Competition and Manipulation in Derivative Contract Markets

Anthony Lee Zhang

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Abstract

This paper studies manipulation in cash-settled derivative contract markets. In my model, agents hedge factor risk exposures using derivative contracts tied to the auction price of a spot good. Agents can manipulate contract payoffs by trading the spot good; in equilibrium, manipulation can make all agents worse off. I define two simple measures of manipulation-induced welfare losses: manipulation-induced basis risk and expected manipulation rents. Both can be estimated using data on spot traders’ auction bids, and the variances and covariances of traders’ contract and inventory positions. Using these measures, I estimate how large manipulation-induced distortions would be if COMEX gold futures were cash-settled using the London Bullion Market Association gold price benchmark.

Keywords: derivatives, manipulation, regulation

JEL classifications: D43, D44, D47, G18, K22, L40, L50

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University of Chicago Booth School of Business, 5807 S Woodlawn Ave, Chicago, IL 60637; Anthony.Zhang@chicagobooth.edu.
1 Introduction

This paper studies manipulation in cash-settled derivative contract markets. If a trader wants to buy or sell exposure to some risk factor, such as US equities, interest rates, volatility, or energy, the simplest way to do so is often to use a cash-settled derivative contract. Some examples of these contracts are S&P 500 futures, LIBOR and SOFR derivatives, VIX futures and options, and various oil, gas and electricity derivatives. Cash-settled derivatives constitute some of the world’s largest markets: the total notional size of the interest rate derivatives market alone is over $100 trillion USD.

Practically, cash-settled derivatives are simply contracts with payoffs determined by some price benchmark, which is constructed based on trade prices of some spot good. If a price benchmark accurately reflects a given risk factor, contracts settled using the benchmark can be used to trade exposure to this risk factor. For example, a long VIX futures position pays its holder some multiple of the CBOE Volatility Index at settlement, which is calculated based on prices of S&P 500 options set in a settlement auction. VIX futures are useful for trading volatility to the extent that the option settlement prices are representative of US equity volatility.

Derivative contract holders who are also active in spot markets have incentives to distort price benchmarks in order to increase contract payoffs. A trader who is long VIX futures can increase her futures payoffs by buying S&P 500 options at the settlement auction to raise the VIX settlement value. If the trader’s futures position is large, her increased futures profits may outweigh any losses incurred by buying S&P 500 options at elevated prices. If many traders bid this way, however, their bids would add noise to the VIX at settlement, creating nonfundamental risk for all agents holding VIX futures contracts.

Legally, trading spot goods to move settlement prices and influence derivative payoffs is considered contract market manipulation, and is illegal in the US and many other jurisdictions. Regulators have imposed billions of dollars of fines on market participants for manipulation in the past two decades alone. However, manipulation is poorly understood from the perspective of economic theory. We do not know how manipulation affects the welfare of different classes of market participants: whether it is Pareto disimproving, or
simply creates transfers from non-manipulators to manipulators. We do not understand what makes contract markets vulnerable to manipulation, so we do not know how to empirically measure manipulation risk in contract markets.

This paper studies a simple model of contract market manipulation. In the model, manipulation is a market failure caused by price impact: agents who hold contract positions, and who have price impact in the spot good market, have incentives to trade non-fundamentally in spot markets to increase their contract payoffs. The size of manipulation-induced market distortions depends on the size of spot traders’ contract positions and the competitiveness of spot markets. Manipulation always decreases the welfare of contract holders who cannot trade the spot good; in some cases, manipulation can even be Pareto disimproving. Using the model, I derive two simple measures of manipulation risk, and I show how to estimate these measures in an empirical application. Regulators can use these measures to determine whether manipulation-induced distortions are likely to be large in any given contract market.

I assume that a large number of risk-averse agents have exogeneous exposures to a common risk factor. Agents cannot contract on the risk factor directly, but can trade cash-settled derivative contracts in a competitive market, which are tied to the auction price of a spot good. The spot good is traded by a finite number of spot traders, and the marginal value of the spot good is equal to the risk factor for all traders. If spot traders behaved competitively, the spot good market would clear with no trade, the spot auction price would be exactly equal to risk factor, and derivative contracts would allow all agents to perfectly share factor risk.

In equilibrium, however, first-best risk sharing is not attainable because the spot market is not perfectly competitive. Spot traders have price impact in the spot market, so they have incentives to trade the spot good in a way that increases their derivative contract payoffs. For example, a spot trader with a long contract position has higher incentives to buy the spot good, since her purchases increase the spot auction price and thus her contract payoffs.

Manipulation causes auction prices to become noisy signals of the risk factor, creating non-fundamental basis risk for all contract holders. This affects spot traders’ contract purchasing decisions: traders tend to buy less contracts because of increased basis risk,
but tend to buy more because of anticipated profits from manipulating the settlement price. In equilibrium, spot traders may even over-hedge, purchasing larger derivative positions than their total factor risk exposures.

Manipulation also distorts outcomes in spot markets: positive quantities of the spot good are traded even though spot traders perfectly agree on its value. Market structure in the spot market determines how vulnerable the contract market is to manipulation. Manipulation-induced distortions are larger when spot markets are less competitive, and when traders have high spot good holding costs, so that the price impact of spot good trades is larger. Distortions are also larger when spot traders hold larger contract positions, as this gives them larger incentives to manipulate settlement prices.

The effects of manipulation on spot traders’ welfare are complex, combining a number of externalities. Spot traders receive a net positive transfer, because they can move prices in favor of their contract positions on average, but they also face increased risk due to increased settlement price variance. The negative effects are strong enough that manipulation can make all spot traders worse off relative to the competitive-bidding benchmark. In some cases, spot traders would be better off if all traders could commit to buying less contracts, so a regulator could create a Pareto improvement by imposing taxes or limits on the size of spot traders’ contract positions.

Pure hedgers – agents who hold contract positions, but do not trade the spot good – are unambiguously harmed by manipulation, since they suffer from nonfundamental settlement price risk, and also lose money to spot traders on average. I show that hedgers’ welfare losses from manipulation depend on two simple metrics: basis risk, which is manipulation-induced variance in contract settlement prices; and manipulation rents, which are the net transfers from pure hedgers to spot traders created by spot traders’ price impact.

I then generalize the spot market auction model: I allow spot traders to have asymmetric holding costs for the spot good, to receive arbitrarily distributed inventory shocks for spot goods, and to have arbitrarily distributed derivative contract positions. I show that there exists a unique equilibrium in the general model, and that basis risk and manipulation rents can be estimated using observable quantities: spot traders’ bid slopes in settlement auctions, and the variances and covariances of spot traders’ inventory shocks.
and contract positions.

I then apply the model to answer the following question: how large would basis risk and manipulation rents be, if COMEX gold futures were cash-settled using the London Bullion Market Association (LBMA) gold price benchmark? The LBMA gold price is set in a static auction which uses a dynamic multi-round price-setting mechanism. I show that, in a simple model of this dynamic auction, there exists an equilibrium equivalent to the unique equilibrium of a static auction. Assuming agents play this equilibrium, using data on auction round prices and volumes, I can recover the slope of aggregate demand in each auction I observe.

Since I only observe aggregate bidding data, not data on individual bidders, I then apply a bounds approach. I simulate a large number of individual bid slopes and inventory variances which are consistent with observed aggregate bid slopes and volumes, and I calculate the maximum and minimum values of basis risk and manipulation rents across all simulated individual parameters. I also calculate both risk metrics under a variety of assumptions on auction aggregate slopes and volumes, the number of auction participants, and variances and covariances between contract positions and inventory positions. Across all these sources of uncertainty, I find that both manipulation risk metrics are fairly low. Even if all spot traders held contract positions with standard deviation equal to half the COMEX futures position limit, manipulation rents would be at most 2.88 basis points per dollar of hedgers’ notional gold exposures, and basis risk would be at most only 2.09% of total monthly gold price variance.

1.1 Related literature

This paper makes two main contributions to the literature. To my knowledge, this is the first paper to analyze the welfare effects of contract market manipulation, microfounding behavior in both the contract and spot markets. This is also the first paper to propose empirically implementable measures of contract market manipulation risk.

There are a number of related strands of literature. There is a growing literature theoretically and empirically analyzing market and benchmark manipulation. Abrantes-Metz et al. (2012), Gandhi et al. (2015), and Bonaldi (2018) analyze LIBOR manipulation, Griffin et al.
and Shams (2018) analyzes VIX manipulation, Birge et al. (2018) study manipulation in electricity markets, and Nozari, Pascutti and Tookes (2019) study the related phenomenon of profitable price impact in convertible bond markets. A number of theoretical papers analyze the question of optimal benchmark design, such as Duffie, Dworczak and Zhu (2017), Duffie and Dworczak (2018), Eisl, Jankowitsch and Subrahmanyam (2017), Coulter, Shapiro and Zimmerman (2018) and Baldauf, Frei and Mollner (2018). Duffie and Dworczak (2018) and Baldauf, Frei and Mollner (2018) propose using volume-weighted average price schemes, whereas Coulter, Shapiro and Zimmerman (2018) proposes an incentivized announcement scheme somewhat similar to an auction. In contrast to these papers, I do not attempt to find an optimal mechanism for benchmark determination in this paper; instead, I adopt a reduced-form auction model of benchmark setting in order to quantify agents’ manipulation incentives.


The spot market auction model of this paper builds on the literature on linear-quadratic double auctions (Vayanos, 1999; Rostek and Weretka, 2015; Du and Zhu, 2017; Duffie and Zhu, 2017). In particular, the proof of equilibrium in the generalized spot market model of section 5 builds on a theorem of Du and Zhu (2012). To my knowledge, however, the contract market model of this paper, in which agents hedge factor risk using contracts settled based on an auction price, is new to the literature.

1.2 Outline

The remainder of the paper proceeds as follows. Section 2 discusses some history and institutional details of contract market manipulation. Section 3 introduces the model, and
section 4 derives the main theoretical results. Section 5 generalizes the spot market model, and derives general expressions for my two risk metrics. Section 6 applies these methods to measure manipulation risk for the LBMA gold price. Section 7 discusses implications of my findings, and section 8 concludes. Proofs, derivations and other supplementary material are presented in the appendix.

2 Institutional background

2.1 Cash-settled derivative contracts and manipulation

Cash-settled derivative contracts entitle contract holders to cash payments linked to price benchmarks, which are set based on trade prices of some spot (or underlying) good. The main benefit of trading cash-settled derivatives is that market participants can trade price risk associated with underlying spot good, without incurring various costs – capital costs, physical transportation and storage costs – associated with trading the spot goods directly.

The first cash-settled contract was the Eurodollar futures contract, introduced on the Chicago Mercantile Exchange in 1981. Futures and options contracts for livestock and many dairy products are financially settled based on USDA-published average transaction prices for underlying commodities. Many energy derivatives, such as oil, gas, and electricity, are settled using price benchmarks calculated based on government or industry sources. Many financial derivatives are also cash-settled using benchmarks, such as LIBOR and SOFR for interest rates, the WM/Reuters fix as well as other FX

1 Market Begins Trading In Eurodollar Futures
2 CME Lean Hog Futures Contract Specs and Feeder Cattle Futures Contract Specs
3 CME Class III Milk Futures Contract Specs and Cash-Settled Cheese Futures Contract Specs
4 ICE Brent Crude Futures and Platts Dubai Swap
5 ICE Fixed Price Swap - Henry Hub - Tailgate, Louisiana
6 ICE ERCOT North 345KV Real-Time Peak Daily Fixed Price Future
7 Financial Stability Board (2018)
8 Financial Stability Board (2014)
indices for equities, commodities, FX, volatility and many others.

Derivative contract markets are often much more liquid than spot markets: the volume of trade in derivatives settled using a given price benchmark is often much larger than the volume of trade in spot assets used to set price benchmarks. For example, the Platts Inside FERC Houston Ship Channel natural gas price benchmark is based on gas trades totalling around 1.4 million MMBtus of natural gas in a week, open interest in the ICE HSC basis future, which is financially settled based on the Platts benchmark, is more than 75 million MMBtus for many delivery months. The Secured Overnight Financing Rate (SOFR), designed to replace USD LIBOR as an interest rate benchmark, is based on average daily volumes of approximately $1 trillion in overnight treasury-backed repo loans as of 2014, the total notional volume of contracts linked to USD LIBOR was estimated to be greater than $160 trillion.

As a result of this liquidity mismatch, spot market participants who hold large derivative contract positions may have incentives to trade spot assets for non-fundamental reasons, in order to move price benchmarks and influence contract payoffs. If a gas trader holds a large position in ICE Houston Ship Channel (HSC) basis futures, she can increase

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9 CME Korean Won futures are settled based on a benchmark price reported by Seoul Money Brokerage Service Limited, which represents activity in the Korean Won spot market, and the CME Chinese Renminbi futures are settled based on a similar benchmark reported by the Treasury Markets Association, Hong Kong.

10 FTSE 100 Index

11 Bloomberg Commodity Index

12 U.S. Dollar Index

13 VIX

14 Two reasons this may be true are because derivative contract activity tends to be concentrated in a small number of contracts for any given commodity, likely because of increasing returns to liquidity; and because derivatives tend to be highly levered, so the capital costs of trading any given amount of notional exposure to underlying assets is lower than the cost of trading the spot asset itself.

15 See “Liquidity in North American Monthly Gas Markets” on the Platts website. In 2018, daily trade volume during “bid week,” the five business days during which index prices are set, was between 50 to 280 thousand MMBtu per day; multiplying by 5, we get an upper bound of 1.4 million. Volume in other years is much larger: in 2008-2010, many delivery months have total trade volume over 10 million MMBtu.

16 ICE Report Center. End of Day reports for HSC basis futures, as of October 24th, 2018. Open interest is above 30,000 contracts for many delivery months, and the contract multiplier is 2,500 MMBtus.

17 NY Fed's Secured Overnight Financing Rate Data.

18 Financial Stability Board (2018). Note that the $160 trillion number measures the total volume of outstanding contracts across expiration dates; the volume of interest rate derivatives expiring on any given settlement date will be substantially smaller.
her futures payoff by buying spot gas at HSC to increase the benchmark price. The trader may lose money in the spot market, but her profits on her futures position may be much larger than her spot market losses.

2.2 Legal background

Regulators have policed contract market manipulation aggressively in the last few decades. UBS was fined $15 million by the CFTC in 2018 for attempting to manipulate gold futures contracts. The CFTC and the FERC have fined traders millions of dollars for manipulating oil, gas, electricity, precious metals, and propane derivatives.\(^\text{19}\) Fines for financial derivative manipulation are orders of magnitude larger: banks have been fined over $10 billion for FX manipulation\(^\text{20}\), over $8 billion USD for manipulation of LIBOR and other interest rate benchmarks\(^\text{21}\), and over $500 million for manipulation of the ISDAFIX interest rate swap benchmark\(^\text{22}\).

In the US, manipulation and attempted manipulation of contract markets are illegal under the Commodity Exchange Act of 1936\(^\text{23}\), but the act does not define manipulation in any way. The CFTC’s formalized operational definition of manipulation, developed through case law, essentially states that trades made with the intent to create artificial prices are manipulative\(^\text{24}\). This definition is still vague, and there has been substantial disagreement in both the economic and legal literatures, both on what constitutes manipulation, and what should constitute manipulation from a welfare perspective.

In recent times, the legal literature has largely moved away from the “artificial price”

\(^{19}\)See, for example, CFTC Press Release 6041-11, 128 FERC 61,269, CFTC Press Release 4555-01, CFTC Press Release 7683-18, and CFTC Press Release 5405-07.

\(^{20}\)Levine (2015)

\(^{21}\)Ridley and Freifeld (2015)

\(^{22}\)Leising (2017)

\(^{23}\)17 CFR Part 180, “Prohibition on price manipulation”, states: “It shall be unlawful for any person, directly or indirectly, to manipulate or attempt to manipulate the price of any swap, or of any commodity in interstate commerce, or for future delivery on or subject to the rules of any registered entity.”

\(^{24}\)17 CFR Part 180 “the Commission reiterates that, in applying final Rule 180.2, it will be guided by the traditional four-part test for manipulation that has developed in case law arising under 6(c) and 9(a)(2): (1) That the accused had the ability to influence market prices; (2) that the accused specifically intended to create or effect a price or price trend that does not reflect legitimate forces of supply and demand; 128 (3) that artificial prices existed; and (4) that the accused caused the artificial prices.”
notion, focusing instead on “intent” as the standard of proof for manipulation. Perdue (1987) argues that manipulation should be defined as conduct which “would be uneconomical or irrational, absent an effect on market price.” Fischel and Ross (1991), similarly, argue that manipulation can only reasonably be defined based on the intent of the trader.

Regulatory authorities have also largely used proof of intent as the primary basis for prosecuting manipulators. Charges are brought based on “smoking gun” evidence that trades were made with the intention of moving benchmark prices, rather than for any fundamental reason. Levine (2014) quotes a number of trader chat messages used in FX manipulation lawsuits. Other examples include the CFTC’s lawsuits brought against Parnon Energy, Inc. and others for crude oil manipulation, Energy Transfer Partners, L.P. and others for natural gas manipulation, and Barclays for ISDAFIX manipulation. The standard of intent is somewhat puzzling from an economic perspective: trades should have the same positive or negative effects on market quality regardless of the intent of the trader, so it is puzzling that trades should be prosecuted on the basis of bad intentions.

