empirically implement this approach in section 4 below.

3.5 Manipulation incentives in equilibrium

The previous subsection characterizes manipulation incentives in terms of the slope of residual supply, d_i, which is an equilibrium object; this subsection characterizes equilibrium manipulation incentives as a function of model primitives. Following the literature, I define a linear ex-post equilibrium as a collection of bid curves for all agents, such that each agent is best-responding to the residual supply functions induced by other agents’ bid curves.

**Definition 2.** Bid curves z_{Bi} (p; y_{Di}, y_{Ci}) constitute a linear ex-post equilibrium if:

1. As in (6), each agent’s bid curve is an ex-post best response given the slope of residual supply d_i she faces.

2. The slope of residual supply facing agent i is the sum over all other agents’ bid slopes:

   \[ d_i \equiv z'_{RSi}(p) = - \sum_{j \neq i} z'_{Bj}(p; y_{Dj}, y_{Cj}) \]  

   Du and Zhu (2012), and Malamud and Rostek (2017) in a more general model, show that there is a unique linear ex-post equilibrium for any collection of demand slopes κ_1 . . . κ_N, characterized by a unique collection of bid slopes b_i and residual supply slopes d_i that satisfy definition 2. While their models do not include derivative contracts, their proofs immediately extend to my model, leading to the following proposition.
Proposition 2. There is a unique linear ex-post equilibrium in the asymmetric model, in which $i$ submits the bid curve:

$$z_{Bi}(p; y_{Ci}, y_{Di}) = \frac{b_i}{\kappa_i} y_{Di} + \frac{b_i}{\sum_{j \neq i} b_j} y_{Ci} - b_i (p - \pi)$$  \hspace{1cm} (8)$$

and the unique equilibrium price is

$$p_B - \pi = \frac{1}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} \left( \frac{b_i}{\kappa_i} y_{Di} + \frac{b_i}{\sum_{j \neq i} b_j} y_{Ci} \right) \right]$$  \hspace{1cm} (9)$$

Where bid slopes $b_i$ satisfy:

$$b_i = \frac{B + 2\kappa_i - \sqrt{B^2 + 4\kappa_i^2}}{2}$$  \hspace{1cm} (10)$$

and $B = \sum_{i=1}^{n} b_i$ is the unique positive solution to the equation

$$B = \sum_{i=1}^{n} \frac{2\kappa_i + B - \sqrt{B^2 + 4\kappa_i^2}}{2}$$  \hspace{1cm} (11)$$

Proposition 2 shows that, in equilibrium, the manipulation coefficient of agent $i$ is

$$\frac{b_i}{\sum_{j \neq i} b_j}$$

This is a function of bid slopes, which are functions of demand slopes $\kappa_i$ through expressions (10) and (11). From (10), $b_i$ is an increasing function of $\kappa_i$ fixing $B$, thus manipulation coefficients are larger for agents with higher demand slopes.

Appendix A.3 shows that only the relative sizes of agents’ demand slopes matter: multiplying all agents’ demand slopes by a constant factor does not affect manipulation coefficients. Thus, manipulation incentives depend on competitiveness, the relative sizes of agents’ demand slopes, rather than liquidity, the absolute size of demand slopes. An asset can be very liquid, with low absolute trading costs, but manipulation incentives will still be high if one agent controls most of the market’s capacity to trade the underlying asset. Conversely, absolute trading costs can be very high without creating large manipulation.
incentives, as long as the market’s capacity to trade the underlying asset is divided among many agents.

If the market is fairly competitive, we can obtain a simple approximation to agents’ optimal bid curves. Define agent i’s capacity share $s_i$ as i’s demand slope $\kappa_i$ divided by the sum of all agents’ demand slopes, that is:

$$s_i \equiv \frac{\kappa_i}{\sum_{i=1}^{n} \kappa_i}$$

(12)

In words, agent i’s capacity share $s_i$ is defined as i’s demand slope divided by the sum of all agents’ demand slopes. Define the largest capacity share across agents, $s_{\text{max}}$, as:

$$s_{\text{max}} \equiv \max_i s_i$$

Proposition 3. i’s equilibrium manipulation coefficient satisfies:

$$s_i \leq b_i \leq \left(1 + \frac{s_{\text{max}}}{1 - 2s_{\text{max}}} \right) s_i$$

Proposition 3 implies that i’s manipulation coefficient is within a factor 

$$\left(1 + \frac{s_{\text{max}}}{1 - 2s_{\text{max}}} \right)$$

of her capacity share, $s_i$; thus, when $s_{\text{max}}$ is small, manipulation coefficients are approximately equal to capacity shares. This implies that manipulation incentives are small when markets are competitive and agents’ capacity shares are low. For example, if all n agents have identical slopes of demand, proposition 3 implies that manipulation coefficients are approximately $\frac{1}{n}$, so each contract position induces only approximately $\frac{1}{n}$ units of trade in the underlying asset. Thus, in competitive markets, agents’ contract positions can be quite large – order $\frac{1}{s_{\text{max}}}$ times as large as the volume of the underlying asset that they trade – before manipulation incentives begin to dominate agents’ trading decisions.

In the following section, I show how to take the model to data to predict the size of manipulation-induced distortions to price benchmarks.
4 Empirical implementation

This section shows how to empirically measure manipulation risk. In subsection 4.1, I describe how, if we can measure agents’ demand slopes and contract positions, we can measure manipulation-induced bias and variance in benchmarks. In subsection 4.2, I describe how we can construct a simpler index of manipulation risk using only data on the total volume of derivative contracts and the total volume of trade in underlying assets used to construct benchmarks.

In order to map the model to data, I extend the baseline model to a time-series context. Assume that $\pi_t$ is a martingale, so that $E[\pi_t - \pi_{t-1}] = 0$. Assume that demand shocks $y_{Di}$ and $y_{Ci}$ are independent over time, across and within agents, though their distributions can be arbitrary. The set of $n$ agents participating in benchmark setting, and their demand slopes $\kappa_1 \ldots \kappa_n$, are fixed over time. Given these assumptions, expression (9) implies that the benchmark price $p_{Bt}$ will be:

$$p_{Bt} - \pi_t = \frac{1}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} \left( \frac{b_i}{\kappa_i} (y_{Di}) + \frac{b_i}{\sum_{j \neq i} b_j} y_{Ci} \right) \right]$$

(13)

4.1 Measuring manipulation-induced bias and variance

Manipulation causes two kinds of distortions in benchmarks: benchmark prices are biased relative to average values $\pi_t$, and benchmark prices have additional variance around $\pi_t$. To measure manipulation-induced benchmark bias, take expectations over $y_{Di}$, $y_{Ci}$ in (13). Using the normalization made in subsection 3.1 that $\sum_{i=1}^{n} E[y_{Di}]=0$, we have:

$$E[p_{Bt} - \pi_t] = \frac{\sum_{i=1}^{n} \frac{b_i}{\kappa_i} E[y_{Di}] + \sum_{j \neq i} \frac{b_i}{b_j} E[y_{Ci}]}{\sum_{i=1}^{n} b_i}$$

(14)

Thus, the contribution of contract positions to benchmark bias is:

$$\frac{1}{\sum_{i=1}^{n} b_i} \left( \sum_{i=1}^{n} \frac{b_i}{\sum_{j \neq i} b_j} E[y_{Ci}] \right)$$

(15)
Expression (15) says that the net effect of manipulation on benchmark prices depends on the sum of expected contract positions, weighted by manipulation coefficients, across agents. Agents with higher demand slopes have higher manipulation coefficients and manipulate more per unit contract that they hold; thus, even if expected contract positions sum to 0 across agents, if contract positions are correlated with agents’ demand slopes, benchmark prices will be biased.

If market participants enter into contract settlement with \textit{ex ante} uncertain contract positions, manipulation will also add variance to benchmark prices, making contracts less effective for hedging against uncertainty in economic fundamentals. We can write:

\[ p_{\text{B}_t} - p_{\text{B}_{t-1}} = \pi_t - \pi_{t-1} + \frac{1}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} b_i \left( \frac{y_{D_i,t} - y_{D_i,t-1}}{\kappa_i} + \frac{b_i (y_{C_i,t} - y_{C_i,t-1})}{\sum_{j\neq i} b_j} \right) \right] \]

(16)

We can thus decompose the variance of benchmark price innovations, \( p_{\text{B}_t} - p_{\text{B}_{t-1}} \), into a component attributable to shifts in fundamentals, \( \pi_t - \pi_{t-1} \), and noise created by demand shocks and contract positions:

\[ \text{Var} (p_{\text{B}_t} - p_{\text{B}_{t-1}}) = \text{Var} (\pi_t - \pi_{t-1}) + \frac{2}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} b_i \text{Var} (y_{D_i,t}) + \frac{b_i}{\sum_{j\neq i} b_j} \text{Var} (y_{C_i,t}) \right] \]

If agents are holding derivative contracts linked to \( p_{\text{B}_t} \) to hedge against changes in \( \pi_t \), expression (16) shows that agents are exposed to basis risk: \( p_{\text{B}_t} \) does not track \( \pi_t \) exactly over time, because of demand shocks and contract positions. As a measure of how much manipulation increases basis risk, we can define the manipulation variance ratio:

\[ \text{MVR} = \frac{\sum_{i=1}^{n} b_i \sum_{i=1}^{n} \frac{b_i \text{Var} (y_{C_i,t})}{\sum_{j\neq i} b_j}}{\mathbb{E} \left[ (p_{\text{B}_t} - p_{\text{B}_{t-1}})^2 \right]} \]

(17)

Intuitively, MVR measures what fraction of the variance in time-series innovations in price benchmarks is attributable to manipulation. If MVR is small, manipulation does not substantially increase basis risk, so price benchmarks are useful for hedging as long


\[ ^{34} \text{Section 4 of the online appendix formalizes this, in a model where agents endogenously choose their derivative contract positions to hedge against exogenously determined exposures to } \pi_t. \]
as basis risk from demand shocks is also low. If MVR is close to 1, most of the variation in \( p_{B_t} \) over time is caused by manipulation rather than innovations in \( \pi_t \), so derivative contracts are not useful for hedging against changes in fundamentals.