Some authors have also argued that manipulation should not be illegal. In a text on futures markets, Hieronymus (1977, pg. 328) argues that contract market manipulation will not survive under market competition. Fischel and Ross (1991) argue that “actual trades should not be prohibited as manipulative regardless of the intent of the trader”, similarly arguing that market competition is likely to deter manipulation. Markham (1991) similarly proposes to abandon the concept of manipulation, and to instead empower the CFTC to take a broader set of actions to maintain fair and orderly markets.

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25 See the CFTC’s website for Parnon Energy, Inc., Energy Transfer Partners, L.P., and Barclays.

26 “Manipulation is its own best cure. To manipulate a price is to put it where it doesn’t belong. The over-priced inventory or the underpriced commitment invariably leads to losses… In the actively speculated markets the forces of countervailing power effectively prevent manipulation. It is only in the thin markets that power plays cause minor distortions which are profitable. A speculator of moderate scale commented, “Show me a market that someone has distorted and I will show you a way to make money, both with him and against him.” The greater the level of speculation, the less is the amount of hanky-panky. Let them trade out.”
3 Model

There are two kinds of agents. There is a continuum of pure hedgers, who buy derivative contracts, but do not participate in spot markets. There are also \( n \) identical spot traders, who can trade both derivative contracts and spot goods. I assume that spot traders are negligibly small relative to pure hedgers. All agents are risk-averse with CARA utility over consumption, with identical risk aversions; that is, agent \( i \)'s utility if she attains consumption \( C \) is:

\[
U_i (C) = -e^{-\alpha C}
\]  

(1)

Agents play a 4-stage game:

1. Agents draw factor exposures \( x_i \sim N \left( 0, \sigma_x^2 \right) \).
2. Agents buy derivative contracts \( c_i (x_i) \).
3. The risk factor \( \psi \) is realized.
4. Spot traders bid for the spot good in an auction, and derivative contracts pay all agents based on the spot auction price.

In period 1, each spot trader and each pure hedger is endowed with \( x_i \) net units of a productive asset, where

\[
x_i \sim N \left( 0, \sigma_x^2 \right)
\]

In period 4, agents receive \( x_i \psi \) units of consumption; that is, the consumption value of each unit of the productive asset depends on a normally distributed risk factor \( \psi \), where:

\[
\psi \sim N \left( \mu_\psi, \sigma_\psi^2 \right)
\]

I thus refer to \( x_i \) as agents' factor exposures. The risk factor \( \psi \) represents, for example, future oil prices, volatility, stock prices, or interest rates; agents may be positively or negatively exposed to each of these sources of risk. I assume that factor exposures \( x_i \) are privately observed. Purely for analytical convenience, I renormalize utility by assuming that agents who have factor exposure \( x_i \) are also endowed with \(-x_i \mu_\psi\) units of
consumption; this implies that, if agents can perfectly hedge factor risk, they receive total consumption equal to 0.

In period 2, agents can trade derivative contracts to hedge their factor exposures. I use $c_i$ to denote the contract position of agent $i$. I assume that the contract market is competitive, so agents can purchase contracts at the actuarially fair price $\mu_\psi$. Under the assumptions of the model, this is the unique market-clearing price in the contract market. Agents’ contract positions are not observed by other agents. In period 3, the risk factor $\psi$ is drawn and commonly observed by agents, and in period 4 derivative contracts pay agents in consumption units.

Derivative contracts are useful because agents can purchase them prior to the realization of $\psi$, so they can be used to hedge factor exposures. If agents could purchase contracts directly tied to the risk factor – that is, contracts which pay $\psi$ consumption units per contract in period 4 – an agent with contract position $c_i$ would receive total consumption:

$$x_i \psi - x_i \mu_\psi + \psi c_i - p c_i$$

At price $p = \mu_\psi$, all agents would optimally hedge perfectly, setting $c_i = -x_i$. Since there is a continuum of agents and factor exposures are mean-0, markets for derivative contracts would clear, and agents would achieve the first-best outcome of perfect risk sharing.

In practice, however, agents cannot contract on abstract risk factors, such as oil prices, volatility, stock prices, or interest rates, directly. Agents instead trade contracts which are settled based on price benchmarks set in markets for spot goods, such as the Brent or WTI oil price indices, the VIX, the S&P 500 and Russell 2000, and LIBOR or SOFR. I model this by assuming that there is a single spot good, and a spot trader who purchases $z_i$ net units of the spot good attains consumption value:

$$\psi z_i - \frac{1}{2\kappa} z_i^2$$

The $\psi z_i$ component of (2) implies that the marginal consumption value of each spot trader for the spot good depends on the risk factor $\psi$. The quadratic component, $\frac{1}{2\kappa} z_i^2$,
implies that spot traders’ marginal consumption value of the spot good is decreasing. This can be thought of as a holding cost term: for physical goods such as oil or gas, these costs correspond to physical storage and infrastructure costs; for financial assets such as FX, interest rate swaps, or repo, these costs correspond to capital or balance sheet costs. Derivative contracts are useful because they are more leveraged than underlying assets, so they have no physical holding costs and substantially lower capital and balance sheet costs. For simplicity, I thus assume there is no holding cost for agents’ derivative positions, so agents’ contract positions are only limited by their ability to bear settlement price risk.

Since all spot traders agree that the marginal consumption value of the spot good is equal to $\psi$ when $z_i = 0$, a benchmark for $\psi$ can be constructed using any market mechanism which elicits spot traders’ marginal values for spot goods. I assume spot traders trade the spot good in a multi-unit double auction: in period 4, after contract positions $c_i$ are determined and $\psi$ is realized and observed, spot traders submit bid curves specifying how much of the spot asset they are willing to purchase at any given price (where prices and payments are denominated in units of consumption consumption). The auction clears at some price, traders trade the spot good, and the auction clearing price is used to determine payments on traders’ contract positions.

Combining all terms, a spot trader’s total consumption, over her factor exposures and outcomes in both the contract and spot markets, is:

$$C_{\text{spot}}(x_i, c_i, z_i, p, \psi) = \psi x_i - \mu \psi x_i + \mu \psi c_i + p c_i + z_i \psi - \frac{1}{2} \kappa z_i^2 - p z_i$$

A hedger’s total consumption is:

$$C_{\text{hedger}}(x_i, c_i, p, \psi) = \psi x_i - \mu \psi x_i + \mu \psi c_i + p c_i$$
3.1 The competitive-bidding benchmark

If all traders behaved competitively in the spot market, derivative contracts would implement the first-best outcome of perfect risk sharing. To see this, note that in period 4, each spot trader’s marginal consumption value of the spot good is the derivative of (2) with respect to \( z_i \):

\[ \psi - \frac{z_i}{k} \]  

(5)

If spot traders behaved competitively, bidding to equate their marginal consumption values of the spot good to prices, all traders would bid:

\[ z_{Bi}(p) = \kappa (\psi - p) \]  

(6)

The unique auction clearing price would be \( p = \psi \), so the auction price would exactly equal the risk factor. Since traders’ bids are identical, there would be no trade of the spot good; this is efficient, since traders’ valuations for the spot good are identical. Anticipating that \( p = \psi \), all agents would buy contracts fully hedge their factor exposures, and agents could thus implement first-best risk sharing with no trade in spot markets.

The first-best outcome is not attainable in equilibrium, however, because spot traders are strategic and recognize that they have price impact in the spot market. A spot trader who has a negative factor exposure \( x_i \) will buy a positive contract position \( c_i \) to hedge factor risk; she then has an incentive to raise her bid in the spot auction to increase \( p \) and thus her contract payoffs. I analyze how these incentives play out in equilibrium in the following section.

If the auction price \( p \) differs from the risk factor \( \psi \), there is no way for hedgers, who are only active in contract markets, to fix this supposed mispricing, because cash-settled derivative contracts are only linked to the risk factor \( \psi \) through the auction price benchmark \( p \). A cash-settled derivative contract, at settlement, is simply a bet on the price benchmark \( p \). If spot traders are able to distort \( p \) upwards relative to \( \psi \), then the fair price of the derivative contract at settlement is the distorted benchmark price.
3.2 Discussion of model assumptions

The baseline model is intentionally stylized in order to illustrate the main intuitions behind the results, but the model can be extended in many ways. In section 5, I allow spot traders to have heterogeneous holding capacities for the spot good, arbitrary and heterogeneous distributions of contract positions, and arbitrarily distributed inventory shocks for the spot good. Agent could purchase contract positions to speculate rather than to hedge factor risk exposures: appendix B.3 allows agents to have heterogeneous beliefs about the mean $\mu_\psi$ of the risk factor, and shows that this generates contract purchasing decisions which are isomorphic to factor exposures $x_i$. This complicates welfare analysis for spot traders, as gains from trade in the contract market depend on whether we believe contracts are purchased to hedge true factor risks, or because of disagreement about the future distribution of factor realizations.

I assume that agents can purchase contracts at the actuarially fair price of $\mu_\psi$. Under the assumptions of the model – there are a continuum of hedgers, spot traders are negligibly small relative to hedgers, both spot traders and hedgers have no information about the future realization of $\psi$, and factor exposures of all agents have mean 0 – $\mu_\psi$ is the unique price which clears the contract market. These assumptions are a reasonable approximation for many contract markets in practice: subsection 2.1 shows that contract markets are often much larger than spot markets, and section 6 shows that, in the gold market, position limits imply that spot traders in the LBMA gold auction could not hold more than around 2% of total contract volume. If spot traders are large enough that their trades move contract prices, appendix B.4 shows that, as long as contract purchases have linear price impact, spot traders’ optimal contract purchases remain linear functions of their factor exposures. However, I have not solved for equilibrium of a model in which agents have price impact in the contract market.

I use auctions as a reduced-form model for benchmark setting. Some benchmarks, such as the CBOE VIX and the LBMA gold price, are in fact set using uniform-price auctions. Other benchmark setting mechanisms may be reasonably approximated by auctions: for example, benchmarks for oil and gas, the WM/Reuters FX benchmark and the ISDAFIX interest rate swap benchmark are based on trades of relatively homogeneous commodities over relatively short periods of time. Auctions are a less
appropriate model for benchmarks based on underlying markets with large distortions other than market power, and the analysis of this paper may not be suitable for these settings. Some assets are traded in decentralized markets with large search frictions; the assumption that all trades happen at the same price is not appropriate. In other markets, such as the interbank loan market which the LIBOR interest rate benchmark is based on, trades are difficult to verify, so false reporting is an important concern. I briefly discuss the extent to which my model applies to different benchmark-setting mechanisms in subsection 7.2.

Throughout the paper, I take the number of spot market participants, $n$, and their spot good holding capacities, $\kappa$, as exogeneous; I am essentially analyzing contract market manipulation holding fixed spot market structure. Spot market structure is not static in practice – in the long run, agents can enter spot markets, and make investments to change their spot holding capacity $\kappa$. However, in most cases, these changes are costly and take time. For physical goods such as oil, gas and electricity, spot trading capacity depends on expensive infrastructure investments, such as pipelines and storage facilities, which are not flexible over short time horizons. For financial assets, many benchmarks, such as the LBMA gold price, SOFR, SONIA, and the WM/Reuters fix, are set based on prices in inter-dealer markets, which do not allow free entry, and market participants’ holding capacities for spot assets depend on exogeneous factors such as leverage constraints and balance sheet costs, which are also hard to quickly adjust. Regulators could use the model and metrics of this paper to assess whether a spot market, under its current structure, is vulnerable to manipulation. These metrics would need to be updated over time as entry, exit, and capacity shifts change the structure of spot markets.

On a related note, I use a static model of the spot good market: I do not allow spot traders to buy and store the spot good prior to the realization of $\psi$. For some goods, such as electricity, storage is impossible, so this is a realistic assumption. For goods that can be stored, spot traders would generally prefer to hedge factor risk using derivatives rather than to hold the spot good directly, because spot goods generally have

\[27\] A large literature has demonstrated that dealers’ leverage constraints and balance sheet costs affect their ability to provide liquidity in various markets; see, for example, [Du, Tepper and Verdelhan (2018)] in FX derivatives markets, [Adrian, Boyarchenko and Shachar (2017)] in corporate bond markets, and [Adrian, Etula and Muir (2014)] and [He, Kelly and Manela (2017)] for evidence on a variety of asset classes.
higher holding costs. Storing physical goods such as oil and gas requires costly physical infrastructure, and due to the higher leverage of derivatives, buying and holding financial assets well in advance of contract settlement generally requires more capital than holding the corresponding derivative contract.  

For goods that can be stored, dynamic interactions between the spot and contract markets could lead to a variety of interesting effects. Spot traders who expect contract markets to be vulnerable to manipulation might instead hedge factor risk by holding spot goods directly, despite their higher holding cost. A spot trader who intends to manipulate contract settlement prices might also store spot goods well in advance to increase spot trading capacity, effectively increasing the trader’s holding capacity $\kappa$ at the time of settlement. These are interesting directions to explore, but I leave these to future work.

## 4 Equilibrium

Proposition 1 describes equilibrium values of spot traders’ equilibrium bid curves, auction prices, spot traders’ and hedgers’ contract purchasing decisions, and spot trader and hedger welfare.

**Proposition 1.** For any $\alpha, \sigma^2_\psi, \kappa, \sigma^2_c, n$, there is a unique equilibrium, in which traders’ spot market bids are:

$$z_{Bi}(p; c_i, \psi) = \frac{1}{n-1} c_i - \frac{n-2}{n-1} \kappa (p - \psi)$$

Spot auction prices are:

$$p - \psi = \frac{\sum_{i=1}^{n} c_i}{n(n-2)\kappa} \sim N\left(0, \frac{\sigma_c^2}{n(n-2)^2\kappa^2}\right)$$

---

28Note that derivatives are not the only way to attain leverage: for example, market participants can also lever up purchases by buying assets and borrowing against them. For example, many market participants in fixed income achieve leverage by funding purchases in repo markets. Repo is often the most efficient way to fund specific fixed income purchases, whereas exchange-traded derivatives are more efficient for trading broad sources of factor risk. For example, a trader who wants to lever up a specific mortgage-backed security or corporate bond would likely buy it and borrow against it; a trader who wanted to speculate or hedge on interest rates more generally would likely trade interest rate swaps, or Eurodollar or Treasury futures.
Spot traders’ contract positions $c_i (x_i)$ are linear in traders’ factor exposures $x_i$, so contract positions are normally distributed with mean 0 and variance $\sigma^2_c$, where:

$$c_i (x_i) = -tx_i, \quad \sigma^2_c = t^2 \sigma^2_x$$  \hspace{1cm} (9)

where $t$ satisfies:

$$t \equiv \left( 1 + \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_\psi \kappa) ((n^2 - 2n) \kappa + \alpha \sigma^2_n)} \right)^{-1}$$  \hspace{1cm} (10)

and $\sigma^2_n$ is the unique positive value satisfying:

$$\sigma^2_n = \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_\psi \kappa) ((n^2 - 2n) \kappa + \alpha \sigma^2_n)}$$  \hspace{1cm} (11)

$$\sigma^2_n > \frac{\kappa - (\alpha \sigma^2_\psi) (n^2 - 2n) \kappa^2}{\alpha (1 + \alpha \sigma^2_\psi \kappa)}$$  \hspace{1cm} (12)

Spot traders’ expected welfare, as a function of $t$, is:

$$-\sqrt{\frac{(n^2 - 2n) \kappa}{\alpha (\frac{\sigma^2_x}{n-1} + (n^2 - 2n) \kappa)}} \left( 1 - \alpha \sigma^2_x \left( \alpha \sigma^2_\psi (1 - t)^2 + \frac{\alpha \left( \frac{\sigma^2_x}{n-1} \right) - \kappa}{(\alpha \sigma^2_\psi \kappa) ((n^2 - 2n) \kappa + \alpha \left( \frac{\sigma^2_x}{n-1} \right))} \right) t^2 \right)$$  \hspace{1cm} (13)

Pure hedgers’ contract positions are mean-0 normally distributed, satisfying:

$$c_i (x_i) = -\frac{\mathnormal{\sigma^2_\psi}}{\mathnormal{\sigma^2_\psi} + \mathnormal{Var (p - \psi)}^2} x_i$$  \hspace{1cm} (14)

Pure hedgers’ consumption has mean 0, and hedgers’ consumption variance, over uncertainty in
\( \psi \) and all \( x_i \)'s, is:

\[
\frac{\text{Var}(p - \psi)}{\text{Var}(p - \psi) + \sigma_\psi^2 x_i^2}
\]

4.1 Spot market distortions and manipulation-induced basis risk

Spot traders’ equilibrium bids, (7), differ from their competitive bids, (6), in two ways. First, as is known in the double-auctions literature, traders shade their bids due to their price impact. The slope of traders’ bid curves with respect to prices is lower than the slope of their competitive bids, by a factor:

\[
\frac{n - 2}{n - 1}
\]

Second, traders’ equilibrium bid curves depend on traders’ contract positions \( c_i \), even though these do not affect traders’ marginal consumption value for spot goods. Intuitively, if a spot trader holds contracts whose payoff depends on an auction price, and the trader can move the auction price by trading the spot good, she has an incentive to trade the spot good in a way that increases her profits from her contract position. I will refer to this phenomenon as manipulation.

Expression (7) shows that increasing a trader’s contract position by 1 unit causes her to increase her spot good bid curve by \( \frac{1}{n - 1} \) units; as \( n \) increases, contract positions affect bids less. Intuitively, when \( n \) is large and auctions are competitive, agents need to buy more of the spot good to move prices a given amount; the cost of manipulation is higher, so spot traders manipulate less per unit contract that they hold. In the limit as \( n \) grows large, expression (7) converges towards traders’ competitive bids, (6): if \( n \) is infinite and traders have no price impact, they simply bid honestly in the auction, and their contract positions do not affect their bids.

Since auction prices depend on contract positions \( c_i \), which depend on traders’ random factor exposures \( x_i \), in equilibrium, the auction price \( p \) is a noisy signal of the risk factor \( \psi \). The difference \( p - \psi \) is normally distributed, and expression (8) characterizes its variance; I call this variance manipulation-induced basis risk. Basis risk is higher when the variance in spot traders’ contract positions \( \sigma_c^2 \) is larger, when the number of traders \( n \) is
smaller, and when the traders’ spot good holding capacities \( \kappa \) are lower. Intuitively, this is because basis risk is the product of how large spot traders’ contract positions are, and how much traders move prices per unit contract that they hold; the former is simply \( \sigma_c^2 \), and the latter depends \( n \) and \( \kappa \). Basis risk decreases rapidly as markets become more competitive, declining cubically in \( n \). This is because increasing \( n \) both decreases how much contract positions affect bids through (7), and also increases the slope of aggregate auction demand, so a given change in bids affects auction prices less.

### 4.2 Spot traders’ contract purchases

Expression (9) of proposition 1 shows that, in the unique equilibrium of the model, agents’ optimal contract purchases \( c_i \) are linear in their factor exposures \( x_i \). The coefficient of proportionality, defined as \( t \) in expression (10), can be interpreted as a hedging aggressiveness parameter. If spot markets are perfectly competitive, all agents fully hedge their factor exposures, so \( t = 1 \). The equilibrium value of \( t \) can be greater or smaller than 1. On the one hand, from a given trader’s perspective, manipulation by other traders create basis risk, lowering her incentive to buy contracts. On the other hand, each trader anticipates that they can make profits from moving spot prices in favor of their contract positions, increasing their incentive to buy contracts. When the second force dominates, so \( t > 1 \), spot traders buy contract positions which are larger than their risk exposures \( x_i \), incurring additional factor risk so that they can hold larger contract positions, allowing them to generate higher profits from manipulation.

From expression (10), \( t \) is greater or smaller than 1 depending on whether:

\[
\kappa > \alpha \sigma^2_{\eta}
\]

where \( \sigma^2_{\eta} \), from (11), is the expected variance of residual supply in the spot market. Intuitively, when residual supply has low variance, spot traders’ risk aversion \( \alpha \) is low, and spot traders’ holding capacity \( \kappa \) is high, the expected profits from manipulation are large relative to the costs, so spot traders will tend to buy larger contract positions.

Increasing \( n \), holding other parameters fixed, causes \( t \) to rapidly converge to its competitive value of 1: appendix B.1 shows that \( t - 1 \) converges to 0 at rate \( \frac{1}{n^2} \).
4.3 Spot trader welfare

Expression (13) shows spot traders’ expected welfare, over factor risk as well as all agents’ factor exposures, as a function of the hedging aggressiveness parameter \( t \). This allows us to answer two questions. First, we can compare spot traders’ equilibrium welfare to their welfare in the competitive benchmark, to see whether manipulation makes spot traders better off. Second, we can study the effects of other counterfactual values of \( t \) on spot traders’ welfare. Varying \( t \) in expression (13) can be thought of as a limited social planner’s problem: (13) is what spot traders’ welfare would be if a planner could force all spot traders to buy \( t \) contracts for each unit of their factor exposures \( x_i \), but could not influence traders’ bids in the spot market. We can analyze whether spot traders, correctly anticipating manipulation in the spot market, would collectively prefer a larger or smaller value of \( t \) than its equilibrium value.

As appendix B.4 shows, regulators could in fact implement any positive choice of \( t \) by imposing quadratic taxes or subsidies on spot traders’ contract positions, charging spot traders \( k c_i^2 \) for buying \( c_i \) contracts. To my knowledge, these kinds of taxes are not used in practice, but regulators and exchanges do impose position limits on the size of market participants’ contract positions; restricting \( t \) to below its equilibrium value can be thought of as an approximate model for these position limits.\(^{29}\)

The three panels of figure 1 shows spot traders’ welfare as a function of \( t \), alongside the equilibrium and competitive values of \( t \) and spot trader welfare, for three different sets of input parameters. A given spot trader’s manipulation generates both positive and negative externalities on other spot traders, so a number of different outcomes are possible: spot traders may gain or lose on average from manipulation, the equilibrium \( t \) may be greater or smaller than 1, and spot traders may hedge more or less than is optimal for spot traders’ welfare.

In the left panel of figure 1, manipulation increases spot traders’ welfare: welfare at the equilibrium \( t \) is greater than -1, which is spot traders’ welfare in the competitive benchmark. Spot traders also over-hedge, choosing \( t > 1 \) in equilibrium. The orange line is to the left of the blue line, so spot traders would do even better as a group if they

\(^{29}\)I do not directly analyze contract position limits because my model requires agents’ contract positions to be Gaussian, so bounds on the size of agent’s contract positions would be intractable.
could collectively commit to holding smaller contract positions. Spot traders face a kind of prisoner’s dilemma: manipulation by one spot trader creates a negative externality on other spot traders by increasing basis risk, so all spot traders would prefer a lower value of \( t \) than the equilibrium value. A lower value of \( t \) would also benefit pure hedgers, since it would decrease basis risk, so a regulator could create a Pareto improvement by imposing taxes on spot traders’ contract positions.

In the middle panel of figure 1, spot traders are once again over-hedging – the equilibrium \( t \) is greater than 1 – but spot traders would actually collectively prefer an even higher value of \( t \). This is because manipulation by one spot trader actually creates a positive externality on other spot traders in spot markets. When a manipulator trades non-fundamentally in spot markets to move settlement prices, other spot traders profit as market makers, buying low and selling high in the spot auction. In this case, spot traders face a kind of coordination problem rather than a prisoner’s dilemma: since manipulation by one trader benefits other traders, all traders would benefit if everyone would commit to buying more contracts and manipulating more. Spot traders would thus benefit from subsidies to traders’ contract positions; however, this would increase basis risk and thus decrease hedgers’ welfare, and would not be Pareto improving.

In the right panel of figure 1, spot traders’ equilibrium welfare is below the competitive value of \(-1\); that is, spot traders’ welfare losses from increased basis risk outweigh their gains from extracting profits from hedgers. Since hedgers are always unambiguously worse off in equilibrium relative to the competitive outcome, this example shows that manipulation can be Pareto disimproving, decreasing welfare for both spot traders and hedgers. The orange line lies to the left of the blue line; thus, analogous to the leftmost panel, spot traders would collectively benefit from hedging less aggressively, and taxes on spot traders’ contract positions would be Pareto improving.

### 4.4 Comparative statics

Figure 2 illustrates the effects of varying input parameters on equilibrium outcomes. When spot traders’ risk aversion \( \alpha \) is low, traders are more willing to bear factor risk in order to attain manipulation profits, so the equilibrium and spot-trader-optimal values
Notes. Expected welfare of spot traders, as a function of hedging aggressiveness parameter $t$. The green lines denote the competitive values of $t$ and welfare, which are always equal to 1 and -1 respectively. The blue line denotes the unique equilibrium value of $t$, and the orange line denotes the value of $t$ which maximizes spot traders’ welfare (the black line). The parameters for the left panel are $n = 3, \alpha = 1, \kappa = 0.5, \sigma^2_\psi = 0.3, \sigma^2_\chi = 0.5$; for the middle panel, they are $n = 3, \alpha = 1, \kappa = 0.8, \sigma^2_\psi = 1.5, \sigma^2_\chi = 0.05$; for the right panel, they are $n = 5, \alpha = 1, \kappa = 0.05, \sigma^2_\psi = 0.9, \sigma^2_\chi = 1$.

of $t$ increase, causing price variance to increase. When the spot good holding capacity $\kappa$ decreases, price variance increases. The welfare-maximizing value of $t$ for spot traders tends to be lower than the equilibrium $t$ when $\kappa$ is low, because the negative basis risk externalities from manipulation are larger.

Decreasing the variance of the risk factor, $\sigma^2_\psi$, makes spot traders more willing to buy large contract positions and manipulate, which increases $t$ in equilibrium and increases basis risk. Increasing agents’ factor exposure variance, $\sigma^2_\chi$, increases price variance, but actually decreases the equilibrium and socially optimal values of $t$, as it becomes more costly for agents to deviate from full hedging. Larger factor exposures also imply that spot traders’ welfare losses are larger, and because negative basis risk externalities are larger, the spot-trader welfare maximizing $t$ tends to fall below the equilibrium $t$ when $\sigma^2_\chi$ is large. Finally, increasing $n$ causes all parameters to converge rapidly to their competitive values: the equilibrium $t$ converges to 1, and price variance and net spot trader welfare losses from manipulation converge to 0.
Figure 2: Comparative statics

Notes. Comparative statics of the equilibrium and spot trader welfare-maximizing values of \( t \), spot traders’ equilibrium welfare gain (minus the competitive equilibrium value of -1), and equilibrium price variance, \( \text{Var}(p - \psi) \), as input parameters vary. The baseline values that parameters are varied around are \( n = 3, \alpha = 1, \kappa = 0.8, \sigma^2_\psi = 1.5, \sigma^2_\chi = 0.05 \).
4.5 Hedger welfare

4.5.1 The basis risk coefficient $\rho$

Figures [1 and 2] show that the effects of manipulation on spot traders’ welfare are complex, as there are multiple channels of effect through the spot and contract markets. The effects of manipulation on pure hedgers are much simpler. Manipulation makes benchmark prices noisier signals of $\psi$, so derivatives are less effective for hedging factor exposures. Expression (14) shows that pure hedgers respond by hedging less per unit factor exposure they have, and expression (15) shows how manipulation increases pure hedgers’ consumption variance.

From expression (15), we can also derive a simple summary statistic for pure hedgers’ welfare losses from basis risk. In the absence of derivative contracts, an agent with factor exposure $x_i$ is exposed to consumption variance

$$\sigma^2_{\psi} x_i^2$$

(16)

If derivatives were settled directly based on $\psi$, agents would hedge perfectly and face no consumption variance (conditional on $x$). With basis risk, agents can only partially hedge. Based on (15), we can define a simple measure of hedgers’ welfare losses from manipulation.

**Definition 1.** Define the *basis risk coefficient* $\rho$ as:

$$\rho \equiv \frac{\text{Var}(p - \psi)}{\text{Var}(p - \psi) + \sigma^2_{\psi}}$$

(17)

From (15), hedgers’ consumption variance can be written as:

$$\rho \sigma^2_{\psi} x_i^2$$

hence, $\rho$ is simply the fraction of total factor risk that hedgers are exposed to, assuming they purchase contract optimally given anticipated basis risk. $\rho = 0$ when basis risk is 0 and contracts pay exactly $\psi$, and $\rho = 1$ when basis risk is infinitely large, so derivatives
are useless for hedging factor risk. Since hedgers have CARA utility and prices are normally distributed, and since the expectation of hedgers’ contract payoffs is always equal to \( \psi \), in the context of the model, \( \rho \) fully captures hedgers’ welfare losses from manipulation.

Using expression (8) for \( \text{Var} (p - \psi) \), we can write \( \rho \) as:

\[
\rho = \frac{\sigma_c^2}{n(n-2)\kappa^2} + \frac{\sigma^2}{n(n-2)\kappa^2}
\]

Qualitatively, \( \rho \) is high when, relative to factor risk \( \sigma^2 \), spot traders’ contract positions \( \sigma_c^2 \) are large; spot traders’ holding capacities \( \kappa \) are low, so the price impact of spot trades is higher; and \( n \) is lower, so auctions are less competitive.

4.5.2 The expected manipulation rent \( \tau \)

Another simple welfare metric for hedgers is the expected net transfer from pure hedgers to spot traders.

Definition 2. Define the expected manipulation rent \( \tau \) as:

\[
\tau \equiv E \left[ (p - \mu_\psi) \sum_{i=1}^{n} c_i \right] \tag{18}
\]

Expression (18) is simply the expectation of the sum of ex-post payoff of spot traders’ contract positions, \( pc_i \), minus the ex-ante cost, \( \mu_\psi c_i \); it thus measures the expected accounting profits that spot traders as a group extract from hedgers, because they can move spot prices in the direction of their aggregate contract position. These profits do not, as in standard informed-trader models, come from better signals or foreknowledge of the risk factor \( \psi \); they arise purely from spot traders’ ability to move the basis, \( p - \psi \). \( \tau \) is also not a complete measure of spot traders’ net economic profits from manipulation, since spot traders suffer inventory costs in spot markets in order to move prices. \( \tau \) simply measures the total expected losses of pure hedgers as a result of manipulation.

In the context of the model, since we have assumed that spot traders are negligibly
small and that all contracts are traded at price $\mu_\psi$, manipulation rents are infinitesimally diffused among hedgers, so only basis risk matters for hedgers’ welfare. In practice, if spot traders’ contract positions are a small fraction of the market, basis risk will have a larger impact on hedgers’ welfare than manipulation rents, because manipulation rents are diffused among hedgers and basis risk is not. However, the manipulation rent metric may still be practically useful, for example, as a simple summary metric of spot traders’ manipulation-induced damages in legal proceedings. In the empirical application to gold futures contracts in section 6, I show that manipulation rents are nontrivially large in absolute terms, but are quite small on a per-contract basis, and are around an order of magnitude smaller than the standard deviation in contract payoffs created by basis risk.

Appendix B.2 shows that manipulation rents can be written as:

$$\tau = \frac{\sigma^2_c}{(n - 2) \kappa} \quad (19)$$

Qualitatively, the forces that affect $\tau$ and $\rho$ in the symmetric model are similar: both are increasing in $\sigma^2_c$, so distortions are larger when spot traders’ contract positions are larger, and are decreasing in $n$ and $\kappa$, so distortions are smaller when the spot market is more competitive and agents’ spot good holding capacities are larger.

5 Generalizations

In this section, I focus on spot traders’ bidding decisions in period 4 of the model, taking their contract positions as exogeneous random variables. This allows me to generalize the spot market model substantially: in subsection 5.1 I allow traders’ holding capacities for the spot good to differ, and traders’ contract positions to have heterogeneous variances and covariances; in subsection 5.2 I allow traders to receive inventory shocks for the spot good, so that not all trade in the spot good is generated by manipulation. In the more general model, I show that the basis risk coefficient $\rho$, and manipulation rents $\tau$, are a function of agents’ bid slopes, and the variances and covariances of agents’ contract positions and inventory shocks.
5.1 Asymmetric agents

First, I relax symmetry of spot traders. Assume that spot market participants’ consumption value for buying \( z_i \) units of the spot good at price \( p \) is:

\[
C_{\text{spot},i}(c_i, z_i, p) = pc_i + \psi z_i - \frac{z_i^2}{2\kappa_i} - pz_i \tag{20}
\]

Comparing (20) to (3), I have omitted the factor exposure and contract price terms in consumption, as they are fixed and sunk costs once contract positions are determined and \( \psi \) is realized. Expression (20) allows spot market participants to have heterogeneous holding capacities, \( \kappa_i \), for the spot good. Certain market participants may have lower capital costs, or lower infrastructure costs for storing spot goods, allowing them to buy and sell large quantities of spot goods more easily.

Second, I require spot traders’ contract positions to have mean 0, full support, and finite second moments, but otherwise I allow them to be arbitrarily distributed. As the previous section shows, spot traders’ contract purchasing decisions depend in complex ways on factor exposures, basis risk, and expected profits from manipulation. However, in the period 4 spot market, once \( \psi \) is realized, spot traders’ manipulation incentives depend only on their net contract positions. We can thus measure spot traders’ manipulation incentives, and the impact of manipulation on hedger welfare, taking spot traders’ contract positions as given, without explicitly modelling contract purchasing decisions.