Expressions (15) and (17) can be estimated in data. We need to observe contract positions \( y_{Ci,t} \) over time, to estimate \( \text{Var}(y_{Di,t}) \) for each agent, and benchmark prices \( p_{B_t} \) to estimate the variance of price innovations, \( \mathbb{E} \left[ (p_{B_t} - p_{B,t-1})^2 \right] \); both contract positions and benchmark prices are commonly observed by regulators. What is harder is that we need to observe either bid slopes \( b_i \) or demand slopes \( \kappa_i \). If auction data is available, bid slopes can be estimated and plugged into expression (17) directly. Alternatively, it may be possible to estimate demand slopes using structural models of traders’ behavior, or supply shock instruments.

### 4.2 The manipulation index

An alternative way to measure manipulation risk is to ask whether manipulation-induced variance is large, relative to non-fundamental noise in benchmarks generated by demand shocks. First, analogously to expression (17), define the demand variance ratio as:

\[
DVR = \frac{\sum_{i=1}^{n} b_i \text{Var}(y_{Di,t})}{\mathbb{E} \left[ (p_{B_t} - p_{B,t-1})^2 \right]} \tag{18}
\]

We can define the manipulation-demand variance ratio as:

\[
\text{MDVR} = \frac{\text{MVR}}{\text{MVR} + \text{DVR}} = \frac{\sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var}(y_{Di}) + \sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var}(y_{Ci})}{\sum_{i=1}^{n} \left( \frac{b_i}{\kappa_i} \right)^2 \text{Var}(y_{Di}) + \sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var}(y_{Ci})} \tag{19}
\]

Expression (19) implies that:

\[
\text{MVR} = \left( \frac{\text{MDVR}}{1 - \text{MDVR}} \right) (\text{DVR}) \tag{20}
\]
Thus, MVR is low as long as both DVR and MDVR are low. MDVR measures what fraction of total noise in benchmarks is attributable to manipulation; if MDVR is low, then most benchmark noise is coming from demand shocks, which would exist even in the absence of derivative contract positions and manipulation.

Under some conditions, there is a simple formula which provides an approximate upper bound for MDVR. I impose two additional assumptions about the distributions of demand shocks \( y_{Di} \) and contract positions \( y_{Ci} \).

**Assumption 1.** Demand shocks \( y_{Di} \) and contract positions \( y_{Ci} \) are normally distributed:

\[
y_{Di} \sim N\left(0, \sigma_{Di}^2\right), \ y_{Ci} \sim N\left(0, \sigma_{Ci}^2\right)
\]

Assuming that contract positions are mean 0 rules out the possibility that market participants hold systematically long or short contract positions, and assuming that demand shocks are mean 0 implies that market participants are equally likely to be buyers or sellers of the underlying asset.

**Assumption 2.** The variance of demand shocks and the variance of contract positions are proportional across agents:

\[
\sigma_{Di}^2 \propto \sigma_{Ci}^2
\]

Assumption 2 can be interpreted as saying that agents are primarily differentiated by size: larger agents both hold larger contract positions and receive proportionally larger demand shocks.

Now, define \( v_i \) as \( i \)'s unsigned volume of trade in the underlying asset in the benchmark-setting auction, that is,

\[
v_i = |z_{Bi}(p_B; y_{Ci}, y_{Di})|
\]

Define the total unsigned trade volume summed across all agents as:

\[
V = \sum_{i=1}^{n} v_i
\]
and the total unsigned volume of contracts, across all agents, as:

\[ C = \sum_{i=1}^{n} |y_{Ci}| \]

**Proposition 4.** Suppose assumptions [1] and [2] hold, and \( s_{\text{max}} \) is small. An approximate upper bound for MDVR is:

\[ \left( \frac{E[C] s_{\text{max}}}{E[V]} \right)^2 \quad (21) \]

We can construct a simple index of manipulation risk by taking the square root of expression (21).

**Definition 3.** Define the manipulation index, MI, as:

\[ MI = \frac{E[C] s_{\text{max}}}{E[V]} \quad (22) \]

If \( s_{\text{max}} \) is small and assumptions [1] and [2] hold, the square of the manipulation index is an approximate upper bound for MDVR. Thus, if MI is smaller than approximately 0.7, so that \( MI^2 \) is below 0.5, then manipulation-induced benchmark variance is likely to be smaller than demand shock-induced benchmark variance, so total noise in the price benchmark is at most twice as large as it would be if agents did not hold any derivative contract positions tied to price benchmarks. This leads to a simple rule-of-thumb for evaluating whether a contract market may be vulnerable to manipulation: test whether

\[ MI > \sqrt{0.5} \]

A simple approximation is to test whether MI is greater than 0.7.

MI is simpler to estimate than MVR. Expression (22) involves only total contract volume and total trade volume in the underlying asset, both of which are tracked closely in many contract markets, and the maximum capacity share \( s_{\text{max}} \). While estimating agents’ capacity shares exactly requires estimating agents’ demand slopes, if a regulator observes some proxy metric \( x_{i} \), and is willing to assume that \( x_{i} \) is proportional to agents’ demand
slopes $\kappa_i$, agents’ capacity shares can be estimated as:

$$s_i \approx \frac{x_i}{\sum_{j=1}^{n} x_j}$$

(23)

Some examples of variables that may be reasonable proxy variables include agents’ total contract position size, trade volume, or even variables outside of the model such as agents’ risk limits or total assets under management.

The manipulation index is a crude measure of manipulation risk: it relies on a number of strong assumptions, and it bounds MDVR, rather than MVR. The main benefit of the manipulation index is that it can be calculated using very limited data. It can thus be used as a first-pass metric, similar to the Herfindahl-Hirschman Index (HHI) in antitrust regulation, to identify contract markets which may be vulnerable to manipulation; better data can then be collected, and the more precise methods described in subsection 4.1 can be used to estimate manipulation-induced benchmark distortions.

5 The LBMA Gold Price

5.1 Background

In this section, I will estimate demand slopes of participants in the auctions used to set the LBMA gold price. I then evaluate whether COMEX gold futures contracts could be cash-settled using the LBMA gold price without high manipulation risk. The LBMA gold price is a benchmark price for gold, set twice each business day at 10:30AM and 3:00PM London time. It is used to settle a variety of contracts; while the popular COMEX gold futures contract (ticker symbol GC) is physically settled, other exchanges have introduced cash-settled gold futures and options contracts tied to the LBMA gold price, and some OTC gold derivatives are settled using the LBMA gold price \(\text{[FCA] 2014}\). The LBMA gold price is also used by other market participants, such as miners, central banks, and jewellers, for purposes such as inventory valuation \(\text{[Aspris et al.] 2015}\).

Prior to 2015, the price was set in a private teleconference auction between five members; this took between 10-15 minutes to conclude, and data on auction bidding...
was not made public. In 2014, the five banks involved in setting the LBMA gold price were accused of manipulation \cite{ReutersStaff2014}; the UK Financial Conduct Authority fined Barclays $43.9 million for bidding strategically to move benchmark prices, in order to avoid paying USD $3.9m to a customer who held a benchmark-linked option contract \cite{FCA2014}. In 2015, the ICE Benchmark Administration (IBA) took over the administration of the LBMA gold price; IBA moved to an electronic auction system, allowing more participants to enter, and began publishing detailed information about bids in intermediate rounds of the auction.

5.2 Data

The LBMA gold price is determined in a multi-round dynamic auction. During each round, IBA publishes a round price, and participants have 30 seconds to enter how much gold they want to buy or sell at the announced price. If the difference between buying and selling is within an imbalance threshold – during the time period that my data covers, usually 10,000oz – the auction concludes. Otherwise, IBA adjusts the price in the direction of the difference between total buy and sell volume, and a new round begins. The primary data source I use is the IBA Gold Auction Historical Transparency Reports.\footnote{36 I accessed the data at the \url{ICE website}} The data covers daily morning and afternoon auctions; the full dataset covers 1650 auctions over the period 2015-03-20 to 2018-06-29.

Let auctions be indexed by $a \in \{1 \ldots A\}$, and suppose that auction $a$ lasts for $R_a$ rounds, indexed by $r$. For each round $r$ of each auction $a$, I observe the number of participants, $n_{ar}$, the round price, $p_{ar}$, and the total volume of gold that auction participants wish to buy and sell, respectively $b_{ar}$ and $b_{ar}$. For estimating my model, I filter to auctions with at least 3 rounds, $R_a \geq 3$, as I will estimate slopes of demand by regressing round buy and sell volume on round prices. I also filter to auctions in which the number of participants $n_{ar}$ is constant over the course of the auction. This reduces the estimation sample to 509 auctions. Table 1 shows features of my sample and the full dataset. Most auctions have 6-9 participants and conclude in 3-6 rounds. The range of prices between rounds for any given auction is small, relative to variation in gold prices between auctions: the difference between the highest and lowest round prices is around $1 USD/oz on average.
Approximately 168,000oz of gold is traded on average in each auction in my estimation sample.

Define the *volume imbalance* in round $r$ of auction $a$ as the difference between buy and sell volume, that is:

$$i_{ar} \equiv b_{ar} - s_{ar}$$

The auction clearing price, buy and sell volume, and volume imbalance at the final round $R_a$ of auction $a$ are:

$$p_{aR_a}, b_{aR_a}, s_{aR_a}, i_{aR_a}$$

Define the total trade volume at the end of the auction as the sum of buy and sell volume:

$$v_{aR_a} = b_{aR_a} + s_{aR_a}$$

For simplicity, when discussing final-round quantities, I will omit the round subscript $R_a$; thus, I will write $p_a, v_a, i_a$ to mean the final auction price, trade volume, and volume imbalance in auction $a$. Let $n_a$ represent the number of participants in auction $a$; this is uniquely defined within my estimation subsample, since I filter to auctions in which participation is constant.

Figure 1 shows a plot of volume imbalance against prices; each data point is an auction round. The $x$-axis shows $p_{ar} - p_{aR_a}$, the difference between auction prices in round $r$ and in the final round $R_a$ of auction $a$. The $y$-axis shows $i_{ar}$, the difference between buy and sell volume in round $r$ of auction $a$. There is a clear decreasing trend: when the auction round price is higher, auction participants want to sell more and buy less. The colored lines in the left panel of figure 1 are linear regression lines, separately estimated for auctions with different numbers of participants $n_a$. Auctions with larger numbers of participants generally have higher aggregate bid slopes, although the slope when $n_a = 9$ is slightly lower than the slope when $n_a = 8$. Thus, the slope of aggregate auction demand appears to be an increasing but concave function of $n_a$. This suggests that marginal participants add to the aggregate demand, but marginal participants contribute less for higher values of $n_a$.

The right panel shows total auction volume versus the number of bidders. Volume increases approximately linearly in the number of bidders; this suggests that, while later
participants may contribute little to the aggregate bid slope, they have demand shocks of similar size to earlier participants. Appendix 2.1 shows that these stylized facts about the relationship between aggregate bid slope, auction volume, and participant count are robust to controlling for various other observable features of auctions. These are the core stylized facts that allow me to identify parameters in the model: I will choose agents’ demand slopes to match the observed aggregate bid slopes, and I will choose agents’ demand shock variances to match observed trade volume.

[Table 1 about here.]

[Figure 1 about here.]

5.3 Parametrization and estimation

I assume that the observed auction data is generated by a fixed set of $N = 11$ participants, with demand slopes $\kappa_i$ which are fixed over time and commonly known to all agents. In each auction, agent $i$ receives an i.i.d. mean-0 normal demand shock with variance $\sigma_{Di}^2$, which is also fixed over time. These assumptions are plausible for the LBMA gold price, as only a small number of agents are allowed to participate in LBMA gold auctions, and their relative sizes are likely to move slowly over time.

While estimating the model, I assume agents are holding no contract positions, so I attribute all trade volume in the auction to demand shocks. This is immaterial for the primary goal of the counterfactual, which is to calculate how much benchmark variance would increase if agents held additional cash-settled derivative contracts tied to the LBMA gold price; the magnitude of this increase only depends on agents’ demand slopes, so it does not matter whether existing trade volume in the auction arises from demand shocks or contract positions. Existing contract positions will, however, change the interpretation of the demand variance ratio, DVR; we can think of my estimates of demand shock-induced variance as encompassing both true shocks to agents’ marginal utility and manipulation incentives from existing contract positions.

I assume that the number of participants $n_a$ in each auction $a$ is exogenously determined and known to all participants prior to bidding. Thus, in an auction with $n_a$

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37 A list of participants is available under the “Current Auction Participants” heading at the ICE website.
participants, if the auction mechanism were a static auction game, agent $i$ would submit the following bid curve:

$$z_{Bi}(p_{ar}, y_{Cai}, y_{Dai}) = \frac{b_{ai}}{\kappa_i} y_{Dai} + \frac{b_{ai}}{\sum_{j \neq i} b_{aj}} y_{Cai} - b_{ai} (p_a - \pi_a)$$

(24)

where the bid slopes $(b_{a1} \ldots b_{an_a})$ satisfy expressions (10) and (11) of proposition 2. The LBMA gold auction, however, is a dynamic game, in which agents are submit bids for multiple prices until a approximate market-clearing price is found. I will simply assume that agents simply bid their static equilibrium bid curves in each auction round: that is, in an auction round with price $p_{ar}$, agent $i$ bids $z_{Bi}(p_{ar}, y_{Cai}, y_{Dai})$. In section 2.2 of the online appendix, I construct a stylized model of the dynamic auction game, and show that these bidding strategies constitute an equilibrium in the dynamic auction game. Given this assumption, in round $r$ of auction $a$, the aggregate volume imbalance will be:

$$i_{ar} = \sum_{i=1}^{n} z_{Bi}(p_{ar}, y_{Dai}) = \sum_{i=1}^{n} \left[ \frac{b_{ai}}{\kappa_i} y_{Dai} - (p_{ar} - \pi_a) b_{ai} \right]$$

(25)

This implies that we can measure the aggregate bid slope, $B_a \equiv \sum_{i=1}^{n} b_{ai}$, using any auction $a$ with more than one round, since for any two rounds $r$ and $\tilde{r}$, we have:

$$\frac{i_{ar} - i_{a\tilde{r}}}{p_{ar} - p_{a\tilde{r}}} = -\sum_{i=1}^{n} b_{ai} = -B_a$$

In any auction with more than two rounds, the aggregate bid slope is overidentified. I construct an estimate the slope of demand $\hat{B}_a$ for auction $a$ as the coefficient from regressing volume imbalance on prices:

$$\hat{B}_a \equiv -\frac{\sum_{r=1}^{R_a} (i_{ar} - \bar{i}_a) (p_{ar} - \bar{p}_a)}{\sum_{r=1}^{R_a} (p_{ar} - \bar{p}_a)}$$

(26)

where $\bar{i}_a$ and $\bar{p}_a$ are the averages of $i_{ar}$ and $p_{ar}$, respectively, within auction $a$.

I choose agents’ demand slopes $\kappa_i$ and demand shock variances $\sigma_{di}^2$ to match the estimated aggregate bid slopes $\hat{B}_a$ and trade volumes $v_a$. Since I observe only aggregate
bid slopes and trade volumes, not participant-level data, I need to impose two simplifying assumptions to estimate the model. First, I assume that agents participate in order: every auction with \( n_a \) participants involves agents \( i = 1 \ldots n_a \). This essentially implies that the demand slope of the 7th agent, \( \kappa_7 \), is identified by the difference between the aggregate bid slopes for auctions with 6 and 7 participants; likewise \( \sigma^2_{d7} \) is identified by the difference between average trade volume for auctions with 6 and 7 participants. Even under this assumption, the model is still underidentified, as I do not observe auctions with \( n_a < 4 \), so I have three fewer moments than parameters for both \( \kappa_i \) and \( \sigma^2_{Di} \). Thus, the second simplifying assumption I make is that the demand slopes and demand shock variances of agents \( i = 1 \ldots 4 \) are equal. In addition, I assume that \( \log (\kappa_i) \) and \( \log (\sigma^2_{Di}) \) are second-order polynomials in \( i \), and I estimate polynomial coefficients using GMM. These assumptions are clearly unrealistic, but are needed given the limitations of my data. As a robustness check, in section 2.4 of the online appendix, I assume two alternative functional forms for estimating demand slopes and demand shock variances, and show that manipulation-induced benchmark variance remains small.

I fit the model by choosing the polynomial coefficients parametrizing agents’ demand slopes \( \kappa_i \) and demand shock variances \( \sigma^2_{Di} \) to match the relationship between the number of auction participants \( n_a \), estimated demand slopes \( \hat{B}_a \), and auction trade volume \( v_a \). Details of the moment matching procedure are described in appendix C.1. Figure 2 shows that the fitted model is able to match both sets of input moments fairly well. The left panel of figure 2 shows the average auction slope \( \hat{B}_a \) from the data for each \( n_a \), along with predicted bid slopes from the estimated model. The blue line in the left panel shows that, as noted in the left panel of figure 1, the estimated aggregate bid slope is an increasing but concave function of the number of participants. The mean estimated bid slope actually begins to decrease for \( n_a > 8 \), which cannot be rationalized in my model. However, the model-predicted values of \( \hat{B}_a \) lie within the 95% pointwise confidence interval for the mean of \( \hat{B}_a \) in the data. The right panel of figure 2 shows average actual volume \( v_a \) for each \( n_a \), together with volume predicted from the estimated model; predicted volume also falls within pointwise 95% confidence intervals for observed trade volume in the data.

[Figure 2 about here.]
5.4 Manipulation risk with cash-settled gold futures

Using the estimated parameters of the model, we can predict how much manipulation would distort the LBMA gold price, if agents held cash-settled contract positions settled using the LBMA gold price, and bid optimally to maximize profits from both the auction and their derivative contract positions. I assume that all agents hold monthly cash-settled contracts which are normally distributed with mean 0 and standard deviation 150,000oz, equal to half the COMEX gold futures position limit. These are fairly large contract positions; the average volume of trade among all agents in any given auction is only approximately 168,000oz, so the total counterfactual size of contract positions, summed across all agents, is approximately 4-10 times larger than the average volume of gold traded in the auction. I calculate the size of gold price innovations in the data as the standard deviation of monthly auction price differences; this works out to $47.69/oz. Using the estimated model parameters, I can then calculate the quantities MVR, DVR, and MDVR defined in section 4; details of these calculations are described in appendix C.3.

Table 2 shows the results. The second column of table 2 shows that the standard deviation of shocks to benchmark prices induced by manipulation ranges from $1.76/oz to $3.23/oz. This is a very small fraction of monthly gold price innovations; column 3 shows that MVR ranges from 0.0014 to 0.0046. Column 4 calculates the standard deviation of benchmark price shocks induced by agents’ demand shocks; this is similarly small, ranging from $1.95/oz to $2.21/oz USD/oz; as a result, DVR is also very low, ranging from 0.0016-0.0021. Manipulation-induced variance is similar in magnitude to demand-shock induced variance: column 5 shows that the ratio MDVR ranges from 0.40-0.68.

Methodologically, this section demonstrates how auction bidding data can be used to my proposed measures of manipulation risk. Practically, my analysis suggests that the LBMA gold price could be used to settle gold futures contracts without creating high manipulation risk.38

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38My analysis may in fact overestimate manipulation risk. If a particular auction each month were used for contract settlement, this would attract additional entry into the auction, both from manipulators attempting to profit by moving prices, and from non-manipulative arbitrageurs who profit from arbitrage when manipulators move prices; this would increase competition and decrease manipulation risk further. I analyze a simple model demonstrating that manipulation creates incentives for entry in section 6 of the
6 Evidence from other contract markets

In this section, I estimate the manipulation index and apply my rule-of-thumb test to three cash-settled derivative contract markets: ICE Brent crude futures, CME feeder cattle futures, and ICE HSC basis futures. I show that, under current contract position limits, the Brent crude futures and CME feeder cattle contracts pass my rule-of-thumb test, but the ICE HSC basis contract fails the test, suggesting that it is potentially vulnerable to manipulation. These predictions appear to be consistent with the data: the HSC basis contract has been the subject of a high-profile manipulation case, whereas the Brent crude and feeder cattle contracts have not. Finally, in subsection 6.3, I use the manipulation index to determine how large position limits for SOFR-linked interest rate derivatives could be without creating high risk of manipulation.