**Proposition 2.** There is a unique linear ex-post equilibrium in the asymmetric model, in which \( i \) submits the bid curve:

\[
z_{Bi}(p; c_i) = \frac{b_i}{\sum_{j \neq i} b_j} c_i - b_i (p - \psi) \tag{21}
\]

The spot auction price is

\[
p - \psi = \frac{1}{\sum_{i=1}^{n} b_i} \sum_{i=1}^{n} b_i c_i \tag{22}
\]

**bid slopes** \( b_i \) satisfy:

\[
b_i = \frac{B + 2\kappa_i - \sqrt{B^2 + 4\kappa_i^2}}{2} \tag{23}
\]
and $B = \sum_{i=1}^{n} b_i$ is the unique positive solution to the equation

$$B = \sum_{i=1}^{n} \frac{2\kappa_i + B - \sqrt{B^2 + 4\kappa_i^2}}{2}$$

(24)

From claim 5 in appendix C.1, agents’ optimal bids can alternatively be written as:

$$z_{Bi}(p; c_i) = \frac{\kappa_i}{\kappa_i + d_i} c_i - \frac{\kappa_i d_i}{\kappa_i + d_i} (p - \psi)$$

When agents are asymmetric, the coefficient $\frac{\kappa_i}{\kappa_i + d_i}$ on $c_i$ is larger for agents with higher $\kappa_i$. Since higher $\kappa_i$ agents are larger relative to the market, price impact is cheaper in relative terms, hence they shift their bids more per unit contract that they hold. As a result, expression (31) shows that equilibrium prices depend on a weighted sum of $c_i$ and $y_i$, where the weights on $c_i$ are higher for agents with larger values of $\kappa_i$: agents with higher spot good holding capacities have larger effects on auction prices.

The variance of auction prices is:

$$\text{Var} (p - \psi) = \text{Var} \left[ \frac{1}{\sum_{i=1}^{n} b_i} \sum_{i=1}^{n} \left( \frac{b_i}{\sum_{j \neq i} b_j} \right) c_i \right]$$

(25)

To express (25) in more intuitive notation, define

$$B \equiv \sum_{i=1}^{n} b_i$$

as the sum of agents’ bid slopes $b_i$, as in proposition 2; define the vector $k_c$ of coefficients as:

$$k_c = \begin{pmatrix} \frac{b_1}{\sum_{j \neq 1} b_j} \\ \vdots \\ \frac{b_n}{\sum_{j \neq n} b_j} \end{pmatrix}$$
And define the covariance matrix of agents’ contract position, $\Sigma_{cc}$, with elements:

$$\Sigma_{cc}(i, j) = \text{Cov}(c_i, c_j)$$

We can then write expression (25) for basis risk as:

$$\text{Var}(p - \psi) = \frac{1}{B^2} (k_c' \Sigma_{cc} k_c) \quad (26)$$

Since the coefficient vector $k_c$ is a function of agents’ bid slopes, only two quantities are required to estimate expression (26) for basis risk: traders’ spot auction bid slopes, $b_i$, and the variances and covariances of spot traders’ contract positions. By law in the US, commodity derivatives must be traded on regulated exchanges, and most financial derivatives are now also subject to fairly stringent reporting requirements; thus, regulators can conceivably estimate the variances and covariances of agents’ contract positions $c_i$ using data on agents’ historical contract positions. We can also estimate the variance of factor risk, $\sigma_{\psi}^2$, simply as the volatility of prices over a given hedging horizon. We can then construct the basis risk coefficient $\rho$, as in subsection 4.5, as:

$$\rho \equiv \frac{\text{Var}(p - \psi)}{\text{Var}(p - \psi) + \sigma_{\psi}^2} \quad (27)$$

Similarly, expression (18) for expected manipulation rents, in matrix form, can be written as:

$$\tau = \frac{1}{B} (k_c' \Sigma_{cc} 1) \quad (28)$$

where $1$ is a length-$n$ unit vector. Expression (28) can also be estimated using only the vector of bid slopes $b_i$, and the contract position covariance matrix $\Sigma_{cc}$.

## 5.2 Inventory shocks

Suppose that spot market participants’ consumption value for buying $z_i$ units of the spot good at price $p$ is:
Expression (29) assumes that agents enter the spot market with some existing inventory position, \( y_i \), in the spot asset. I assume that \( y_i \) is independent of the risk factor \( \psi \); as is common in the literature, I assume that each \( y_i \) has mean 0, and I assume each \( y_i \) has full support and finite variances and covariances, but otherwise \( y_i \) can be arbitrarily distributed. Inventory shocks imply that agents will trade the spot good even in the absence of derivative contracts and manipulation, so not all trade volume is due to manipulation.

**Proposition 3.** There is a unique linear ex-post equilibrium in the asymmetric model, in which \( i \) submits the bid curve:

\[
\begin{align*}
Z_{Bi} \left( p; y_i, c_i \right) &= - \frac{b_i}{\kappa_i} y_i + \frac{b_i}{\sum_{j \neq i} b_j} c_i - b_i (p - \psi) \\
\end{align*}
\]

The unique equilibrium price is:

\[
\begin{align*}
p - \psi &= \frac{1}{\sum_{i=1}^{n} b_i} \left[ \frac{b_i}{\kappa_i} y_i + \frac{b_i}{\sum_{j \neq i} b_j} c_i \right] \\
\end{align*}
\]

where bid slopes are described by expressions (23) and (24) in proposition 2.

Once again, define vectors of inventory coefficients as:

\[
k_y = \begin{pmatrix}
\frac{b_1}{\kappa_1} \\
\vdots \\
\frac{b_n}{\kappa_n}
\end{pmatrix}
\]

Inventory shocks \( y_i \) can alternatively be interpreted as preference shocks for the spot good: expanding the quadratic term in (29), we have:

\[
- \frac{z^2}{2\kappa_i} - \frac{y_i z}{\kappa_i} - \frac{y_i^2}{2\kappa_i}
\]

Ignoring the constant \( \frac{y_i^2}{2\kappa_i} \) term, \( y_i \) simply linearly shifts \( i \)'s marginal consumption value of the spot good.
And, define the inventory position covariance matrix $\Sigma_{yy}$, and the inventory-contract covariance matrix $\Sigma_{yc}$, respectively as:

$$
\Sigma_{yc}(i,j) = \text{Cov}(y_i, c_j)
$$

$$
\Sigma_{yy}(i,j) = \text{Cov}(y_i, y_j)
$$

Analogous to (26) and (28), we can express basis risk and manipulation rents as:

$$
\text{Var}(p - \psi) = \frac{1}{B^2} \left( k'_y \Sigma_{yy} k_y - 2k'_y \Sigma_{yc} k_c + k'_c \Sigma_{cc} k_c \right)
$$

$$
\tau = \frac{1}{B} \left( -2k'_y \Sigma_{yc} 1 + k'_c \Sigma_{cc} 1 \right)
$$

Thus, basis risk and manipulation rents are a function of bid slopes $b_i$, holding capacities $\kappa_i$, and the variances and covariances of agents’ contract positions and inventory shocks. Holding capacities $\kappa_i$ can be estimated using all agents’ bid slopes $b_i$, and I show in the next section that, under some assumptions, inventory variances can be estimated using data on trade volumes in spot markets.

The spot market model of this section is more realistic than the baseline model of section 3 as it allows many forms of heterogeneity between spot traders. However, it does not endogenize spot traders’ contract purchasing decisions, so it cannot be used to conduct a full analysis of spot trader welfare. In bringing the model to data, I focus on hedgers’ welfare, rather than attempting to estimate spot traders’ welfare, for two reasons. First, the dollar value of hedgers’ welfare gains and losses is likely larger than the dollar value of spot traders’ welfare changes in most markets, because hedgers’ gross contract positions are often much larger than spot traders’ contract positions and the volume of spot market trade.

Second, empirically estimating spot traders’ welfare is difficult, even in the simple symmetric model of section 3. Spot traders’ welfare depends on their risk aversions and factor exposures, which are difficult to estimate empirically. Moreover, as appendix B.3 shows, spot traders’ contract purchases could be driven by heterogeneous beliefs rather than hedging demand, complicating welfare analysis further. In contrast, manipulation-induced basis risk and manipulation rents are sufficient statistics for hedgers’ welfare; the
variances and covariances of spot traders’ contract positions matter, but not the particular reasons why spot traders entered into contract positions.

6 The LBMA Gold Price

In this section, I address the following question: how large would basis risk and manipulation rents be, if COMEX gold futures contracts were cash-settled using the London Bullion Market Association (LBMA) gold price benchmark? There are many sources of uncertainty in the LBMA gold auctions that may affect manipulation risk, such as the number of auction participants, the absolute and relative sizes of their bid slopes, and correlations between their contract positions and inventory shocks. I show that, across all of these sources of uncertainty, I can estimate informative bounds on basis risk and manipulation rents, which imply that both are small.

6.1 Background

The LBMA gold price is a benchmark price for gold, set twice each business day at 10:30AM and 3:00PM London time. While the popular COMEX gold futures contract (ticker symbol GC) is physically settled, the LBMA gold price is used to settle some exchange-traded gold futures and options as well as some OTC gold derivatives (FCA, 2014). The LBMA gold price is also used by other market participants, such as miners, central banks, and jewellers, for purposes such as inventory valuation (Aspris et al., 2015).

Prior to 2015, the price was set in a private teleconference auction between five members; this took between 10-15 minutes to conclude, and data on auction bidding was not made public. In 2014, the five banks involved in setting the LBMA gold price were accused of manipulation (Reuters Staff, 2014); the UK Financial Conduct Authority fined Barclays $43.9 million for bidding strategically to move benchmark prices, in order to avoid paying USD $3.9m to a customer who held a benchmark-linked option contract (FCA, 2014). In 2015, the ICE Benchmark Administration (IBA) took over the administration of the LBMA gold price; IBA moved to an electronic auction system.

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31 Gold Futures and Gold Options: Further Information
allowing more participants to enter, and began publishing detailed information about bids in intermediate rounds of the auction.

The LBMA gold price is now determined in a static auction which uses a dynamic price setting mechanism. During each round, IBA publishes a round price, and participants have 30 seconds to enter how much gold they would be willing to buy or sell at the announced price. If the difference between buy and sell volume is within an imbalance threshold – during the time period that my data covers, usually 10,000oz – the auction concludes and agents buy and sell the amounts that they bid. Otherwise, IBA adjusts the price in the direction of the difference between total buy and sell volume, and a new round begins. Thus, all trade occurs in the final round, but I observe agents’ bids for early rounds as IBA searches for the market-clearing price, and I can use these bids to estimate the slope of auction demand.

6.2 Data

The primary data source I use is the IBA Gold Auction Historical Transparency Reports. The data covers daily morning and afternoon auctions; the full dataset covers 1,650 auctions over the period 2015-03-20 to 2018-06-29. For each round of each auction, I observe the number of participants, the round price, and the total volume of gold that auction participants wish to buy and sell during the round.

To my knowledge, the formal rules used by ICE for updating prices between auction rounds are not public; however, the data suggest that ICE essentially runs a Walrasian auction. The left panel of figure plots, for non-terminal auction rounds, the volume imbalance in round \( r \) (that is, the difference between buy and sell volume) against the change in prices from round \( r \) to round \( r + 1 \). Price changes correlate with volume imbalances: if there is excess demand, the round price increases, and if there is excess supply, the round price decreases.

The right panel of figure shows that auction participants appear to respond to round price changes in the expected direction: if prices go up, the volume imbalance decreases,

\[ \text{Note that, throughout this section, I will use oz as shorthand for troy ounces.} \]

\[ \text{I accessed the data at the [ICE website](https://www.iceworld.com).} \]
Figure 3: Price and bid behavior in LBMA gold auctions

Notes. The left panel shows the round $r$ volume imbalance – buy minus sell volume – against the price change from round $r$ to round $r + 1$. The right panel shows the round $r$ to round $r + 1$ price change versus the round $r$ to round $r + 1$ change in volume imbalance. Each data point is one auction round. The sample of auctions used is the set of 1,331 auctions which last for more than a single round.

and vice versa. For each auction round $r > 1$, the x-axis shows the change in prices from round $r - 1$ to $r$, and the y-axis shows the change in aggregate volume imbalance from round $r - 1$ to $r$. The relationship between price changes and volume imbalance changes is negative: if the auction price increases from round $r - 1$ to $r$, the difference between buy and sell volume tends to decrease, and vice versa. This relationship is statistically quite robust: round volume imbalance differences have opposite signs to round price changes in 88.4% of all auction rounds. The OLS coefficient from regressing volume differences on price differences is -49,888 oz²/USD, implying that each dollar change in round price is associated with a change in buy-sell imbalance of 49,888 oz, on average across auctions.

### 6.3 Auction demand slope measurement

The first step in the empirical analysis is to map quantities in the data to $B$, the slope of aggregate auction demand in the model. I will essentially identify $B$ based on the pattern observed in the right panel of figure 3 by regressing volume imbalances on prices. This faces a number of challenges.
First, the LBMA gold auction uses a dynamic price setting process and it is unclear that the static bidding game I analyzed in previous sections is an appropriate model. In appendix D.1, I study a stylized model of the ICE dynamic price-setting mechanism, and show that it admits an equilibrium which is isomorphic to the unique static bidding equilibrium: if the round price is $p$, agents simply bid their unique equilibrium bids $z_{Bi}(p; y_i, c_i)$ from proposition 3. The intuition is very simple: if all agents are bidding non-dynamically, according to their static equilibrium strategies, no agent can do better by deviating. Thus, the static bid submission game is at least an internally consistent model of behavior in the dynamic LBMA gold auctions.

I have not proven that the equilibrium is unique, so other equilibria may exist; I must assume that agents are playing the static equilibrium to bring the model to the data. Empirically, however, dynamic behavior does not appear to affect bid slope estimates substantially: figure 5 in the appendix estimates demand slopes using only bids in later rounds of auctions, and shows that these estimates are statistically very similar to my baseline slope estimates.

Second, I must choose how flexibly to allow demand slopes to vary across auctions. Under the assumption of linear equilibrium bid functions, the slope of aggregate demand $B$ is identified using data from any two rounds within an auction: $B$ is simply the ratio of volume imbalance differences to price differences for any two rounds. I thus estimate $B$ separately for each auction $a$ in the dataset; I will use $B_a$ to refer to the estimated demand slope for auction $a$. In any auction with more than two rounds, the slope of demand is overidentified, so I estimate demand slopes by regressing volume imbalances on prices within auctions. Demand slope measurement is described more formally in appendices D.2 and D.3. Appendix D.4 shows that the linear bid model fits the data fairly well, explaining around 80% of the variance in volume imbalances across rounds within auctions.

Third, I must choose a sample of auctions to use for estimation. For the main text, I limit the sample to auctions with a fixed number of participants, and with at least 3 rounds. This restricts the sample to 502 of the 1,331 auctions used in figure 3. This

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34 Note that the LBMA gold auction uses a dynamic process to set prices, but it is a static auction, since all trade occurs in a single period at a single price. It is thus much simpler to analyze than dynamic auctions with multiple trading periods, as papers such as Sannikov (2016) and Du and Zhu (2017) study.
Table 1: Gold auctions descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>P10</th>
<th>P90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds</td>
<td>4.17</td>
<td>1.5</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Participants</td>
<td>7.69</td>
<td>1.21</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Price (USD / oz)</td>
<td>1223.98</td>
<td>75.24</td>
<td>1118.16</td>
<td>1327.47</td>
</tr>
<tr>
<td>Price range (USD / oz)</td>
<td>1.05</td>
<td>0.79</td>
<td>0.45</td>
<td>1.9</td>
</tr>
<tr>
<td>Volume (1000 oz)</td>
<td>167.58</td>
<td>85.26</td>
<td>90.08</td>
<td>244.24</td>
</tr>
<tr>
<td>N</td>
<td>502</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. Each observation is an auction, indexed by a. “Rounds” is the number of rounds the auction took to complete, R_a. “Participants” is the number of auction participants, n_a. “Price” is the auction clearing price, p_a. “Price range” is the difference between the highest and lowest round prices. “Volume” is v_a, the sum of buy volume and sell volume in the final round of the auction.

Restriction helps in interpreting the results, because my model does not formally apply to auctions with a varying number of participants over time, and because auction demand slopes are estimated more precisely for auctions with more rounds. However, this does not quantitatively matter for the results: in appendix D.8 I repeat the analysis using the full sample of all 1,331 auctions which last more than one round, and all results are quantitatively very similar.

The left panel of figure 4 shows the distribution of estimated auction demand slopes, B_a, for each auction a. The modal demand slope is around 50,000 oz^2/USD, but some auctions have slopes as high as 200,000 oz^2/USD or as low as around 6,000 oz^2/USD. Auction demand slopes are somewhat higher for auctions with more participants.

I also use total auction trade volumes to identify the variance of traders’ inventory shocks. The right panel of figure 4 shows the distribution of the total volume of gold traded in auctions. Modal volume is around 150,000oz, but volumes can be as high as 500,000oz. Volumes are also somewhat higher for auctions with more participants.
Figure 4: Gold auctions demand slopes and volumes

Notes. The left panel shows the distribution of estimated aggregate demand slopes, \( B_a \), for different numbers of auction participants \( n_a \), estimated by regressing round volume imbalances on prices within auctions, as described in appendices D.2 and D.3. The right panel shows the distributions of volumes \( v_a \) for different \( n_a \). The dataset used is the primary estimation sample of 502 auctions.