6.1 The CFTC Commitments of Traders reports

To measure contract market competitiveness, I use data from the CFTC’s weekly Commitments of Traders reports on commodity futures from 2010-2016. I focus on the 10 largest contracts by open interest in each contract category in the data. Appendix D.1 describes details of my data cleaning process, and Table 3 reports results. Section 3.1 of the online appendix reports analogous results for financial derivatives.

The “Top 4” columns in table 3 show the shares of long and short contract open interest held by the 4 largest market participants in each market. Contract market competitiveness varies substantially across contract markets. The top 4 share is in the range of 15-30% for agricultural products, compared to around 30-60% for energy products, dairy products, and emissions permits. If we assume that the share of the largest agent is approximately half the average long and short top 4 share, the largest agent seems to account for between...
10-30% of total contract positions. What matters for manipulation incentives, however, is agents’ capacity shares, not their shares of contract positions. As discussed in subsection 4.2 we can calculate agents’ capacity shares using their contract position shares as proxy variables if we are willing to assume that contract position sizes are proportional to agents’ demand slopes. Under this assumption, the COT data suggests that the max capacity share, $s_{\text{max}}$, ranges from approximately 7% to 30% across markets.

[Table 3 about here.]

### 6.2 Estimating the manipulation index

#### 6.2.1 ICE Brent Crude futures

From the CFTC’s COT data, the top 4 shares of total outstanding contracts for ICE Brent crude futures are approximately 44% on the long side and 51% on the short side; taking half the average of these top 4 shares, this suggests that the $s_{\text{max}}$ is in the ballpark of 24%. While these markets are fairly concentrated, the total volume of outstanding contracts for any given month is only around 200 million barrels, around 3 times larger than the volume of trade used to determine the Brent crude index. Moreover, ICE imposes fairly strict position limits on contract holders: during the last five days before contract expiration, each agents’ contract position is restricted to 6,000 contracts, or 6 million barrels of oil. In comparison, the Brent Crude index is based on trades of around 75 million barrels of physical oil. Suppose that there are 4 large oil traders, who hold contract positions approximately half the size of position limits; the manipulation index

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40 This is likely to be somewhat of an underestimate of $s_{\text{max}}$, since there may be many participants in contract markets who do not have substantial capacity to trade the underlying asset.

41 Table 3 also shows that the distribution of long and short open interest is not uniform among different kinds of market participants. In most agricultural, livestock and metals products – producers and merchants hold net short contract positions, and financial participants – swap dealers and managed money participants – are significantly net long. Producers and merchants regularly trade underlying assets for commercial purposes, so they are likely to have higher demand slopes than purely financial market participants; thus, demand slopes will be negative correlated with contract positions in these markets, so price benchmarks are likely to be biased downwards. The prediction that manipulators are likely to be commercial market participants may seem somewhat counterintuitive; section 3.2 of the online appendix provides some support for this prediction, showing that many recent cases of contract market manipulation involve commercial market participants.

42 See “Expiry Limits” on the [ICE Brent Crude Futures](https://www.ice.com) website.
is then:
\[
\frac{E[C] s_{max}}{E[V]} = \frac{(12 \text{ mil}) (0.24)}{(75 \text{ mil})} = 0.038
\]
This is very low, well below the rule-of-thumb cutoff of 0.7, so my model predicts that manipulation will add very little variance to the Brent crude benchmark. In support of this prediction, to my knowledge, there have been no successful lawsuits under US jurisdiction alleging manipulation of the Brent crude benchmark.

### 6.2.2 CME Feeder Cattle futures

An intermediate case is feeder cattle contracts. Using the COT reports, the CME feeder cattle contract market appears to be fairly competitive; computing \( s_{max} \) as half the average top 4 share, we get 7.34%. However, the total volume of outstanding contracts totals around 500 million pounds of cattle, when the total volume of cattle trades used to calculate the CME feeder cattle index is only around 10-20 million pounds. Contract position limits are also fairly large; each agent is allowed to hold 15 million pounds of cattle in contracts. If we assume there are roughly 10 large traders, who hold contracts approximately half the size of position limits, the manipulation index is:
\[
\frac{E[C] s_{max}}{E[V]} = \frac{(75 \text{ mil}) (0.0734)}{(15 \text{ mil})} = 0.367
\]
This is a nontrivially high value for the manipulation index, but is still smaller than the rule-of-thumb cutoff of 0.7, so my model suggests that manipulation risk in this market is not very large. Empirically, to my knowledge, there have been no successful manipulation lawsuits involving the CME feeder cattle contract. The contract was the subject of a 2006 manipulation lawsuit, brought by the CFTC against a cattle trader, Todd J. Delay; the CFTC ultimately lost the case, as the court found that there was insufficient evidence to conclude that Delay’s trades were illegitimate.

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43A class action suit was dismissed in 2017 on the basis of extraterritoriality, and the inability of plaintiffs to prove that they were harmed by alleged manipulation; see in re N. Sea Brent Crude Oil Futures Litig., 256 F. Supp. 3d 298 (S.D.N.Y. 2017)
44Contract position limits are available on the CME website.
45Commodity Futures Trading Commission v. Delay, 7:05CV5026 (D. Neb. Nov. 17, 2006). Another case occurred in 1991, when the feeder cattle contract was settled by physical delivery; the plaintiffs also lost. See Utesch v. Dittmer, 947 F.2d 321 (8th Cir. 1991)
6.2.3  ICE HSC Basis futures

The ICE Houston Ship Channel (HSC) basis futures contract market is fairly concentrated; using the CFTC COT reports, I estimate that $s_{\text{max}}$ is roughly 21.8%. The volume of outstanding contracts for any given month is also quite large, at around 75 million MMBtus for some delivery months, around 50 times larger than the volume of gas trades used to determine the benchmark. Position limits for HSC basis futures are quite lenient, at 13,900 contracts, equivalent to 34.7 million MMBtus of gas. If we assume that roughly 4 large traders hold contracts half the size of position limits, the manipulation index is:

$$
\frac{E\{C|s_{\text{max}}\}}{E\{V\}} = \frac{(87.4\text{mil})(0.218)}{(1.4\text{mil})} = 13.6
$$

This is well above the rule-of-thumb cutoff of 0.7, so my model suggests that the HSC basis futures contract is quite vulnerable to manipulation. In support of this prediction, in 2007 and 2009 respectively, the CFTC and the FERC fined Energy Transfer Partners (ETP) a total of $40 million for manipulation of the HSC index. In September 28, 2005, ETP built up a short HSC basis position of 23.8 Bcf (approximately 23 million MMBtu). HSC then sold approximately 10 million MMBtus of gas in a single day, in an attempt to lower the HSC index to profit from its contract position. The HSC index, which is usually within approximately 10 cents of gas prices in nearby locations, decreased to more than $1.29/MMBtu below nearby points; the HSC index was around $10 during this time period, so this is quite a large price change.

Together, these three case studies provide suggestive evidence that my proposed rule-of-thumb is able to identify contract markets which are vulnerable to manipulation.

6.3  Setting position limits for SOFR-linked interest rate derivatives

The Secured Overnight Financing Rate (SOFR) is an interest rate benchmark, established as an alternative to USD LIBOR, based on interest rates on overnight treasury-backed
repo transactions. We can use the manipulation index rule-of-thumb to determine how large market participants’ SOFR-linked derivative contract positions could be before manipulation risk becomes a concern. The total volume of repo transactions used in calculating SOFR, at the time of writing, was around $1 trillion daily.\textsuperscript{50} The treasury-backed tri-party/GCF repo market also seem to be fairly competitive, as the share of the top 3 dealers falls in the range of 20-45% from 2012 onwards. Suppose, then, that $s_{\text{max}}$ is approximately 0.2. We can calculate the value of dealers’ total contract positions which sets the manipulation index equal to the rule-of-thumb cutoff, by solving for $E[C]$ in:

$$\frac{E[C] s_{\text{max}}}{E[V]} = \frac{E[C] (0.2)}{(1000\text{bil})} = 0.7$$

We find that the total notional size of contract positions held by dealers can be at most $3.5$ trillion in notional value. Dividing this between roughly five large active dealers, this implies a position limit for each dealer of $700$ billion. It seems plausible that a position limit of this size could be imposed on market participants without substantially constraining their ability to engage in legitimate market activity.\textsuperscript{51}

\section{Discussion}

\subsection{Additional theoretical results}

The online appendix contains a number of additional theoretical extensions of my results, generalizing and extending some of the intuitions in the main text. Section 4 endogenizes agents’ contract positions, assuming that agents have exogeneously determined exposures to $\pi$, and hedge against uncertainty in $\pi$ by writing contracts based on auction prices. I show how manipulation decreases welfare by reducing the effectiveness of auction prices for hedging, as well as decreasing allocative efficiency in the auction. Section 5 shows that, if agents’ values are interdependent rather than private, agents will shade bids more

\textsuperscript{50}See the New York Fed’s Secured Overnight Financing Rate Data.

\textsuperscript{51}The total volume of outstanding LIBOR-linked USD interest rate derivatives is two orders of magnitude larger than this position limit, at $160$ trillion (\textit{Financial Stability Board} 2018), but this total sums across all tenors and expiration dates, and includes contract positions held by agents who do not participate in the underlying treasury-backed repo market.
in equilibrium; this decreases the slope of residual supply and increases manipulation incentives. However, as the size of contract positions increases, the extent of adverse selection decreases, increasing the slope of residual supply and decreasing manipulation incentives. The results of this section are related to a recent paper by Lee and Kyle (2018). Section 6 endogenizes agents’ decisions to participate in auctions; I show that increasing contract volume can in fact decrease benchmark variance by increasing auction entry, both from manipulators and non-manipulators. Section 7 shows that manipulation incentives are higher when agents collude, as colluding agents’ bids are influenced more by each unit of contract they collectively hold, since each bidder internalizes the effects of her price impact on all other bidders’ contract positions.