6.4 Estimation

The next step in the estimation is to map the measured aggregate demand slopes \( B_a \), participant counts \( n_a \), and trade volumes \( v_a \) for each auction \( a \) into bidder-level primitives: spot holding capacities \( \kappa_i \) and inventory variances \( \sigma^2_{yi} \) for individual bidders. Then, based on these primitives, and assumptions about the counterfactual variances and covariances of contract positions \( \sigma^2_{ci} \), I can calculate implied basis risk and manipulation rents. In the process, I must address three sources of model uncertainty. First, I observe only aggregate demand slopes and volumes, not \( \kappa_i \) and \( \sigma^2_{yi} \). Second, these bid slopes and volumes vary across auctions. Third, I must take a stance on the covariances between contract positions and inventory shocks to calculate basis risk and manipulation rents.

6.4.1 Uncertainty in \( \kappa_i \) and \( \sigma^2_{yi} \) given \( B \) and \( v \)

To deal with uncertainty in individual parameters within auctions, I use a bounds approach. For each set of target moments \( n, B, v \), I randomly sample 10,000 possible
vectors of individual bidders’ parameters, $\kappa_i$ and $\sigma^2_{yi}$, which are consistent with the target moments. I will then calculate and report the maximum and minimum values of basis risk and manipulation rents over all draws of $\kappa_i$ and $\sigma^2_{yi}$. This tells us how much knowing the aggregate quantities $B$ and $v$ bounds basis risk and manipulation rents, over any possible combination of individual bidders’ parameters $\kappa_i$ and $\sigma^2_{yi}$ which are consistent with $B$ and $v$.

While I can estimate bid slopes $b_i$ and thus spot holding capacities $\kappa_i$ without imposing any distributional assumptions, I assume inventory shocks $y_i$ are independent normal random variables, in order to match $\sigma^2_{yi}$ to observed volumes. Further details of the random sampling process are described in appendix D.3.

As tables 2 and 3 below show, these random sampling bounds are surprisingly tight: for any given choice of $n, B, v$, the max and min values for basis risk and manipulation risk differ by a factor of around 2. Intuitively, holding fixed a given auction demand slope $B$, increasing one agent’s holding capacity $\kappa_i$ implies that some other agent’s $\kappa_j$ must decrease, and the effects of these changes on outcome metrics approximately offset each other.

### 6.4.2 Uncertainty in $B_a$ and $v_a$ across auctions

As figure 4 shows, there is substantial variation in demand slopes and auction volume across auctions. According to my model, auctions with higher slopes $B_a$ and lower volumes $v_a$ have lower manipulation risk, since prices are harder to move, and lower trade volumes imply lower inventory shock-driven basis risk. Calibrating the model separately to each auction is very computationally intensive; instead, I calibrate the model to match three sets of target moments for each $n$: a low manipulation risk, setting $B$ to its 80th percentile and $v$ to its 20th percentile across auctions; medium, setting both $B$ and $v$ to their median values; and high, setting $B$ to its 80th percentile and $v$ to its 20th percentile. Appendix table 4 shows the values of target moments, that is, the p20, p50, and p80 values of $B_a$ and $v_a$ across auctions for each $n_a$. 
6.4.3 Assumptions on contract and inventory covariances

As subsection 5.2 discusses, both basis risk and manipulation rents depend on the variances and covariances between \( c_{ai} \) and \( y_{ai} \). I set the standard deviations of agents’ contract positions to 1,500 contracts, worth 150,000oz of gold exposure, which is half the CME position limit of 300,000oz, implying that agents’ contract positions will be below position limits with approximately 95% probability. I then consider three possible cases for covariances, ranked in order of increasing size of manipulation-induced distortions. First, I assume contracts \( c_{ai} \) are independent of inventory shocks \( y_{ai} \), as well as all other agents’ \( c_{aj}, y_{aj} \). Second, I assume \( c_{ai} \) is perfectly negatively correlated with \( y_{ai} \), but independent of \( c_{aj}, y_{aj} \); this maximizes basis risk and manipulation rents under the constraint that \( c_{ai}, y_{ai} \) are uncorrelated across agents. Third, I assume \( c_{ai} \) is independent of \( y_{ai} \), but is perfectly correlated with all other agents’ contract positions \( c_{aj} \); this maximizes basis risk and manipulation rents under the constraint that \( c_{ai}, y_{ai} \) are uncorrelated. Since I require inventory shocks to be independent and Gaussian to match the auction trade volume moment, for internal consistency, I do not allow correlations between inventory shocks \( y_{ai} \) across agents. Appendix D.7 derives expressions for basis risk under each of these three cases.

6.5 Results

Tables 2 and 3 show my estimates of basis risk and manipulation rents across all sources of uncertainty: estimates are shown separately for different numbers of participants, for the low, medium and high choices of target slope and volume moments, and for different assumptions about contract and inventory covariances. Each cell reports the median value of basis risk or manipulation rents over all draws of \( \kappa_i \) and \( \sigma_{yi} \), with the minimum and maximum values in parentheses. Column 3 assumes contract positions are independent across agents; column 4 assumes contract positions and inventory shocks are perfectly negatively correlated for each agent, but independent across agents; and column 5 assumes contract positions are perfectly correlated across agents, but are independent of inventory shocks.

All four sources of uncertainty – \( N \), target moments, \( \kappa_i \) and \( \sigma_{yi} \), and contract covari-
ances – affect both outputs, but the effects of each source of uncertainty are relatively small. Increasing \( N \) has a non-monotone effect on both outcomes; basis risk and rents are lowest for auctions with 8-9 participants, and increase somewhat for 10. The inflection at 10 is because, as appendix table 4 shows, aggregate demand slopes are actually somewhat lower for auctions with 10 participants, so price impact is cheaper in these auctions.

As we move from slope and volume target moments with low to high manipulation risk, (the “B+v type” column), both risk metrics increase by a factor of around 2. Similarly, the max and min of both outcomes, over different draws of \( \kappa_i \) and \( \sigma_{yi} \), differ by a factor of around 2-3, depending on other parameters. Finally, the different assumptions about contract covariances change basis risk by a factor of around 2-3, and manipulation rents by a factor of around 10. Collectively, this implies that the max and min values in tables 2 and 3, over all sources of uncertainty, are not extremely far apart: basis risk is between 0.795 to 9.916 USD/oz. The range of possible manipulation rents is somewhat larger, varying from $348,272 to $12,954,987 USD.

Position limit-sized contract positions are fairly large: a 150,000oz gold contract position is around a $225 million USD exposure to gold prices. However, this is a very small fraction of the gold futures market as a whole: on average from 2015-2016, the most liquid COMEX gold contract month has around 300,000 outstanding contracts, equal to a total gold exposure of around $45 billion USD. Dividing total manipulation rents by the total value of outstanding contracts, hedgers lose only between 0.077 to 2.88 basis points of their notional gold exposure to manipulation rents; while these bounds are fairly wide, even the upper bound is very low.

Basis risk, on the other hand, can create quantitatively large transfers from longs to shorts. Multiplying basis risk per ounce by the volume of outstanding gold exposure, basis risk creates net transfers from longs to shorts with a standard deviation between $23.8 million to $297.5 million USD; this is large in absolute terms, although it is a transfer mostly from hedgers to other hedgers. To evaluate how much this basis risk increases hedgers’ consumption variance, I calculate the basis risk coefficient \( \rho \). Since gold futures

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35These calculations effectively assume all 300,000 outstanding contracts are held to expiration. In practice, contract holders will rarely hold their contracts to expiration, as many will roll into the next month close to expiration, and manipulation would only directly affect agents holding to expiration, so the numbers here are likely to overestimate manipulation-induced distortions.
contracts expire once per month, I estimate factor risk, $\sigma_{\psi}^2$, as the variance of monthly gold futures prices, over the period 2015-2018; the standard deviation of these differences is $67.94$ USD/oz. Plugging price variance and the upper and lower bounds for factor risk into expression (27), I find that the basis risk coefficient $\rho$ is between 0.000137 and 0.0209.

In words, even under the unrealistic assumption that all spot traders hold position-limit-sized contract positions, which are perfectly correlated across agents, manipulation-induced basis risk only exposes hedgers to 2.09% of total monthly variance in gold prices. Under more realistic assumptions, basis risk exposes hedgers to well under 1% of total monthly factor risk. Together, these results suggest that, under current contract position limits, the LBMA gold could be used to settle COMEX gold futures contracts, with very small welfare losses to hedgers from manipulation.

### 6.6 Discussion of assumptions

The analysis omits a number of factors. First, participants in the LBMA gold price can submit bids on behalf of their clients if dealers simply perfectly pass through their clients’ bids to the auction, the actual number of effective auction participants may be much larger than the 5-10 dealers who formally bid in the auction. Increasing the number of auction participants while holding fixed the aggregate demand slope has two offsetting effects on manipulation risk. When $n$ is higher, auctions are more competitive, so each agent has lower manipulation incentives; however, since contract position limits apply to each independent agent, the total number of contracts held by all auction participants can increase.

In appendix D.9 I analyze the effect of increasing $n$, while holding fixed agents’ contract position variances $\sigma_c^2$ and the aggregate bid slope $B$, on basis risk and manipulation rents. Basis risk decreases linearly in $n$, while total manipulation rents are asymptotically constant. Thus, my results will tend to overestimate basis risk, and would be within a constant factor of manipulation rents, if the number of effective participants is larger than the observed $n_a$ values.

36See, for example, [London Gold Fix](#).
### Table 2: Gold auctions basis risk estimates

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>1.76 (1.36-2.89)</td>
<td>2.43 (2.05-3.15)</td>
<td>3.31 (3.03-4.30)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>2.55 (1.97-4.19)</td>
<td>3.96 (3.38-5.16)</td>
<td>4.80 (4.39-6.23)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>4.06 (3.13-6.67)</td>
<td>4.48 (3.68-6.75)</td>
<td>7.64 (6.99-9.92)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>1.38 (1.04-2.28)</td>
<td>1.91 (1.64-2.55)</td>
<td>2.77 (2.54-3.52)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>2.36 (1.78-3.90)</td>
<td>3.72 (3.19-4.69)</td>
<td>4.74 (4.35-6.02)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>3.22 (2.43-5.34)</td>
<td>3.60 (2.96-5.48)</td>
<td>6.47 (5.94-8.22)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>1.30 (0.984-2.19)</td>
<td>1.88 (1.61-2.48)</td>
<td>2.78 (2.57-3.48)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>1.68 (1.27-2.83)</td>
<td>2.81 (2.38-3.55)</td>
<td>3.59 (3.32-4.50)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>2.54 (1.92-4.25)</td>
<td>2.89 (2.36-4.37)</td>
<td>5.41 (5.00-6.78)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>1.13 (0.853-1.95)</td>
<td>1.75 (1.52-2.30)</td>
<td>2.53 (2.36-3.15)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>1.45 (1.09-2.50)</td>
<td>2.56 (2.23-3.27)</td>
<td>3.25 (3.02-4.03)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>2.03 (1.53-3.51)</td>
<td>2.37 (1.97-3.66)</td>
<td>4.55 (4.24-5.66)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>1.05 (0.795-1.80)</td>
<td>1.63 (1.43-2.13)</td>
<td>2.48 (2.32-3.05)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>1.46 (1.10-2.50)</td>
<td>2.63 (2.34-3.29)</td>
<td>3.44 (3.22-4.23)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>2.46 (1.86-4.21)</td>
<td>2.78 (2.26-4.32)</td>
<td>5.80 (5.43-7.13)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>1.23 (0.94-2.16)</td>
<td>2.15 (1.90-2.69)</td>
<td>3.02 (2.85-3.61)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>1.68 (1.28-2.95)</td>
<td>3.35 (2.98-4.10)</td>
<td>4.13 (3.89-4.94)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>2.95 (2.25-5.16)</td>
<td>3.24 (2.62-5.30)</td>
<td>7.23 (6.81-8.64)</td>
</tr>
</tbody>
</table>

**Notes.** Estimated standard deviations of basis risk, in units of prices (USD/oz). Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 4. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
Table 3: Gold auctions manipulation rent estimates

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>0.497 (0.454-0.645)</td>
<td>0.762 (0.675-0.865)</td>
<td>2.484 (2.272-3.223)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>0.720 (0.659-0.935)</td>
<td>1.261 (1.095-1.412)</td>
<td>3.601 (3.295-4.674)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>1.146 (1.049-1.487)</td>
<td>1.329 (1.218-1.619)</td>
<td>5.730 (5.243-7.437)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>0.415 (0.381-0.528)</td>
<td>0.647 (0.584-0.725)</td>
<td>2.491 (2.285-3.165)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>0.710 (0.652-0.903)</td>
<td>1.284 (1.147-1.415)</td>
<td>4.262 (3.911-5.416)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>0.971 (0.891-1.233)</td>
<td>1.153 (1.062-1.386)</td>
<td>5.824 (5.344-7.401)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.417 (0.385-0.522)</td>
<td>0.687 (0.618-0.750)</td>
<td>2.916 (2.698-3.657)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>0.539 (0.498-0.675)</td>
<td>1.041 (0.909-1.136)</td>
<td>3.770 (3.489-4.727)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>0.811 (0.750-1.017)</td>
<td>0.994 (0.928-1.164)</td>
<td>5.676 (5.252-7.117)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.380 (0.354-0.473)</td>
<td>0.685 (0.619-0.755)</td>
<td>3.041 (2.832-3.780)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>0.487 (0.453-0.605)</td>
<td>1.014 (0.904-1.099)</td>
<td>3.894 (3.626-4.841)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0.683 (0.636-0.849)</td>
<td>0.866 (0.817-1.013)</td>
<td>5.466 (5.090-6.795)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.372 (0.348-0.457)</td>
<td>0.672 (0.623-0.734)</td>
<td>3.348 (3.134-4.114)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>0.516 (0.484-0.635)</td>
<td>1.100 (1.009-1.193)</td>
<td>4.648 (4.352-5.713)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>0.870 (0.815-1.070)</td>
<td>1.053 (0.994-1.227)</td>
<td>7.834 (7.335-9.627)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.454 (0.427-0.542)</td>
<td>0.941 (0.867-1.015)</td>
<td>4.535 (4.275-5.419)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>0.620 (0.584-0.740)</td>
<td>1.477 (1.348-1.593)</td>
<td>6.197 (5.841-7.403)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>1.084 (1.022-1.295)</td>
<td>1.267 (1.207-1.467)</td>
<td>10.843 (10.220-12.955)</td>
</tr>
</tbody>
</table>

Notes. Estimated expected manipulation rents, in units of millions of US dollars. Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 4. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
I estimate the slope of demand using relatively small price movements – table I shows that, within a single auction, the range of prices tends to be around $0.5 to $1.9 USD/oz. One concern is that the slope of market demand for large price movements could be different from this local slope. On the one hand, if a manipulator were able to “corner” the gold market, buying enough gold to exhaust the supplies of all other auction participants, gold supply could become much more inelastic, and the manipulator could increase prices essentially without bound. On the other hand, large price movements might cause auction participants who are normally inactive to enter the market, so the slope of market supply could also be larger for large price movements.

The data I have do not allow me to distinguish between these two possibilities. I can only state that, within the range of demand slopes I observe in my sample, basis risk and manipulation rents are fairly low. As figure 4 shows, these slope estimates vary by a factor of 3-4 across auctions. So long as demand slopes for large price movements are not substantially lower than the lowest demand slopes observed in my sample, my conclusions that manipulation risk is low should be robust for larger price movements.

The LBMA gold auctions are currently run twice daily, while COMEX gold futures contracts settle only once per month. If a single auction each month were used to settle COMEX gold futures, this auction would likely attract more participants than standard daily auctions. Entry would decrease manipulation risk, by increasing competition and the slope of aggregate demand: expression (8) of proposition I shows that basis risk decreases at rate $1/n^3$, and expression (19) shows that manipulation rents decrease at rate $1/n^{37}$.

Finally, my analysis estimates manipulation risk assuming the standard deviation of agents’ contract positions is half the current position limit of 3,000 contracts. The model could also be used to measure basis risk under alternative position limits by plugging in different values for $\sigma_c^2$. A regulator could thus use this analysis to determine how large

---

37 As I discuss in subsection 3.2, the costs of entry vary widely across different kinds of spot markets. Some assets, such as the S&P 500 options underlying the VIX, or the stocks underlying S&P 500 futures, are traded on public-facing exchanges, so there is essentially no entry cost. For goods such as oil, gas and electricity, entry is effectively impossible over short time horizons, since spot market participants require physical infrastructure – pipelines and storage facilities – which are costly and take time to build. For other goods, such as gold, FX, ISDAFIX and repo loans, benchmarks are set using inter-dealer trade prices, often on trade platforms with restricted participation, so there are institutional and regulatory constraints to entry in spot markets.
position limits could be, while ensuring that basis risk and manipulation rents stay below certain target values.

7 Discussion

7.1 Implications for contract market regulation

Besides attempting to catch traders admitting to attempted manipulation, contract market regulators use two main structural tools in contract markets: position limits and benchmark regulation. My results have implications for how regulators should apply both of these tools.