7.2 Mechanisms for benchmark setting

This paper uses uniform-price double auctions as a reduced-form model of price benchmarks. Some benchmarks, such as VIX and the LBMA gold price discussed in this paper, are determined using actual auctions. Derivative contracts for many equity indices are also settled based on exchange opening or closing auction prices. Other benchmark-setting mechanisms may produce outcomes similar to uniform-price double auctions. The WM/Reuters FX fixing and the ISDAFIX interest rate swap benchmark (now the ICE swap rate) are set using exchange prices within a few minutes; if agents submitted fixed bid curves at the start of the fixing period and do not adjust bids through the course of the auction, outcomes will coincide with static uniform-price double auction outcomes. Some benchmarks for commodities such as oil and gas are set using volume-weighted average prices in specific geographical locations, over relatively short time spans; if the underlying goods are relatively homogeneous, market outcomes can be approximated by supply function competition between dealers (Klemperer and Meyer, 1989), which is...
equivalent to a uniform-price double auction.

Other benchmarks are less well approximated by auctions. Some benchmarks are based on trades of underlying assets in markets with large search or transportation frictions. For example, the CME Feeder Cattle Index is based on US-wide cattle trade prices; the price of cattle traded in New York on any given day may differ substantially from the price of cattle traded in California. Other markets may have nontrivial network structure; a number of recent papers analyze trading networks with central dealers trading with peripheral counterparties (Wang, 2016; Duffie and Wang, 2016). In these markets, the ability of different agents to influence price benchmarks may vary depending on their network positions and resultant bargaining power. Other benchmarks are not based on prices of verifiable trades, but rely on market participants to self-report trades or potential trades; for example, LIBOR is based on banks’ announcements of their borrowing costs and some natural gas benchmarks are based on reports of trades which are rarely verified. Such benchmarks can be manipulated without trading the underlying asset, as agents can simply make reports to move benchmarks in the desired direction. Auctions are potentially a reasonable model for market structures in which the primary distortion is market power. Settings in which there are significant frictions other than market power, such as search frictions, network structure, or concerns about trade falsification, are less well-approximated by my model.

A normative interpretation of my results is that auctions are a relatively robust way to set price benchmarks. Auctions have a number of benefits besides those discussed in this paper. They are anonymous, so any offer to buy or sell can be taken up by any other agent; thus, agents cannot distort benchmarks by trading at artificially high or low prices with favored counterparties. They are straightforward to run; unlike average-price benchmarks, they do not require costly infrastructure to track all trades, and thus cannot be manipulated by falsifying trades. Regulators could encourage broader use of auctions for benchmark setting; even assets which are usually traded in OTC markets could be traded in periodic auctions in order to determine benchmark prices.

56ICE LIBOR
57CFTC Press Release 5409-07
58As an example, the CFTC brought charges against many companies for false reporting of natural gas trades to price index compilers; see CFTC Press Release 5300-07
7.3 Settlement by physical delivery

Many derivative contracts are settled, not by cash payments, but by physical delivery of the underlying asset. Under normal market conditions, physical delivery and cash-settled contracts function similarly, because most holders of physical delivery contracts close out their positions for cash payments prior to delivery. [Kyle (2007)] derives a set of conditions under which cash-settled contracts and physical delivery contracts are economically equivalent; one important condition is that agents can trade an arbitrary quantity of the underlying asset at the cash settlement price. The auction model of this paper satisfies the conditions for equivalence, so the metrics and methods proposed in this paper may also be applicable to measuring manipulation risk in some physical delivery contract markets.

However, there are some differences between the practical mechanisms of manipulation in physical delivery and cash-settled contract markets. Physical delivery contract markets have historically been manipulated in a few different ways. “Squeezes” are similar to cash-settled contract manipulation: a long manipulator buys the underlying asset to create a shortage of the underlying asset, raising prices of the underlying asset and increasing the prices at which shorts can close out their contracts. “Corners” are somewhat different: a long manipulator aims to buy up enough of the underlying asset that it is practically impossible for shorts to fulfill their delivery requirements; the long manipulator then uses the threat of default to extract large payments from shorts to close out contracts. [Markham (2014, pg. 3)] Corners thus require that long contract holders can threaten to take delivery, and that delivery for shorts is logistically impossible at any cost. The assumptions of the [Kyle (2007)] equivalence result are violated in the case of corners, because shorts cannot purchase arbitrary quantities of underlying assets at settlement prices – since shorts owe more of the underlying asset than the total deliverable supply in the market, settlement prices are essentially undefined. Thus, in some real-world settings, physical delivery contract manipulation appears to have substantive differences from cash-settled contract manipulation so further work is needed to determine precise conditions under which the metrics developed in this paper can also be applied to contract markets settled

[Markham (2014, pg. 3)]
by physical delivery.

8 Conclusion

This paper has constructed methods and metrics for measuring manipulation risk in derivative contract markets. If regulators observe data which is sufficiently rich to estimate agents’ demand slopes, I show how to estimate the fraction of benchmark price variance over time which is attributable to manipulation. With more limited data, regulators can estimate a simple manipulation index, which appears to correctly identify manipulable contract markets in a few case studies that I analyze. The methods I propose can be applied using data which are regularly observed by contract market regulators, and thus can potentially be used as the basis for a more structural, theory-based approach to regulating manipulation in derivative contract markets.

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Appendix

A Supplementary material for section 3

A.1 Proof of proposition 1

Assume residual supply is:

\[ z_{RSi}(p, \eta) = d_i(p - \pi) + \eta_i \]

Define \( p^*(\eta) \) as the optimal choice of \( p \) for any given \( \eta_i \), that is:

\[ p^*(\eta_i) \equiv \arg \max_p U_i(z_{RSi}(p, \eta_i), p; y_{DL}, y_{CI}) \]

\[ = \arg \max_p \pi z_{RSi}(p, \eta_i) + \frac{y_{DL} z_{RSi}(p, \eta_i)}{\kappa_i} - \frac{z_{RSi}(p, \eta_i)^2}{2\kappa_i} + y_{CI}p - z_{RSi}(p, \eta_i)p \]

Since \( z_{RSi}(p, \eta) \) is affine and increasing in \( p \), the objective function concave in \( p \), thus

the first-order condition is necessary and sufficient for \( p^*(\eta_i) \) to be optimal. Taking the

derivative with respect to \( p \) and setting to 0, and using that \( z'_{RSi}(p, \eta_i) = d_i \), we have:

\[ \frac{y_{DL}d_i}{\kappa_i} - \frac{z_{RS}(p^*(\eta_i), \eta_i)}{\kappa_i} d_i + y_{CI} - z_{RS}(p^*(\eta_i), \eta_i) - (p^*(\eta_i) - \pi) d_i = 0 \] (27)

Hence, any pair \((p^*(\eta_i), z_{RS}(p^*(\eta_i), \eta_i))\) – that is, any point \((p, z)\) which is the agent’s

optimal choice for some \( \eta_i \) – satisfies (27). Hence, the unique bid curve which passes

through the set of all ex-post optimal points is the curve implicitly defined by (27). Solving

for \( z_{RS}(p^*(\eta_i), \eta_i) \), we get expression (6) of proposition 1.

A.2 Proof of proposition 2

This proof is based on Appendix A.4 of [Du and Zhu (2012)], with notational modifications

to suit the context of this paper.

We seek a vector of demand and residual supply slopes \( b_i \) which satisfy the following

equations for all \( i \):

\[ d_i = \sum_{j \neq i} b_i = B - b_i \] (28)

\[ b_i = \frac{d_i \kappa_i}{\kappa_i + d_i} \] (29)
Rearranging, we have:
\[
\begin{align*}
    b_i \kappa_i + b_i d_i &= d_i \kappa_i \\
    d_i (\kappa_i - b_i) &= b_i \kappa_i \\
    d_i &= \frac{b_i \kappa_i}{\kappa_i - b_i}
\end{align*}
\] (30)

Combining (28) and (30), we have:
\[
\sum_j b_j - b_i = \frac{b_i \kappa_i}{\kappa_i - b_i}
\]

Defining \( B \equiv \sum_j b_j \), we have
\[
(\kappa_i - b_i) (B - b_i) = b_i \kappa_i
\]

This has two solutions:
\[
b_i = \frac{B + 2 \kappa_i \pm \sqrt{B^2 + 4 \kappa_i^2}}{2}
\]

In order for \( B > b_i \), we must pick:
\[
b_i = \frac{2 \kappa_i + B - \sqrt{B^2 + 4 \kappa_i^2}}{2}
\] (31)

This is (10) of proposition 2. B must satisfy:
\[
B = \sum_j b_j = \sum_{i=1}^{n} \frac{2 \kappa_i + B - \sqrt{B^2 + 4 \kappa_i^2}}{2}
\] (32)

By multiplying the top and bottom of the RHS by \( 2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2} \) and simplifying, this becomes:
\[
B = \sum_{i=1}^{n} \frac{2 \kappa_i B}{2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2}}
\]

Or,
\[
B \left( -1 + \sum_{i=1}^{n} \frac{2 \kappa_i}{2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2}} \right) = 0
\] (33)
Now, define
\[ f(B) = -1 + \sum_{i=1}^{n} \frac{2\kappa_i}{2\kappa_i + B + \sqrt{B^2 + 4\kappa_i^2}} \]

In order for \( B \) to solve (33) when \( B > 0 \), we need \( f(B) = 0 \). Now, \( f(0) > 0 \), \( f(B) \to -1 \) as \( B \to \infty \), and \( f'(B) < 0 \) for \( B > 0 \). Hence, \( f(B) = 0 \) at some unique \( B \), hence there is a unique value of \( B \) which solves (33), and thus there is a unique linear equilibrium for any demand slopes \( \kappa_1 \ldots \kappa_n \).