7.1.1 Position limits

Under the Commodity Exchange Act, the CFTC has the authority to impose speculative position limits on contract market participants, to limit the risk of large price movements. These limits apply not only to individual market participants, but also to groups of participants who are coordinating their trades. Position limit violations are a very common class of enforcement actions. In a famous 1979 case, seven Hunt family members and an associated corporation coordinated to purchase soybean futures positions totalling over 23 million bushels of soybeans, well over the position limit of 3 million

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38Section 4a(1) of the Commodity Exchange Act, 7 U.S.C. § 6a(1) states that: “Excessive speculation in any commodity... made on or subject to the rules of contract markets or derivatives transaction execution facilities... causing sudden or unreasonable fluctuations or unwarranted changes in the price of such commodity, is an undue and unnecessary burden on interstate commerce in such commodity. For the purpose of diminishing, eliminating, or preventing such burden, the Commission shall, from time to time... proclaim and fix such limits on the amounts of trading which may be done or positions which may be held by any person, including any group or class of traders, under contracts of sale of such commodity for future delivery on or subject to the rules of any contract market or derivatives transaction execution facility... In determining whether any person has exceeded such limits, the positions held and trading done by any persons directly or indirectly controlled by such person shall be included with the positions held and trading done by such person; and further, such limits upon positions and trading shall apply to positions held by, and trading done by, two or more persons acting pursuant to an expressed or implied agreement or understanding, the same as if the positions were held by, or the trading were done by, a single person.”

39Commodity Futures Trading Com’n v. Hunt, 591 F.2d 1211 (7th Cir. 1979)
bushels. More recent cases involving wheat, soybean, and other markets are documented on the CFTC’s press release website.

While the primary stated purpose of position limits, according to both the CEA and the CFTC’s website, is to prevent excessive futures price volatility, the results of this paper suggest that position limits can also be effective as a tool for reducing manipulation risk. In fact, in the context of the baseline model of section 3, perfect risk sharing can be achieved simply by banning spot traders from holding derivative contract positions; spot traders will then bid identically in the spot auction, and the spot price will be identically equal to $\psi$. This is obviously infeasible in practice, since spot traders tend to be active in contract markets and have nonzero welfare weight; however, in some circumstances, my results imply that even spot traders would prefer to limit the size of spot traders’ contract positions.

Interestingly, the CFTC currently applies position limits more harshly for pure hedgers than spot traders. This is justifiable if the goal is to limit the risk of default, or margin calls and large price movements resulting from large unhedged positions: a spot trader with offsetting contract and spot positions has less factor risk exposure than a pure hedger with only the contract position. However, this conclusion is reversed if the goal is to limit manipulation risk: spot traders with offsetting spot and contract factor risk exposures still have large incentives to trade spot goods to move the basis, $p - \psi$, whereas pure hedgers who are unable to trade in spot markets do not increase manipulation risk, regardless of the size of their contract positions. Thus, my results suggest that, in markets where manipulation risk is a first-order concern, position limits should actually be applied more harshly to spot traders than pure hedgers.

40CFTC Press Release 7955-19
41CFTC Press Release 8021-19
42CFTC Press Release 8002-19
43The CFTC’s stated purpose for speculative position limits is to “protect futures markets from excessive speculation that can cause unreasonable or unwarranted price fluctuations”; hence position limits do not exist solely to combat manipulation, although according to my theory they can be an effective tool for doing so.
44Contract position limits do not apply for market participants who have bona fide commercial risks to hedge; see the CFTC’s website on Speculative Limits.
7.1.2 Benchmark regulation

Benchmark regulation is a comparatively new form of structural market intervention. Following the LIBOR manipulation scandal, there has been a concerted effort across jurisdictions to move hundreds of trillions of dollars of interest rate derivatives towards new, less manipulable interest rate benchmarks\textsuperscript{45} since then, principles for financial benchmarks have been released by the International Organization of Securities Commissions (IOSCO (2013)), the FCA began regulating a number of benchmarks,\textsuperscript{46} and then in 2018 EU law was revised to include benchmark regulation, along similar principles to the IOSCO report\textsuperscript{47}.

The IOSCO (2013) report outlines many details of benchmark governance, calculation methodology, transparency, and accountability; it does not discuss features of market structure which influence the robustness of benchmarks. This paper shows that, even if benchmarks are based on correctly reported prices of actual trades, if spot traders have market power and price impact, they have incentives to distort price benchmarks to profit from their derivative positions. A number of manipulation cases in practice may in fact be linked to market structure: while the LIBOR scandal was effectively caused by governance problems relating to trade misreporting, instances of manipulation in other markets, such as FX and ISDAFIX\textsuperscript{48}, involved actual and correctly reported trades made by market participants in auction or limit-order-book markets. Governance, measurement, and other interventions have limited effectiveness in markets which are structurally vulnerable to manipulation. Regulators should instead combat and limit manipulation through market structure: by encouraging entry to make spot markets for benchmark setting more competitive, by monitoring market participants’ spot capacity costs to ensure that the price impact of spot trades remains small, and by limiting the size of spot traders’ contract positions.

Practically, the tools developed in this paper could be applied to many of these markets. While bid slopes are not always easy to measure, especially for benchmarks which are not set in static auctions, the analysis of the LBMA gold price in section 6

\textsuperscript{45}See, for example, Duffie and Stein (2015).
\textsuperscript{46}FCA to regulate seven additional financial benchmarks
\textsuperscript{47}FCA (2016)
\textsuperscript{48}See, respectively, Levine (2014) for FX and Leising (2017) for ISDAFIX
shows that it is possible to estimate informative bounds on manipulation risk using much simpler data: the number of spot market participants and the slope of aggregate demand in the spot market. Regulators observe the number of participants in most markets, and a generalized version of (the inverse of) aggregate demand slope for different market structures is the cost of price impact – Kyle’s lambda – which can be estimated using a variety of methods in the empirical microstructure literature, so the metrics developed in this paper could plausibly be applied to a many other benchmarks.

7.2 Mechanisms for benchmark setting

This paper uses uniform-price double auctions as a reduced-form model of price benchmarks. Some benchmarks, such as VIX and the LBMA gold price discussed in this paper, are determined using actual auctions. Derivative contracts for many equity indices are also settled based on exchange opening or closing auction prices. Other benchmark-setting mechanisms may produce outcomes similar to uniform-price double auctions. The WM/Reuters FX fixing and the ISDAFIX interest rate swap benchmark (now the ICE swap rate) are set using exchange prices within a few minutes; if agents submitted fixed bid curves at the start of the fixing period and do not adjust bids through the course of the auction, outcomes will coincide with static uniform-price double auction outcomes. Some benchmarks for commodities such as oil and gas are set using volume-weighted average prices in specific geographical locations, over relatively short time spans; if the underlying goods are relatively homogeneous, market outcomes can be approximated by supply function competition between dealers (Klemperer and Meyer, 1989), which is equivalent to a uniform-price double auction.

Other benchmarks are less well approximated by auctions. Some benchmarks are based on trades of underlying assets in markets with large search or transportation

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49 Understanding the Special Opening Quotation (SOQ)  
50 WM/Reuters FX Benchmarks  
51 ICE Swap Rate  
52 Since the WM/Reuters fixing period is only a few minutes long, there is likely little arrival of information during the fixing, and thus no fundamental reason for participants to change their bids over time. However, there may be strategic reasons to adjust bids dynamically; Du and Zhu (2017) argue that the frequency of auctions does in fact affect market outcomes even in the absence of information arrival.
frictions. For example, the CME Feeder Cattle Index is based on US-wide cattle trade prices; the price of cattle traded in New York on any given day may differ substantially from the price of cattle traded in California. Other markets have nontrivial network structure, with central dealers trading with peripheral counterparties (Wang, 2017; Duffie and Wang, 2016). In these markets, agents’ effective ability to move price benchmarks may depend on their network positions and resultant bargaining power. Other benchmarks are not based on prices of verifiable trades, but rely on market participants to self-report trades or potential trades; for example, LIBOR is based on banks’ announcements of their borrowing costs and some natural gas benchmarks are based on reports of trades which can be falsified. Such benchmarks can be manipulated simply by making false reports about trades. Auctions are potentially a reasonable model for market structures in which the primary distortion is market power; settings in which there are significant frictions other than market power are less well-approximated by my model.

A normative interpretation of my results is that auctions are a relatively robust way to set price benchmarks, so regulators could encourage broader use of auctions for benchmark setting. Auctions have a number of benefits besides those discussed in this paper. They are anonymous, so any offer to buy or sell can be taken up by any other agent; thus, agents cannot distort benchmarks by trading at artificially high or low prices with favored counterparties. They are also straightforwards to run; unlike average-price benchmarks, they do not require costly infrastructure to track all trades, and thus cannot be manipulated by falsifying trades.

7.3 Settlement by physical delivery

Many derivative contracts are settled, not by cash payments, but by physical delivery of the underlying asset. Under normal market conditions, physical delivery and cash-settled contracts function similarly, because most holders of physical delivery contracts close out their positions for cash payments prior to delivery. Kyle (2007) derives a set of conditions
under which cash-settled contracts and physical delivery contracts are economically equivalent; one important condition is that agents can trade an arbitrary quantity of the underlying asset at the cash settlement price. The auction model of this paper satisfies the conditions for equivalence, so the metrics and methods proposed in this paper may also be applicable to measuring manipulation risk in some physical delivery contract markets.

However, there are some differences between the practical mechanisms of manipulation in physical delivery and cash-settled contract markets. Physical delivery contract markets have historically been manipulated in a few different ways. “Squeezes” are similar to cash-settled contract manipulation: a long manipulator buys the underlying asset to create a shortage of the underlying asset, raising prices of the underlying asset and increasing the prices at which shorts can close out their contracts. “Corners” are somewhat different: a long manipulator aims to buy up enough of the underlying asset that it is practically impossible for shorts to fulfill their delivery requirements; the long manipulator then uses the threat of default to extract large payments from shorts to close out contracts. Corners thus require that long contract holders can threaten to take delivery, and that delivery for shorts is logistically impossible at any cost.

The assumptions of the Kyle (2007) equivalence result are violated in the case of corners, because shorts cannot purchase arbitrary quantities of underlying assets at settlement prices – if shorts owe more of the underlying asset than the total deliverable market supply, settlement prices are essentially undefined. Thus, in some real-world settings in which the Kyle (2007) equivalence conditions do not hold, physical delivery contract markets may be manipulable in ways that are outside the scope of this paper’s model, so the metrics developed in this paper may not be appropriate.

8 Conclusion

This paper studies a simple model of manipulation in cash-settled derivative contract markets. I show that manipulation harms market participants by creating basis risk, distorting allocations of spot assets, and transferring wealth from hedgers to spot market participants. If the incremental basis risk borne by spot traders outweighs their gains

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Markham, 2014, pg. 3

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from increased contract payoffs, manipulation can actually decrease the welfare of all market participants. I show how to measure two simple summary statistics of hedgers’ welfare losses from manipulation, and I empirically estimate bounds for both statistics, assuming COMEX gold futures were cash-settled using the LBMA gold price.

The regulation of contract market manipulation is a contentious topic in both academic and policy circles. Essentially the only internally consistent way to define manipulation is trading with the intent to move prices, and this is effectively the definition used by regulators. This definition creates substantial regulatory uncertainty for market participants. Traders in imperfectly competitive spot markets know that they move prices – the role that they play in these markets is precisely to manage and optimize the price impact of their trades. Under the current regulatory environment, a trader who is caught by regulators acknowledging that his trades move prices, and that he maximizes profits cognizant of his impact on prices, can be subject to billions of dollars in fines.

The methods developed in this paper suggests that regulators could take a different approach. Assuming that spot traders maximize profits across contract and spot market, I show how to estimate, using commonly observed market data, manipulation-induced welfare losses for pure hedgers. If predicted welfare losses are high, regulators can use structural policy tools, such as position limits and benchmark redesign, to limit market participants’ manipulation incentives and improve hedger welfare. If predicted manipulation risk is low, regulators can take a more hands-off approach, as competition will likely discipline the exercise of market power without much need for case-by-case regulatory intervention. The tools of this paper can thus be used as the basis for a more quantitative, theory-based approach to regulating manipulation in derivative contract markets.
References


FCA. 2014. “Barclays fined £26m for failings surrounding the London Gold Fixing and former Barclays trader banned and fined for inappropriate conduct.”

FCA. 2016. “EU Benchmarks Regulation.”


A Proof of proposition 1

Repeating (3), spot traders’ total consumption can be written as:

\[
C_{\text{spot}}(x_i, c_i, z_i, p, \psi) = \underbrace{\psi x_i - \mu \psi x_i}_\text{Factor exposure} - \underbrace{\mu \psi c_i}_\text{Contract price} + \underbrace{p c_i}_\text{Contract payoff} + \underbrace{z_i \psi - \frac{1}{2 \kappa} z_i^2}_\text{Spot good payoff} - \underbrace{p z_i}_\text{Spot good price} \tag{34}
\]

A.1 Spot market bidding

To solve the spot market auction, I adopt the standard solution concept of equilibrium in ex-post optimal bid curves. A bid curve is ex-post optimal if it is optimal for any realization of other agents’ bid curves which occurs in equilibrium.

From the perspective of agent \(i\), the spot auction defines a residual supply curve, \(z_{RSi}(p)\), specifying the number of units of the underlying asset that \(i\) is able to trade at price \(p\). This is the negative of the sum of all other agents’ bid curves:

\[
z_{RSi}(p) = - \sum_{j \neq i} z_{Bj}(p; c_j, \psi) \tag{35}
\]

In equilibrium, given my assumptions on agents’ utility functions, residual supply functions will be affine with a fixed slope:

\[
z_{RSi}(p) = d(p - \psi) + \eta_i \tag{36}
\]

Where the random intercept \(\eta_i\) depends on uncertainty in other traders’ bid curves, resulting from uncertainty in their contract positions \(c_i\). I solve the spot auction model using the standard Kyle (1989) trick: I assume agents can choose the quantity they want to purchase for every possible realization of \(\eta_i\), then show that these choices can be
implemented by an affine demand schedule.

Claim 1. In the spot market, given \( d \), agents’ optimal bid curves are:

\[
z_{Bi}(p; c_i, \psi) = \frac{\kappa}{d + \kappa} c_i - \frac{\kappa d}{d + \kappa} (p - \psi) \tag{37}
\]

Proof. Spot trader \( i \)'s consumption value is given by (34). Trader \( i \) chooses her bid curves after \( x_i \) and \( \psi \) are realized, so I analyze agents’ choices conditional on \( x_i \) and \( \psi \). Assume the agent faces a residual supply curve as described in (36), and rearrange to get the inverse residual supply function:

\[
p_{RS}(z_i; \eta_i, \psi) = \psi + \frac{z_i - \eta_i}{d} \tag{38}
\]

Suppose \( i \) can condition her purchase decision on \( \eta_i \). We can write (34) as:

\[
C = \psi x_i - \psi \mu_x + p_{RS}(z_i; \eta_i, \psi) c_i - \mu_x c_i + z_i \psi - \frac{1}{2 \kappa} z_i^2 - z_i p_{RS}(z_i; \eta_i, \psi) \tag{39}
\]

All components of (39) are known to \( i \), so \( i \) simply chooses her purchase quantity \( z_i \) to maximize (39). Differentiate with respect to \( z_i \):

\[
p'_{RS}(z_i; \eta_i, \psi) c_i + \psi - z_i p'_{RS}(z_i; \eta_i, \psi) - p(z_i; \eta_i, \psi) - \frac{z_i}{\kappa} = 0 \tag{40}
\]

From (38), we have \( p'_{RS}(z_i; \eta_i) = \frac{1}{d} \), hence (40) becomes:

\[
\frac{c_i}{d} + \psi - \frac{z_i}{d} - p_{RS}(z_i; \eta_i, \psi) - \frac{z_i}{\kappa} = 0 \tag{41}
\]

Expression (41) implicitly defines the optimal choice of \( z_i \) given \( \eta_i \). Expression (41) defines an affine bid curve; solving for \( z_i \), we attain expression (37). Since (37) passes through exactly all pairs \((z_i, p)\) which are \( i \)'s optimal choices for some realization of \( \eta_i \), \( i \) can do no better than submitting bid curve (37). \( \square \)

Now, note that from (35), the slope of residual supply \( d \) facing any given agent is
equal to the sum of all \( n - 1 \) other agents’ bid slopes. Thus, in equilibrium, we must have:

\[
d = (n - 1) \frac{\kappa d}{d + \kappa}
\]

Solving for \( d \), we get:

\[
d = (n - 2) \kappa
\] (42)

Plugging this into (37), we get (7) of proposition 1. This implies that agents’ optimal bids are:

\[
z_{Bi}(p; c_i, \psi) = \frac{1}{n - 1} c_i - \frac{n - 2}{n - 1} \kappa (p - \psi)
\]

proving (7).

To get prices, sum bids and add to 0:

\[
\sum_{i=1}^{n} \frac{1}{n - 1} c_i - \frac{n - 2}{n - 1} \kappa (p - \psi) = 0
\]

Solving for \( p - \psi \), and using that agents’ contract positions are normally distributed with variance \( \sigma^2 \), we get (8).

**A.2 Spot trader welfare conditional on \( x_i \)**

Now, I analyze how much utility \( i \) achieves in expectation, if she has factor exposure \( x_i \).

Suppose an agent with factor exposure \( x_i \) is bidding against a residual supply curve of the form (36).

**Claim 2.** Agent \( i \)’s expected utility given \( \alpha, \sigma^2, \kappa, x_i, c_i, \sigma^2 \), \( d \) is:

\[
\sqrt{\frac{d^2 + 2\kappa d}{\alpha \kappa \sigma^2 + d^2 + 2\kappa d}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\frac{\alpha \sigma^2}{\kappa} (c_i + x_i)^2 - \frac{\alpha \sigma^2 - \kappa}{\alpha \kappa \sigma^2 + d^2 + 2\kappa d} c_i^2 \right) \right) \right)
\] (43)

**Proof.** To calculate expected utility over uncertainty in \( \eta_i \) and \( \psi \), we first write expected utility from the auction as a function of \( \eta_i \), fixing \( c_i \). Rearranging residual supply from (36), we have:

\[
p = \psi + \frac{z_i + \eta_i}{d}
\] (44)
Consumption is:

\[ C = \psi x_i - \mu \psi x_i - \mu \psi c_i + pc_i + z_i \psi - \frac{1}{2\kappa} z_i^2 - z_i p \]

Plugging in (44) for prices and rearranging, we have:

\[ C = \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{z_i (\eta_i; c_i)}{d} \left( \frac{(z_i (\eta_i; c_i))^2}{2\kappa} - \frac{\eta_i z_i (\eta_i; c_i)}{d} \right) \]

(45)

Now, to find an expression for \( z_i (\eta_i; c_i) \), we eliminate prices from expression (37) for optimal bid curves and expression (36) for residual supply, to get \( z_i \) as a function of \( \eta_i \):

\[ z_i (\eta_i; c_i) = \frac{\kappa}{d + 2\kappa} (c_i - \eta_i) \]

(46)

Plugging (46) into expression (45) for consumption, and simplifying, we have that consumption is:

\[ \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{(c_i - \eta_i)^2 \kappa}{2d^2 + 4\kappa d} \]

Given our assumption of CARA utility, agents’ utility is:

\[ -\exp \left( -\alpha \left( \psi x_i - \mu \psi x_i - \mu \psi c_i + \psi c_i + \frac{\eta_i c_i}{d} + \frac{(c_i - \eta_i)^2 \kappa}{2d^2 + 4\kappa d} \right) \right) \]

(47)

We first integrate (47) over uncertainty in \( \eta_i \), assuming that \( \eta_i \) is normally distributed with mean 0 and variance \( \sigma_{\eta_i}^2 \), to get:

\[ -\sqrt{\frac{d^2 + 2\kappa d}{\alpha \kappa \sigma_{\eta_i}^2 + d^2 + 2\kappa d}} \exp \left[ -\alpha \left( \psi x_i - \mu \psi x_i + \psi c_i - \mu \psi c_i \right) + \frac{\alpha}{2} \left( \frac{\alpha \sigma_{\eta_i}^2 - \kappa}{\alpha \kappa \sigma_{\eta_i}^2 + d^2 + 2\kappa d} \right) c_i^2 \right] \]

(48)
This gives expected utility over uncertainty in $\eta_i$, conditional on $\psi$. Now, we integrate (48) over uncertainty in $\psi$, which is normally distributed with mean $\mu_\psi$ and variance $\sigma^2_\psi$, to get:

$$
\sqrt{\frac{d^2 + 2\kappa d}{\alpha \kappa \sigma^2_\eta + d^2 + 2\kappa d}} \left( \exp \left( -\frac{\alpha}{2} \left( -\alpha \sigma^2_\psi (c_i + x_i)^2 - \frac{\alpha \sigma^2_\psi \kappa \alpha \kappa \sigma^2_\eta}{\alpha \kappa \sigma^2_\eta + d^2 + 2\kappa d c_i^2} \right) \right) \right)
$$
as desired. \qed

A.3 Optimal hedging

Using claim 2, we can find spot traders’ optimal choice of $c_i$.

Claim 3. If:

$$1 + \frac{\alpha \sigma^2_\eta - \kappa}{\alpha \sigma^2_\psi (\alpha \kappa \sigma^2_\eta + d^2 + 2\kappa d)} > 0 \quad (49)$$

then spot traders’ objective function is strictly concave in $c_i$, and there is a unique optimal choice of $c_i$, which satisfies:

$$c_i \left( 1 + \frac{\alpha \sigma^2_\eta - \kappa}{\alpha \sigma^2_\psi (d^2 + 2\kappa d + \alpha \kappa \sigma^2_\eta)} \right) = -x_i \quad (50)$$

Proof. Take conditional expected utility from (43). Since only the exponent depends on $c$, and the function $-\exp \left( -\frac{\alpha}{2} (x) \right)$ is increasing in $x$, we choose $c_i$ to maximize:

$$-\alpha \sigma^2_\psi (c_i + x_i)^2 - \frac{\alpha \sigma^2_\eta - \kappa}{\alpha \kappa \sigma^2_\eta + d^2 + 2\kappa d c_i^2}$$

(51)

Taking the second derivative, note that the problem is only concave if:

$$1 + \frac{\alpha \sigma^2_\eta - \kappa}{\alpha \sigma^2_\psi (d^2 + 2\kappa d + \alpha \kappa \sigma^2_\eta)} > 0$$

proving (49). Assuming (49) holds, differentiate (51) with respect to $c_i$ and rearrange to
get (50). Claim 3, plugging in \( d = (n - 2) \kappa \) from (42), gives expressions (9) and (10) of proposition 1.

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hence, using representation (36) of residual supply, we have:

\[ \eta_i = - \sum_{j \neq i} \frac{1}{n-1} c_i \]

and all factor exposures \( x_i \) are independent. Hence, the variance of the residual supply intercept, \( \eta_i \), is:

\[ \sigma^2_{\eta_i} = \frac{\sigma^2_c}{n-1} \] (55)

Combining this with (54), and plugging in \( d = (n-2) \kappa \) from (42), we obtain (52). Note also that, using the definition of \( t \) in (9), we can write (55) as:

\[ \sigma^2_{\eta_i} = \frac{t^2 \sigma^2_x}{n-1} \] (56)

By claim 3 in order for (11) to solve agents’ optimal contract purchasing problem, the concavity condition (49) must also hold; plugging in \( d = (n-2) \kappa \) to (49), we get:

\[ 1 + \frac{\alpha \sigma^2_{\eta_i} - \kappa}{\left( \alpha \sigma^2_{\psi} \kappa \right) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_{\eta_i} \right)} > 0 \]

Solving for \( \sigma^2_{\eta_i} \), we get (53).

To show that there is a unique value of \( \sigma^2_{\eta_i} \) satisfying (52) and (53), rearrange (52) to:

\[ (n-1) \sigma^2_{\eta_i} = - \left( 1 + \frac{\alpha \sigma^2_{\eta_i} - \kappa}{\left( \alpha \sigma^2_{\psi} \kappa \right) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_{\eta_i} \right)} \right)^{-2} \sigma^2_x \] (57)

The LHS of (57) is increasing in \( \sigma^2_{\eta_i} \) from 0 to \( \infty \). As \( \sigma^2_{\eta_i} \) varies from the lower bound (53) to 0, the RHS decreases from \( \infty \) towards the finite quantity:

\[ - \left( 1 + \frac{1}{\alpha \kappa \sigma^2_{\psi}} \right)^{-2} \sigma^2_x \]

Hence, there is exactly one positive value of \( \sigma^2_{\eta_i} \) greater than the lower bound (53) which...
equates the LHS and RHS of (57), proving that there is a unique equilibrium value of \( \sigma^2 \).

This proves claim (11) and (12) of proposition 1.

### A.5 Expected welfare over uncertainty in factor exposures

Plugging in \( c_i = tx_i \), \( d = (n - 2) \kappa \) to (43) of claim 2, we get expected utility conditional on \( x_i \), for any linear contract purchasing rule:

\[
\sqrt{\frac{(n^2 - 2n) \kappa}{\alpha \sigma^2_n + (n^2 - 2n) \kappa}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\alpha \sigma^2_{\psi} (1 - t)^2 - \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_{\psi} \kappa)} \left( (n^2 - 2n) \kappa + \alpha \sigma^2_n \right) t^2 \right) \right) \right)
\]

Integrating this against uncertainty in \( x_i \), with mean 0 and variance \( \sigma^2_x \), we get:

\[
- \frac{1}{\sqrt{\frac{(n^2 - 2n) \kappa}{\alpha \sigma^2_n + (n^2 - 2n) \kappa}}} \sqrt{1 - \alpha \sigma^2 \left( \alpha \sigma^2_{\psi} (1 - t)^2 + \frac{\alpha \sigma^2_n - \kappa}{(\alpha \sigma^2_{\psi} \kappa) \left( (n^2 - 2n) \kappa + \alpha \sigma^2_n \right)} t^2 \right)}
\]

Substituting for \( \sigma^2_n \) using (56), we get (13).

### A.6 Pure hedgers

Repeating (4), hedgers’ consumption is:

\[
C_{\text{hedger}}(x_i, c_i, p, \psi) = \psi x_i - \mu_{\psi} x_i - \mu c_i + pc_i
\]

Since the expectation of \( p \), over uncertainty in \( \psi \) and all \( x_i \)’s, is \( \mu_{\psi} \), the expectation of hedgers’ consumption is always equal to 0. To find the variance, adding and subtracting \( \psi c_i \), we can write this as:

\[
= \psi x_i - \mu_{\psi} x_i - \mu c_i + \psi c_i + (p - \psi) c_i
\]
Since the mean of hedgers’ consumption is independent of $c_i$, hedgers simply purchase contracts to minimize consumption variance, which is:

$$
= \sigma_x^2 (x_i + c_i)^2 + \sigma_i^2 \text{Var} (p - \psi)^2
$$

(59)

Minimizing with respect to $c_i$, we have (14). Plugging (14) in to (59), we get (15).

**B Supplementary material for section 4**

**B.1 Convergence rate of equilibrium $t$**

From (11) and (10), we have:

$$
\sigma_n^2 = \frac{\sigma_x^2}{n-1} \left( 1 + \frac{\alpha \sigma_n^2 - \kappa}{\left( \alpha \sigma_x^2 \psi \kappa \right) ((n^2 - 2n) \kappa + \alpha \sigma_n^2)} \right)^{-2}
$$

(60)

and:

$$
t = \left( 1 + \frac{\alpha \sigma_n^2 - \kappa}{\left( \alpha \sigma_x^2 \psi \kappa \right) ((n^2 - 2n) \kappa + \alpha \sigma_n^2)} \right)^{-2}
$$

(61)

The RHS of (61) is decreasing in $\sigma_n^2$. Hence an upper bound for the equilibrium $t$ comes from setting $\sigma_n^2$ to 0:

$$
t \leq \left( 1 - \frac{\kappa}{\left( \alpha \sigma_x^2 \psi \kappa \right) ((n^2 - 2n) \kappa)} \right)^{-1}
$$

(62)

Now, we can get an upper bound for $\sigma_n^2$ by plugging the upper bound on $t$, (62) into (60):

$$
\sigma_n^2 \leq \frac{\sigma_x^2}{n-1} \left( 1 - \frac{\kappa}{\left( \alpha \sigma_x^2 \psi \kappa \right) ((n^2 - 2n) \kappa)} \right)^{-2}
$$

(63)
The RHS is decreasing in \( n \), hence, we can set \( n = 3 \) in (63) to get:

\[
\sigma^2_n \leq M \equiv \frac{\sigma^2_x}{2} \left( 1 - \frac{\kappa}{(\alpha \sigma^2_\psi \kappa)(3\kappa)} \right)^{-2}
\]

where \( M \) does not depend on \( n \). Now, a lower bound for \( t \) comes from plugging the upper bound \( M \) into (61).

\[
t \geq \left( 1 - \frac{\alpha M - \kappa}{(\alpha \sigma^2_\psi \kappa)((n^2 - 2n) \kappa + \alpha M)} \right)^{-1}
\]

(64)

Together, (62) and (64) bound the equilibrium value of \( t \). Now, the function \( \frac{1}{1-x} \) is differentiable at \( x = 1 \) with derivative equal to 1. Hence, (62) converges to 1 at the same rate that

\[
1 - \frac{\kappa}{(\alpha \sigma^2_\psi \kappa)((n^2 - 2n) \kappa)}
\]

converges to 1, which is \( \frac{1}{n^2} \); (64) is analogous. Thus, the equilibrium value of \( t - 1 \) converges to 0 at rate \( \frac{1}{n^2} \).

**B.2 Expected manipulation rents**

Since we have assumed that \( \psi \) is independent of factor exposures \( x_i \), which are linearly related to contract positions \( c_i \), we have:

\[
E \left[ \psi \sum_{i=1}^{n} c_i \right] = E \left[ \psi \right] E \left[ \sum_{i=1}^{n} c_i \right] = E \left[ \mu_\psi \sum_{i=1}^{n} c_i \right]
\]

Hence,

\[
E \left[ (p - \mu_\psi) \sum_{i=1}^{n} c_i \right] = E \left[ (p - \psi) \sum_{i=1}^{n} c_i \right]
\]
Now, plugging in for \( p \) using (8), expected manipulation rents are:

\[
E \left( \frac{\sum_{i=1}^{n} c_i}{n(n-2) \kappa} \frac{\left( \sum_{i=1}^{n} c_i \right)}{} \right) \tag{65}
\]

Since we have assumed agents’ factor exposures \( x_i \) are independent, agents’ contract positions \( c_i \) are also independent, so (65) becomes:

\[
\frac{\sigma_c^2}{(n-2) \kappa}
\]

### B.3 Heterogeneous beliefs

In this appendix, I show that contract purchases can also be generated by dispersion in agents’ beliefs about \( \psi \). Suppose an agent believes the mean of \( \psi \) is \( \beta_{\psi i} \), and its variance is \( \sigma_{\psi}^2 \). The agent can purchase contracts at a fixed price \( \mu_{\psi} \), and has no factor exposure \( x_i \).

From (48), the agent’s utility as a function of \( \psi \), over uncertainty in the spot market, is:

\[
\sqrt{\frac{d^2 + 2 \kappa d}{\alpha \kappa \sigma_{\eta}^2 + d^2 + 2 \kappa d}} \left( -\exp \left( -\alpha \left( \psi c_i - \mu_{\psi} c_i \right) - \frac{\alpha}{2} \left( \frac{\alpha \sigma_{\eta}^2 - \kappa}{\alpha \kappa \sigma_{\eta}^2 + d^2 + 2 \kappa d} \right) c_i^2 \right) \right)
\]

To calculate the agent’s expected utility (under her beliefs), we integrate assuming \( \psi \) has mean \( \beta_{\psi i} \) and variance \( \sigma_{\psi}^2 \). This gives:

\[
= \sqrt{\frac{d^2 + 2 \kappa d}{\alpha \kappa \sigma_{\eta}^2 + d^2 + 2 \kappa d}} \left( -\exp \left( -\alpha \sigma_{\psi}^2 c_i^2 + \frac{\alpha \kappa \sigma_{\eta}^2 - \kappa}{\alpha \kappa \sigma_{\eta}^2 + d^2 + 2 \kappa d} \right) c_i^2 \right)
\]

Only the exponent depends on \( c_i \), so we maximize:

\[
-\alpha \sigma_{\psi}^2 c_i^2 + \left( \beta_{\psi i} - \mu_{\psi} \right) c_i - \frac{\alpha \sigma_{\eta}^2 - \kappa}{\alpha \kappa \sigma_{\eta}^2 + d^2 + 2 \kappa d} c_i^2
\]
Differentiating and solving for \( c_i \), we have:

\[
c_i \left( 1 + \frac{\alpha \sigma^2_{\eta} - \kappa}{\alpha \sigma^2_{\psi}} \right) \left( d^2 + 2d\kappa + \alpha \kappa \sigma^2_{\eta} \right) = \frac{\beta \psi_i - \mu \psi}{\alpha \sigma^2_{\psi}} \tag{66}
\]

Comparing (66) to (9) and (10), a belief shock \( \beta \psi_i \) is isomorphic to a factor exposure \( x_i \) of size:

\[
x_i = -\frac{\beta \psi_i - \mu \psi}{\alpha \sigma^2_{\psi}}
\]
in the sense that it generates the same contract purchasing decisions and spot market behavior. Hence, contract purchases can be motivated either by disagreement or risk-sharing. If we do not observe factor exposures directly, based only on agents’ contract purchases and spot market behavior, we cannot separately identify factor exposures from heterogeneous beliefs, complicating welfare analysis for spot traders.

**B.4 Price impact in the contract market, quadratic taxes/subsidies**

In this appendix, I show that, if spot traders’ contract purchases have price impact, we can still solve for linear optimal contract purchasing rules. Moreover, price impact is isomorphic to subsidies or taxes to spot traders which are quadratic in the size of traders’ contract positions. Thus, a regulator can implement any positive choice of hedging aggressiveness \( t \) in equilibrium using some quadratic subsidy or tax.