From proposition 1, we have that best-response demand functions are:
\[ z_{Bi}(p; y_{Di}, y_{Ci}) = \frac{d_i}{\kappa_i + d_i} y_{Di} + \frac{\kappa_i}{\kappa_i + d_i} y_{Ci} - \frac{\kappa_i d_i}{\kappa_i + d_i} (p - \pi) \] (34)

Substituting \( b_i = \frac{\kappa_i d_i}{\kappa_i + d_i} \), we have:
\[ z_{Bi}(p; y_{Di}, y_{Ci}) = \frac{b_i}{d_i} y_{Di} + \frac{b_i}{\kappa_i} y_{Ci} - b_i (p - \pi) \] (35)

Now \( d_i = \sum_{j \neq i} b_i \), so (35) is the same as (8).

To find benchmark prices, sum demand and equate to 0:
\[ \sum_{i=1}^{n} \left[ y_{Di} \frac{b_i}{\kappa_i} + y_{Ci} - \frac{b_i}{\kappa_i + d_i} \right] = \left( p_B - \pi \right) b_i \]

Solving for \( p_B \), we have:
\[ p_B - \pi = \frac{1}{\sum_{i=1}^{n} b_i} \sum_{i=1}^{n} \left[ \frac{b_i y_{Di}}{\kappa_i} + \frac{b_i y_{Ci}}{\sum_{j \neq i} b_j} \right] \]
proving (9).

A.3 Scale invariance of manipulation coefficients

Given some vector of demand slopes \( \kappa_i \), suppose some \( b_i \) and \( B = \sum_{i=1}^{n} b_i \) satisfy (10) and (11) in proposition 2. For a rescaled set of demand slopes \( \bar{\kappa}_i = c\kappa_i \), bid slopes \( \bar{b}_i = cb_i \) and \( \bar{B} = cB \) also satisfy (10) and (11); that is,
\[ cb_i = \frac{cB + 2c\kappa_i - \sqrt{cB^2 + 4c\kappa_i^2}}{2}, \quad cB = \sum_{i=1}^{n} \frac{2c\kappa_i + cB - \sqrt{cB^2 + 4c\kappa_i^2}}{2} \]
Factoring out $c$ from both sides, we reach (10) and (11). Thus, rescaling all demand slopes by a factor $c$ simply rescales all bid slopes. Now, under the rescaling, manipulation coefficients are unchanged, since:

$$\frac{\tilde{b}_i}{\sum_{j \neq i} b_j} = \frac{c b_i}{\sum_{j \neq i} c b_j} = \frac{b_i}{\sum_{j \neq i} b_j}$$

### B Supplementary material for section 4

#### B.1 Proof of proposition 4

Assuming that contract positions $y_{Ci}$ and demand shocks $y_{Di}$ are both normally distributed, with variances $\sigma_{Ci}^2$, $\sigma_{Di}^2$ respectively, the ratio of manipulation-induced variance to total variance, (17), is:

$$\frac{\sum_{i=1}^n s_i^2 \sigma_{Ci}^2}{\sum_{i=1}^n \sigma_{Di}^2 + \sum_{i=1}^n s_i^2 \sigma_{Ci}^2}$$

(36)

For notational convenience, I will refer to the ratio in (36) as MVR, short for “manipulation variance ratio.” To prove proposition 4, we need to show that:

$$\text{MVR} \leq \left( \frac{s_{\max} E[C]}{E[V]} \right)^2$$

(37)

First, note that contract volume is

$$E[C] = E\left[\sum_{i=1}^n |y_{Ci}|\right] = \sum_{i=1}^n \sqrt{\frac{2 \sigma_{Ci}^2}{\pi}}$$

Appendix B.1.1 below proves that, if $s_{\max}$ is small, trade volume can be approximated as:

$$E[V] = E \left| \sum_{i} v_i \right| \approx \sum_{i=1}^n \sqrt{\frac{2 \left( \sigma_{Di}^2 + s_i^2 \sigma_{Ci}^2 \right)}{\pi}}$$

Under assumption 2, we have $\sigma_{Di}^2 = \theta^2 \sigma_{Ci}^2$. Thus,

$$\sum_{i=1}^n \sqrt{\frac{2 \left( \sigma_{Di}^2 + s_i^2 \sigma_{Ci}^2 \right)}{\pi}} = \sum_{i=1}^n \sqrt{\frac{2 \left( \theta^2 + s_i^2 \right) \sigma_{Ci}^2}{\pi}}$$
Taking the ratio of $E[C]$ and $E[V]$, we get:

$$\frac{E[C]}{E[V]} = \frac{\sum_{i=1}^{n} \sqrt{\sigma_{C_i}^2}}{\sum_{i=1}^{n} \sqrt{(\theta^2 + s_i^2) \sigma_{C_i}^2}}$$

Since this is decreasing in $s_i$, we have:

$$\frac{\sum_{i=1}^{n} \sqrt{\sigma_{C_i}^2}}{\sum_{i=1}^{n} \sqrt{(\theta^2 + s_i^2) \sigma_{C_i}^2}} \geq \frac{\sum_{i=1}^{n} \sqrt{\sigma_{C_i}^2}}{\sum_{i=1}^{n} (\theta^2 + s_{\text{max}}^2) \sigma_{C_i}^2} = \frac{1}{\sqrt{\theta^2 + s_{\text{max}}^2}}$$

Hence,

$$\frac{E[C]}{E[V]} \geq \frac{1}{\sqrt{\theta^2 + s_{\text{max}}^2}}$$

implying that

$$s_{\text{max}} E[C] \geq E[V] = \frac{s_{\text{max}}}{\sqrt{\theta^2 + s_{\text{max}}^2}}$$

Or,

$$\left(\frac{s_{\text{max}} E[C]}{E[V]}\right)^2 \geq \frac{s_{\text{max}}^2}{\theta^2 + s_{\text{max}}^2} \quad (38)$$

Now, we will show that the RHS of inequality (38) is an upper bound for $\text{MVR}$. Taking the derivative of the $\text{MVR}$ as defined in (36) with respect to $s_i$, we have:

$$\frac{\partial \text{MVR}}{\partial s_i} = \frac{2\sigma_{C_i}^2 s_i}{\left(\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2\right)} \left[\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2\right] - \frac{\sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2}{\left(\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2\right)^2} > 0$$

Hence,

$$\frac{\sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2}{\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_i^2 \sigma_{C_i}^2} \leq \frac{\sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2}{\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2}$$

Now, plug in $\sigma_{D_i}^2 = \theta^2 \sigma_{C_i}^2$, to get:

$$\frac{\sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2}{\sum_{i=1}^{n} \sigma_{D_i}^2 + \sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2} = \frac{\sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2}{\sum_{i=1}^{n} \theta^2 \sigma_{C_i}^2 + \sum_{i=1}^{n} s_{\text{max}}^2 \sigma_{C_i}^2} = \frac{s_{\text{max}}^2}{s_{\text{max}}^2 + \theta^2}$$

This implies that

$$\text{MVR} \leq \frac{s_{\text{max}}^2}{s_{\text{max}}^2 + \theta^2}$$
Combining this with \(38\) we have:

\[
\left( \frac{s_{\text{max}} E[C]}{E[V]} \right)^2 \geq \frac{s_{\text{max}}^2}{s_{\text{max}}^2 + \theta^2} \geq \text{MVR}
\]

proving (37), and thus proposition 4.

B.1.1 Proof of volume approximation

To find trade volume, plug equilibrium prices (9) into demand 8, to get:

\[
z_{Bi}(p_B) = y_{Di} + \frac{\kappa_i}{\sum_{j=1}^{n} \kappa_j} y_{Ci} - \kappa_i \left( \frac{\sum_{i=1}^{n} y_{Di}}{\sum_{i=1}^{n} \kappa_i} + \frac{\sum_{i=1}^{n} \kappa_i y_{Ci}}{(\sum_{i=1}^{n} \kappa_i)^2} \right)
\]

We can write this as:

\[
z_{Bi}(p_B) = y_{Di} + s_i y_{Ci} - s_i \sum_{i=1}^{n} y_{Di} - s_i \sum_{j=1}^{n} s_j y_{Cj}
\]

\[
= (1 - s_i) y_{Di} + s_i (1 - s_i) y_{Ci} - s_i \sum_{j \neq i} y_{Di} - s_i \sum_{j \neq i} s_j y_{Cj}
\]

Thus, \(z_{Bi}(p_B)\) has mean 0 and variance:

\[
= (1 - s_i)^2 \sigma_{Di}^2 + s_i^2 (1 - s_i)^2 \sigma_{Ci}^2 - s_i^2 \sum_{j \neq i} \sigma_{Dj}^2 - s_i^2 \sum_{j \neq i} s_j^2 \sigma_{Cj}^2
\]

(39)

Thus, the expectation of the absolute value of \(z_{Bi}(p_B)\) is:

\[
\sqrt{\frac{2 \left[(1 - s_i)^2 \sigma_{Di}^2 + s_i^2 (1 - s_i)^2 \sigma_{Ci}^2 - s_i^2 \sum_{j \neq i} \sigma_{Dj}^2 - s_i^2 \sum_{j \neq i} s_j^2 \sigma_{Cj}^2\right]}{\pi}}
\]

If \(s_{\text{max}}\) is small, then:

\[
(1 - s_i)^2 \sigma_{Di}^2 - s_i^2 \sum_{j \neq i} \sigma_{Dj}^2 \approx \sigma_{Di}^2
\]

\[
s_i^2 \left[(1 - s_i)^2 \sigma_{Ci}^2 - \sum_{j \neq i} s_j^2 \sigma_{Cj}^2\right] \approx s_i^2 \sigma_{Ci}^2
\]
thus, expression (39) is approximately:

\[ \sqrt{2 \left[ \sigma_{Di}^2 + \sigma_{Ci}^2 \right]} \]

as desired.

C Supplementary material for section 5

C.1 Moment matching

I match parameters to moments using a two-step process. First, I choose demand slopes \( \kappa_i \) to match observed aggregate bid slopes. Second, fixing \( \kappa_i \) at the estimated values, I choose demand shock variances \( \sigma_{Di}^2 \) to match observed trade volumes.

As described in (26) the main text, I measure \( \hat{B}_a \) for each auction by regressing volume imbalance on prices:

\[
\hat{B}_a \equiv - \sum_{t=1}^{T_a} (i_{at} - \bar{i}_a) (p_{at} - \bar{p}_a) / \sum_{t=1}^{T_a} (p_{at} - \bar{p}_a)
\]

I assume that \( \kappa_1 \ldots \kappa_4 \) are equal, and that \( \log (\kappa_i) \), for \( i \geq 4 \), is a second-order polynomial in \( i \), with coefficients described by the parameter vector \( \theta_\kappa \). For a given \( \theta_\kappa \), let

\[
B (n_a; \theta_\kappa)
\]

represent the predicted slope of aggregate demand given \( \theta_\kappa \). To calculate \( B (n_a; \theta_\kappa) \), note that for an auction with \( n_a \) participants with demand slopes \( \kappa_1 \ldots \kappa_{n_a} \), we can numerically solve (10) and (11) in proposition 2 to obtain predicted equilibrium bid slopes

\[
b_1 (\kappa_1 \ldots \kappa_{n_a}) \ldots b_{n_a} (\kappa_1 \ldots \kappa_{n_a})
\]

and thus a predicted aggregate bid slope \( B (\kappa_1 \ldots \kappa_{n_a}) \). Thus, I choose a vector of polynomial coefficients \( \theta_\kappa^* \) to minimize the distance between the model-predicted and estimated slopes of aggregate demand:

\[
\theta_\kappa^* = \arg \min_{\theta_\kappa} \sum_{a=1}^{A} [\hat{B}_a - B (n_a; \theta_\kappa)]^2 \tag{40}
\]

Similarly, I choose demand shock variances to match observed trade volume. In appendix C.2, I show that, given the equilibrium vector of bid slopes \( b_i, z_{ Bai} (p_a) \) is a
normal random variable, with mean 0 and variance:

$$\text{Var}(z_{Bai}(p_a)) = \left(\frac{b_{ai}}{\kappa_i} \left(1 + \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}}\right)\right)^2 \sigma_{D_i}^2 + \frac{\sum_{j \neq i} b_{aj} \sigma_{D_j}^2}{\sum_{i=1}^{n} b_{ai}}$$

(41)

Thus, the expected total trade volume for participant $i$ in equilibrium is:

$$E[|z_{Bai}(p_a)|] = \sqrt{\frac{2\text{Var}(z_{Bai}(p_a))}{\pi}}$$

Expected volume for all participants the sum of $E[|z_{Bai}(p_a)|]$ across all participants in auction $a$. This is a function of $n_a$, demand slopes $\kappa_i$, and demand shock variances $\sigma_{D_i}^2$. As with demand slopes, I assume that $\sigma_{D1}^2 \ldots \sigma_{D4}^2$ are equal, and that $\log(\sigma_{D_i}^2)$, for $i \geq 4$, is a second-order polynomial in $i$, with coefficient vector $\theta_d$. Denote predicted volume given $n_a$, $\theta_\kappa$, and $\theta_d$ as:

$$v(n_a; \theta_\kappa, \theta_d) \equiv \sum_{i=1}^{n} E_n[|z_{Bi}(p_a)| \mid \theta_\kappa, \theta_d]$$

Fixing $\theta_\kappa = \theta_\kappa^*$ at the optimal value from (40), I choose $\theta_d^*$ to minimize the distance between model-predicted and observed volume:

$$\theta_d^* = \arg\min_{\theta_d} \sum_{a=1}^{A} [v_a - v(n_a; \theta_\kappa^*, \theta_d)]^2$$

### C.2 Derivation of expected volume

Equating the sum of agents’ bid curves in (24) to 0 and solving for $p_a$, the auction clearing price is:

$$p_a - \pi_a = \frac{1}{\sum_{i=1}^{n} b_{ai}} \left[\sum_{i=1}^{n} \left(\frac{b_{ai} y_{Dai}}{\kappa_i} + \frac{b_{ai} \sum_{j \neq i} b_{aj} y_{Cai}}{\sum_{i=1}^{n} b_{ai}}\right)\right]$$

(42)

Since I assume agents do not hold contract positions when estimating the model, set $y_{Cai} = 0$. are $y_{cai}$

Demand for each agent at the auction clearing price is:

$$z_{Bai}(p_a) = \frac{b_{ai}}{\kappa_i} y_{Dai} - b_{ai} (p_a - \pi)$$
\[
= \left(1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \right) \left( \frac{b_{ai}}{\kappa_i} y_{Da_i} \right) - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{j \neq i} \left( \frac{b_{aj}}{\kappa_j} y_{Da_j} \right) \right]
\]

Thus, \( z_{Bi}(p_a) \) is a normal random variable, with mean 0 and variance:

\[
\text{Var}(z_{Bai}(p_a)) = \left(1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \right)^2 \left( \frac{b_{ai}}{\kappa_i} \right)^2 \sigma_{Di}^2 + \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{j \neq i} \left( \frac{b_{aj}}{\kappa_j} \right)^2 \sigma_{Dj}^2 \right]
\]

### C.3 Calculations for table 2

This appendix describes how I calculate quantities in table 2. From expression (16), if all auctions have \( n \) agents, we have:

\[
E \left[ (p_{Bt} - p_{B,t-1})^2 \right] = E \left[ (\pi_t - \pi_{t-1})^2 \right] + \frac{1}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} \frac{2b_i \text{Var}(y_{Di,t})}{\kappa_i} + \frac{2b_i \text{Var}(y_{Ci,t})}{\sum_{j \neq i} b_j} \right]
\]

\( p_{Bt} - p_{B,t-1} \) represents monthly gold price innovations would be, assuming that agents held derivative contracts with standard deviation 150,000 oz. I estimate that the standard deviation of monthly innovations in gold prices is $47.69/oz; this corresponds to the standard deviation of price innovations when agents are not holding derivative contracts, that is,

\[
E \left[ (\pi_t - \pi_{t-1})^2 \right] + \frac{1}{\sum_{i=1}^{n} b_i} \left[ \sum_{i=1}^{n} \frac{2b_i \text{Var}(y_{Di,t})}{\kappa_i} \right] = 47.69
\]

This implies, from (17), that:

\[
\text{MVR} = \frac{2\sum_{i=1}^{n} \frac{b_i \text{Var}(y_{Ci,t})}{\sum_{j \neq i} b_j}}{47.69 + \frac{2\sum_{i=1}^{n} \frac{b_i \text{Var}(y_{Ci,t})}{\sum_{j \neq i} b_j}}{\sum_{i=1}^{n} b_i}} (43)
\]

Where the denominator is how large the variance of monthly gold price innovations would be if agents optimally manipulated, which is somewhat larger than $47.69/oz; however, since I find that manipulation-induced variance is fairly small, the difference is small. We can estimate (43) are known, since we have assumed that \( \text{Var}(y_{Ci}) = (150,000 \text{oz})^2 \), and \( b_i \) can be calculated for any \( n, i \) using our estimated demand slopes \( \kappa_i \).
Similarly, from (18) and (19), MDVR and DVR are respectively:

$$\text{MDVR} = \frac{\sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var} (y_{Ci})}{\sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var} (y_{Di}) + \sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right)^2 \text{Var} (y_{Ci})}$$

$$\text{DVR} = \frac{2 \sum_{i=1}^{n} \frac{b_i \text{Var} (y_{Di,t})}{\kappa_i}}{47.69 + \sum_{i=1}^{n} b_i}$$

The other terms in Table 2 are the standard deviation of distortions induced by manipulation and demand shocks respectively, defined as:

$$\text{Manip SD} = \sqrt{\frac{2 \sum_{i=1}^{n} \frac{b_i \text{Var} (y_{Ci,t})}{\sum_{j \neq i} b_j}}{\sum_{i=1}^{n} b_i}}, \quad \text{Demand SD} = \sqrt{\frac{2 \sum_{i=1}^{n} \frac{b_i \text{Var} (y_{Di,t})}{\kappa_i}}{\sum_{i=1}^{n} b_i}}$$

### D Supplementary material for section 6

#### D.1 CFTC Commitments of Traders Data

The COT reports contains four concentration metrics: the percentage of net or gross open interest held by the top 4 and top 8 largest agents. Net concentration differs from gross concentration in that it nets out spreading positions which involve offsetting contract positions at different delivery months; for my purposes, what matters is the fraction of open interest held in any particular delivery month, so gross concentration is a more appropriate metric. Likewise, although top 4 and top 8 concentration are strongly correlated, I use top 4 concentration, as markets with $n = 8$ similarly sized participants are quite competitive in the context of my theory. I filter to data from 2010-01-01 to 2016-12-27.

I divide commodity and financial futures into subgroups based on the CFTC’s commodity subgroup codes. The CFTC does not provide a codebook, so I create my own labels, in the “Description” column of table 3. A few categories – in particular, those for oil, gas and electricity – have very large numbers of contracts. There are 92 kinds of oil contracts, 106 kinds of natural gas contracts, and 287 kinds of electricity contracts in my data. However, open interest is very concentrated, and the largest contract markets are much bigger than median contract markets. I must therefore choose a scheme for aggregating over all these contracts. Flat averaging does not reflect differences in size
between contract markets, and weighting contract markets by open interest is undesirable because contract units can differ within commodity categories, and also because estimates would generally be dominated by the largest contract markets. Instead, I use only the 10 largest contracts in each commodity category, based on the total open interest summed across all dates where a contract is observed. The numbers in table 3 then reflect flat averages across the 10 largest contract categories. Thus, my estimates reflect only large contract markets in each category.

The COT report data reports open interest for five categories of traders. For commodity futures, these are “Producers/Merchants/Processors/Users” (“Prod+Merch” in table 3), “Swap Dealers”, “Managed Money” (“Man. Money”), “Other Reportable”, and “Nonreportable”. The distinction between “Other Reportable” and “Nonreportable” is that “Other Reportable” represents parties who hold large enough positions to meet reporting requirements, but do not fall into other categories; nonreportable positions are calculated as a difference between total open interest and total held by reporting parties. For the purposes of my analysis, I combine “Other Reportable” and “Nonreportable” into a single “Other” category. The COT reports also separately reports long, short, and spreading open interest, where spreading open interest represents offsetting long and short contracts with different delivery months. I do not need to separately consider spreading positions for the purposes of my analysis; thus, I add the fraction of spreading positions to both long and short positions. This makes long and short positions summed across participants equal to 100%, up to rounding error, as expected.

“Producers/Merchants/Processors/Users” are participants in markets for the underlying asset – for example, agents who produce, process or store the physical underlying commodity, and are thus exposed to price risk that they hedge in futures markets. “Managed money” participants are commodity trading advisors, commodity pool operators, or other funds, who can be thought of as largely financial speculators purchasing risk. The role of “swap dealers” in these markets is less clear. The CFTC’s explanatory notes state: “A “swap dealer” is an entity that deals primarily in swaps for a commodity and uses the futures markets to manage or hedge the risk associated with those swaps transactions.” A more detailed 2008 CFTC staff report on swap dealers [CFTC 2008] suggests that swap dealers primarily function to bridge between the OTC and exchange-traded derivatives markets; that is, they enter into customized OTC derivative contracts with customers, and hedge the resultant risk on commodity exchanges. The staff report suggests that swap dealers’ customers are both speculators and hedgers. Further description of the different classes of participants is available at the CFTC’s website.
Notes. The left plot shows round volume imbalances and prices for all auctions with \( n_a \in \{6, 7, 8, 9\} \) participants. Each data point is an auction-round; final rounds are excluded. The x-axis shows the round price \( p_{ar} \) minus the auction clearing price \( p_{aR} \), and the y-axis shows the round imbalance \( i_{ar} \). Colored lines are predictions from linear regressions for each \( n_a \). The right plot shows boxplots of total auction volume \( v_a \), versus the number of bidders \( n_a \) participating in the auction.
Notes. The left plot shows the average slope of demand estimated from the data, $\hat{B}_n$, for each $n$ in blue, and the model-predicted slope of aggregate auction demand for each $n$ in red. The right plot shows average volume in the data, $v_n$, for each $n$ in blue, and the model-predicted volume for each $n$ in red. The dotted lines represent 95% pointwise confidence intervals for the data moments.
Table 1: Gold auction descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>P10</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds</td>
<td>4.18</td>
<td>1.52</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Participants</td>
<td>7.71</td>
<td>1.26</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Price (USD / oz)</td>
<td>1223.35</td>
<td>75.11</td>
<td>1118.23</td>
<td>1327.3</td>
</tr>
<tr>
<td>Price range (USD / oz)</td>
<td>1.06</td>
<td>0.8</td>
<td>0.45</td>
<td>1.91</td>
</tr>
<tr>
<td>Volume (1000 oz)</td>
<td>167.79</td>
<td>85.22</td>
<td>90.39</td>
<td>244.78</td>
</tr>
</tbody>
</table>

Notes. Each observation is an auction. “Rounds” is the number of rounds the auction took to complete, $R_a$. “Participants” is the number of auction participants, $n_a$. “Price” is the auction clearing price, $p_a$. “Price range” is the difference between the highest and lowest round prices. “Volume” is $v_a$, the sum of buy volume and sell volume in the final round of the auction.
Table 2: Components of gold auction price variance

<table>
<thead>
<tr>
<th>$n_a$</th>
<th>Manip SD</th>
<th>MVR</th>
<th>Demand SD</th>
<th>DVR</th>
<th>MDVR</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.232 (0.946)</td>
<td>0.457% (0.213%)</td>
<td>2.207 (0.685)</td>
<td>0.2141% (0.1054%)</td>
<td>68.2% (4.82%)</td>
</tr>
<tr>
<td>5</td>
<td>2.242 (0.515)</td>
<td>0.221% (0.079%)</td>
<td>2.032 (0.561)</td>
<td>0.1816% (0.0773%)</td>
<td>54.9% (6.58%)</td>
</tr>
<tr>
<td>6</td>
<td>1.909 (0.350)</td>
<td>0.160% (0.042%)</td>
<td>1.962 (0.493)</td>
<td>0.1693% (0.0632%)</td>
<td>48.6% (7.96%)</td>
</tr>
<tr>
<td>7</td>
<td>1.799 (0.296)</td>
<td>0.142% (0.032%)</td>
<td>1.953 (0.464)</td>
<td>0.1677% (0.0572%)</td>
<td>45.9% (8.74%)</td>
</tr>
<tr>
<td>8</td>
<td>1.768 (0.283)</td>
<td>0.137% (0.030%)</td>
<td>1.970 (0.454)</td>
<td>0.1707% (0.0554%)</td>
<td>44.6% (9.14%)</td>
</tr>
<tr>
<td>9</td>
<td>1.761 (0.280)</td>
<td>0.136% (0.029%)</td>
<td>2.002 (0.455)</td>
<td>0.1763% (0.0557%)</td>
<td>43.6% (9.36%)</td>
</tr>
<tr>
<td>10</td>
<td>1.760 (0.280)</td>
<td>0.136% (0.029%)</td>
<td>2.054 (0.470)</td>
<td>0.1856% (0.0594%)</td>
<td>42.3% (9.60%)</td>
</tr>
<tr>
<td>11</td>
<td>1.760 (0.281)</td>
<td>0.136% (0.029%)</td>
<td>2.160 (0.605)</td>
<td>0.2050% (0.1047%)</td>
<td>39.9% (11.24%)</td>
</tr>
</tbody>
</table>

Notes. Components of gold auction price variance, in units of USD/oz. Values in parentheses are standard errors from a nonparametric bootstrap with 200 repetitions. “Manip SD” is the square root of manipulation-induced variance, “Demand SD” is the square root of demand shock-induced variance, and MVR, DVR, and MDVR are as defined in subsections 4.1 and 4.2. Detailed expressions for all quantities are provided in appendix C.3.
Table 3: Commitments of Traders reports: commodity futures

<table>
<thead>
<tr>
<th>Category</th>
<th>Top 4 Long</th>
<th>Top 4 Short</th>
<th>$s_{\text{max}}$ Long</th>
<th>$s_{\text{max}}$ Short</th>
<th>Swap Dealers Long</th>
<th>Swap Dealers Short</th>
<th>Man. Money Long</th>
<th>Man. Money Short</th>
<th>Other Long</th>
<th>Other Short</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn and Wheat</td>
<td>24%</td>
<td>31%</td>
<td>14%</td>
<td>29%</td>
<td>19%</td>
<td>3%</td>
<td>21%</td>
<td>17%</td>
<td>31%</td>
<td>30%</td>
</tr>
<tr>
<td>Soybean Products</td>
<td>19%</td>
<td>25%</td>
<td>11%</td>
<td>22%</td>
<td>18%</td>
<td>3%</td>
<td>24%</td>
<td>14%</td>
<td>37%</td>
<td>32%</td>
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<tr>
<td>Cotton</td>
<td>18%</td>
<td>29%</td>
<td>12%</td>
<td>14%</td>
<td>33%</td>
<td>8%</td>
<td>32%</td>
<td>15%</td>
<td>21%</td>
<td>19%</td>
</tr>
<tr>
<td>Sugar/Cocoa/Coffee</td>
<td>21%</td>
<td>33%</td>
<td>13%</td>
<td>28%</td>
<td>20%</td>
<td>8%</td>
<td>30%</td>
<td>18%</td>
<td>22%</td>
<td>14%</td>
</tr>
<tr>
<td>Livestock</td>
<td>17%</td>
<td>16%</td>
<td>8%</td>
<td>12%</td>
<td>24%</td>
<td>3%</td>
<td>35%</td>
<td>17%</td>
<td>29%</td>
<td>44%</td>
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<tr>
<td>Dairy</td>
<td>52%</td>
<td>50%</td>
<td>25%</td>
<td>61%</td>
<td>3%</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
<td>33%</td>
<td>47%</td>
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<tr>
<td>Precious Metals</td>
<td>26%</td>
<td>36%</td>
<td>16%</td>
<td>9%</td>
<td>18%</td>
<td>27%</td>
<td>44%</td>
<td>14%</td>
<td>29%</td>
<td>16%</td>
</tr>
<tr>
<td>Base Metals</td>
<td>42%</td>
<td>48%</td>
<td>22%</td>
<td>15%</td>
<td>27%</td>
<td>6%</td>
<td>18%</td>
<td>19%</td>
<td>40%</td>
<td>42%</td>
</tr>
<tr>
<td>Oil</td>
<td>29%</td>
<td>27%</td>
<td>14%</td>
<td>22%</td>
<td>28%</td>
<td>28%</td>
<td>18%</td>
<td>13%</td>
<td>31%</td>
<td>30%</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>34%</td>
<td>37%</td>
<td>18%</td>
<td>35%</td>
<td>27%</td>
<td>43%</td>
<td>20%</td>
<td>17%</td>
<td>18%</td>
<td>17%</td>
</tr>
<tr>
<td>Electricity</td>
<td>60%</td>
<td>67%</td>
<td>32%</td>
<td>74%</td>
<td>20%</td>
<td>22%</td>
<td>4%</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>Lumber</td>
<td>31%</td>
<td>31%</td>
<td>16%</td>
<td>14%</td>
<td>16%</td>
<td>0%</td>
<td>25%</td>
<td>21%</td>
<td>45%</td>
<td>44%</td>
</tr>
<tr>
<td>Emissions Permits</td>
<td>72%</td>
<td>75%</td>
<td>37%</td>
<td>47%</td>
<td>10%</td>
<td>40%</td>
<td>7%</td>
<td>0%</td>
<td>35%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Notes. “Top 4” is the fraction of gross long and short open interest held by the 4 largest market participants, defined as $q_{4l}$ and $q_{4s}$ above. $s_{\text{max}}$ is the sum of $q_{4l}$ and $q_{4s}$ divided by 4. All other columns describe the fraction of long and short open interest held by different classes of market participants. Open interest percentages add to 100%, separately for longs and shorts, across the 4 categories of market participants: “Prod+Merch”, “Swap Dealers”, “Man. Money”, and “Other”. I provide descriptions of categories in Appendix D.1. Since the CFTC does not provide a codebook for subgroup codes, I manually label subgroups in the “Description” column.