Suppose that purchasing \( c_i \) contracts increases the price per contract to:

\[
\mu \psi + kc_i
\]
The total cost of buying \( c_i \) contracts is then:

\[
\mu \psi c_i + kc_i^2 \tag{67}
\]

Expression (67) shows that price impact shows up as a quadratic cost for holding derivative contracts, \( kc_i^2 \). This quadratic holding cost can also be used to model holding costs (most directly, margin capital costs) for derivative contracts. Alternatively, if a regulator
wanted to either decrease or increase the size of spot traders’ contract positions, she could impose a quadratic tax \( (k > 0) \) or subsidy \( (k < 0) \) on spot traders’ contract positions; traders’ contract payments would still be equal to \((67)\).

Combining this cost with \((3)\) and integrating, a spot trader’s conditional expected utility if she has factor exposure \( x_i \) and contract position \( c_i \) is:

\[
\frac{-1}{\sqrt{2\pi}\sigma_\psi^2} \int \sqrt{\frac{d^2 + 2\kappa d}{\alpha\kappa\sigma_\eta^2 + d^2 + 2\kappa d}} \exp \left[ -\alpha \left( \psi x_i - \mu_\psi x_i + \psi c_i - \mu_\psi c_i - kc_i^2 \right) - \frac{\alpha}{2} \left( \frac{\alpha\sigma_\eta^2 - \kappa}{\alpha\kappa\sigma_\eta^2 + d^2 + 2\kappa d} \right) c_i^2 \right] \exp \left( -\frac{(\psi - \mu_\psi)^2}{2\sigma_\psi^2} \right) d\psi
\]

\[
= \sqrt{\frac{d^2 + 2\kappa d}{\alpha\kappa\sigma_\eta^2 + d^2 + 2\kappa d}} \left( -\exp \left( -\frac{\alpha}{2} \left( -\alpha\sigma_\psi^2 (c_i + x_i)^2 - 2kc_i^2 - \frac{\alpha\sigma_\eta^2 - \kappa}{\alpha\kappa\sigma_\eta^2 + d^2 + 2\kappa d} c_i^2 \right) \right) \right)
\]

Hence, agents choose \( c_i \) to maximize:

\[-\alpha\sigma_\psi^2 (c_i + x_i)^2 - 2kc_i^2 - \frac{\alpha\sigma_\eta^2 - \kappa}{\alpha\kappa\sigma_\eta^2 + d^2 + 2\kappa d} c_i^2\]

The optimal choice of \( c_i \) satisfies:

\[
c_i \left( 1 + \frac{\alpha\sigma_\eta^2 - \kappa}{\alpha\sigma_\psi^2} \left( d^2 + 2\kappa d + \alpha\kappa\sigma_\eta^2 \right) + \frac{4k}{\alpha\sigma_\psi^2} \right) = -x_i \tag{68}
\]

Expression \((68)\) shows that spot traders’ optimal contract positions are still linear in their factor exposures \( x_i \). Comparing \((68)\) to \((9)\) and \((10)\) of proposition 1, if \( k > 0 \), the quadratic cost term decreases \( t \), so agents hedge less aggressively, by an amount which depends on the quadratic cost parameter \( k \), agents’ risk aversion \( \alpha \), and factor risk \( \sigma_\psi^2 \). Similarly, a subsidy \( k < 0 \) causes agents to hedge more aggressively.

Expression \((68)\) implies that, using quadratic taxes or subsidies, regulators can implement any desired value of \( t \) as an equilibrium outcome. For any \( t \), from \((56)\) of appendix
we have

$$\sigma_n^2 = \frac{t^2 \sigma_x^2}{n-1}$$

From (68), the quadratic tax/subsidy coefficient $k$ must satisfy:

$$c_i = -tx_i = \left(1 + \frac{\alpha \sigma_n^2 - \kappa}{\alpha \sigma_x^2} \frac{\kappa}{(d^2 + 2d\kappa + \alpha \kappa \sigma_n^2)} + \frac{4k}{\alpha \sigma_x^2} \right)^{-1} x_i$$

Plugging in for $\sigma_n^2$, $k$ must satisfy:

$$t = \left(1 + \frac{\alpha \sigma_x^2}{(\alpha \sigma_x^2) \left(d^2 + 2d\kappa + \alpha \kappa \sigma_x^2 \right)} + \frac{4k}{\alpha \sigma_x^2} \right)^{-1}$$  (69)

As we vary $k$, the RHS of (69) varies from 0 to $\infty$, so for any $t$, and any values of other primitives, there is a unique value of $k$ which satisfies (69), and thus implements $t$ as an equilibrium outcome.

C Supplementary material for section 5

C.1 Proof of proposition 3

We prove proposition 3; proposition 2 is a special case. Claim 5 first characterizes agents’ best responses, given the slope of residual supply, and subsection C.1.2 proves proposition 3 using claim 5.

C.1.1 Best responses

Claim 5. If agent $i$ has inventory position $y_i$ and contract position $c_i$, and the slope of residual supply is $d_i$, then agent $i$’s unique ex-post optimal bid curve is:

$$z_{Bi}(p; y_i, c_i) = -\frac{d_i}{\kappa_i + d_i} y_i + \frac{\kappa_i}{\kappa_i + d_i} c_i - \frac{\kappa_i d_i}{\kappa_i + d_i} (p - \psi)$$  (70)
Proof. Analogously to claim 1, assume that residual supply takes the form:

\[ z_{RSi}(p, \eta) = d_i(p - \psi) + \eta_i \]

We will optimize pointwise in \( \eta_i \). Define \( p^*(\eta_i) \) as the consumption-maximizing choice of \( p \) for any given \( \eta_i \), that is:

\[ p^*(\eta_i) \equiv \arg \max_p C(z_{RSi}(p, \eta_i), p; y_i, c_i) = \arg \max_p \psi z_{RSi}(p, \eta_i) - \frac{y_i^2}{2\kappa_i} - \frac{y_i z_{RSi}(p, \eta_i)}{\kappa_i} - \frac{z_{RSi}(p, \eta_i)^2}{2\kappa_i} + c_i p - z_{RSi}(p, \eta_i) p \]

Since \( z_{RSi}(p, \eta_i) \) is affine and increasing in \( p \), the objective function concave in \( p \), thus the first-order condition is necessary and sufficient for \( p^*(\eta_i) \) to be optimal. Differentiating with respect to \( p \) and setting to 0, and using that \( z'_{RSi}(p, \eta_i) = d_i \), we have:

\[ -\frac{d_i}{\kappa_i} y_i - \frac{z_{RS}(p^*(\eta_i), \eta_i)}{\kappa_i} d_i + c_i - z_{RS}(p^*(\eta_i), \eta_i) - (p^*(\eta_i) - \pi) d_i = 0 \tag{71} \]

Hence, any pair \((p^*(\eta_i), z_{RS}(p^*(\eta_i), \eta_i))\) — that is, any point \((p, z)\) which is the agent’s optimal choice for some \( \eta_i \) — satisfies (71). Hence, the unique bid curve which passes through the set of all ex-post optimal points is the curve implicitly defined by (71). Solving (71) for \( z_{RS}(p^*(\eta_i), \eta_i) \), we have (70).

\[ \square \]

C.1.2 Equilibrium

This proof is based on Appendix A.4 of Du and Zhu (2012), with notational modifications to suit the context of this paper. We seek a vector of demand and residual supply slopes \( b_i \) which satisfy, for all \( i \):

\[ d_i = \sum_{j \neq i} b_i = B - b_i \tag{72} \]

\[ b_i = \frac{d_i \kappa_i}{\kappa_i + d_i} \tag{73} \]
Rearranging, we have:

\[ d_i = \frac{b_i \kappa_i}{\kappa_i - b_i} \quad (74) \]

Combining (72) and (74), we have:

\[ \sum_j b_j - b_i = \frac{b_i \kappa_i}{\kappa_i - b_i} \]

Defining \( B \equiv \sum_j b_j \), we have

\[ (\kappa_i - b_i)(B - b_i) = b_i \kappa_i \]

This has two solutions. In order for \( B > b_i \), we must pick:

\[ b_i = \frac{2 \kappa_i + B - \sqrt{B^2 + 4 \kappa_i^2}}{2} \quad (75) \]

This is (23) of proposition 3. \( B \) must satisfy:

\[ B = \sum_j b_j = \sum_{i=1}^{n} \frac{2 \kappa_i + B - \sqrt{B^2 + 4 \kappa_i^2}}{2} \quad (76) \]

By multiplying the top and bottom of the RHS by \( 2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2} \) and simplifying, this becomes:

\[ B = \sum_{i=1}^{n} \frac{2 \kappa_i B}{2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2}} \]

Or,

\[ B \left( -1 + \sum_{i=1}^{n} \frac{2 \kappa_i}{2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2}} \right) = 0 \quad (77) \]

Now, define

\[ f(B) = -1 + \sum_{i=1}^{n} \frac{2 \kappa_i}{2 \kappa_i + B + \sqrt{B^2 + 4 \kappa_i^2}} \]
In order for $B$ to solve (77) when $B > 0$, we need $f(B) = 0$. Now, $f(0) > 0$, $f(B) \to -1$ as $B \to \infty$, and $f'(B) < 0$ for $B > 0$. Hence, $f(B) = 0$ at some unique $B$, hence there is a unique value of $B$ which solves (77), and thus there is a unique linear equilibrium for any demand slopes $\kappa_1 \ldots \kappa_n$.

Substituting $b_i = \frac{\kappa_i d_i}{\kappa_i + d_i}$ into best-response bid curves from claim 5, we have agents’ equilibrium bids in expression (30). To find prices, sum agents’ demand curves and equate to 0:

$$ \sum_{i=1}^{n} \left[ -y_i \frac{b_i}{\kappa_i} + c_i \frac{b_i}{\sum_{j \neq i} b_j} - (p - \psi) b_i \right] = 0 $$

Solving for $p$, we have (31).

### D Supplementary material for section 6

#### D.1 A dynamic price determination game

In this appendix, I model the LBMA gold auction as a continuous-time dynamic price determination game: agents announce their demand quantities in response to prices, and the game concludes when markets clear. I prove that, while we cannot rule out the existence of other equilibria, the dynamic game always admits an equilibrium equivalent to the unique static auction equilibrium. The intuition is very simple: if all agents besides $i$ are behaving according to their equilibrium strategies in a static bid submission game, then $i$ can do no better than to bid her static equilibrium strategy.

As in the baseline model of section 5, $i \in \{1 \ldots n\}$ agents have types $\kappa_1 \ldots \kappa_n$, which are common knowledge, and inventory positions $y_1 \ldots y_n$ and contract positions $c_1 \ldots c_n$, which are private information. As before, the utility of agent $i$ for purchasing $z$ units of the asset when the price is $p$ is:

$$ \psi z - \frac{(z + y_i)^2}{2\kappa} - pz + pc_i $$

Agents play a continuous-time auction, which I model as a simple differential game. At time $t = 0$, the auction begins at some deterministic price $p(0)$, which is known
to all participants. Participants simultaneously announce initial demands \( z_{Bi}(p(0)) \). Thereafter, at any given time \( t \), the price evolves according to the differential equation:

\[
\frac{dp}{dt} = \begin{cases} 
  k & z_{Bi}(p(t)) > 0 \\
  -k & z_{Bi}(p(t)) < 0 
\end{cases}
\]

This stylized price-setting process matches the stylized fact shown in the left panel of figure 3, that prices decrease when aggregate demand is below 0, and increase when aggregate demand is above 0. Agents can update their demand functions \( z_{Bi}(p(t)) \) as the price \( p(t) \) changes; agents’ demand functions are required to be decreasing in price. The game ends at the first time \( T \) when aggregate demand is exactly equal to 0,

\[
\sum_{i=1}^{n} z_{Bi}(p(T)) = 0
\]

at which point each agent purchases \( z_{Bi}(p(T)) \) units of the good for \( p(T) \) per unit, and is paid \( p(T) \) per unit contract that she holds.

I assume that agents choose the rate at which demand changes with price, \( z'_{Bi}(p(t)) = \frac{dz_{Bi}}{dp} \), rather than the level of demand; this ensures that resultant demand functions are continuous. Since \( \frac{dp}{dt} \) is constant throughout any instance of the game, choosing \( \frac{dz_{Bi}}{dp} \) is equivalent to choosing \( \frac{dz_{Bi}}{dt} \). I require agents’ reported demand slopes to be finite and bounded away from 0:

\[
-M \leq z'_{Bi}(p(t)) \leq -\epsilon < 0 \quad (78)
\]

This guarantees that the game will end in finite time. I will show that the dynamic differential auction game admits an equilibrium which coincides with the equilibrium of the static auction in section 3 since agents’ bid curves in the static game have slopes which are negative, finite and bounded away from 0, for any \( \kappa_1 \ldots \kappa_n, \epsilon \) can always be chosen small enough in magnitude, and \( M \) large enough, that the bounds in (78) are not binding in equilibrium.

\[\text{\textcopyright 2018 IBA's published auctions specification documents do not describe how the auction starting price is chosen. However, the choice of starting price does not significantly affect agents' optimal strategies, so I assume the price is non-random and commonly known for simplicity.}\]
Claim 6. The dynamic auction game always ends in finite time.

Proof. Suppose that $\sum_{i=1}^{n} z_{Bi}(p(0)) > 0$. Then, we have:

$$\frac{dz_{Bi}}{dt} = \sum_{i=1}^{n} \frac{dz_{Bi}}{dt} = \sum_{i=1}^{n} \frac{dz_{Bi}}{dp} \frac{dp}{dt} = k \sum_{i=1}^{n} \frac{dz_{Bi}}{dp} \in [-kM, -k\epsilon]$$

Thus,

$$z_{Bi}(p(t)) \in [z_{Bi}(p(0)) - kMt, z_{Bi}(p(0)) - k\epsilon t]$$

Hence, $z_{Bi}(p(t)) = 0$ for some $t \in \left[\frac{z_{Bi}(p(0))}{kM}, \frac{z_{Bi}(p(0))}{k\epsilon}\right]$, hence the game ends in finite time. The proof of the case where $\sum_{i=1}^{n} z_{Bi}(p(0)) < 0$ is analogous.

The setup of this game fails to capture two features of the ICE gold auction in practice. First, in practice the price does not evolve smoothly, but jumps in increments; second, the auction does not end when supply and demand are exactly equal, but allows for some volume gap. These features are difficult to model tractably; modelling the first would require taking a stance on the price updating rule, which to my knowledge is not publically documented by ICE, and the second induces noise in prices which implies that outcomes in the dynamic game not to correspond one-to-one to outcomes in the static bid submission game. Thus, for analytical tractability, I adopt the simpler monotone differential game model as an approximation to the ICE auction.

A history is a sequence of observed demand slopes $z'_{Bi}(p(t))$ of all agents on the interval $t \in [0, T]$. It is equivalent to assume we observe agents’ demand functions; thus a history at time $t$ can be described as:

$$h_T = \{(z_{D1}(p(t)) \ldots z_{Dn}(p(t))) , t \in [0, T]\}$$

A strategy is a decision $z_{Bi}(p(0))$ about what demand to announce at the starting price $p(0)$, and then a choice of $z'_{Bi}(p(t))$ for every possible history. Both decisions may also depend on the realization of agents’ inventory $y_i$ and contract position $c_i$. Thus, in full generality, strategies can be very complex. I will restrict attention to a class of naive linear strategies, which I define below.
Definition 3. A naive linear strategy for agent i is a strategy in which:

$$z_{Bi}(p(0)) = f_i(y_i, c_i), z'_{Bi}(p(t)) = b_i$$

that is, agent i’s demand function has an intercept which may depend on $y_i$ and $c_i$, and constant slope.

Agents playing naive strategies cannot condition their slopes on the observed behavior of other agents, or their own contract positions and inventory positions – they must commit to a constant slope, independent of the history of the game. Note that (78) implies that we must have

$$b_i \in [-M, -\epsilon]$$

Naive linear strategies are isomorphic to affine bid curves. For any naive linear strategies described by $f_i(y_i, c_i), b_i$, we can construct the equivalent bid curve as:

$$z_{Bi}(p; f_i(y_i, c_i), b_i) \equiv f_i(y_i, c_i) + \int_{p_0}^{p} b_i dp = f_i(y_i, c_i) + b_i(p - p_0) \quad (79)$$

Proposition 4. Naive linear strategies $f_i(y_i, c_i), b_i$ corresponding to equilibrium bidding strategies $z_{Bi}(p; y_i, c_i)$ in the static auction game in proposition 3 also constitute an equilibrium in the dynamic auction game.

I prove proposition 4 in two steps. Claim 7 shows that, if all agents are restricted to naive linear strategies, the dynamic auction game produces exactly the same outcomes as the static auction game. Claim 8 shows that, if all agents other than i are playing naive linear strategies, agent i can do no better than to play a naive linear strategy. Thus, equilibria in naive linear strategies are equilibria in the broader game. This proves proposition 4 as equilibrium strategies in proposition 3 for the static auction game can be implemented as naive linear strategies in the dynamic auction game.

Claim 7. If all agents play naive linear strategies, the outcomes of the dynamic auction game are exactly the outcomes of a static auction game in which in which agents submit the bid functions specified by (79).

Proof. Suppose agents are playing the naive linear strategies $f_i(y_i, c_i), b_i$. Construct the
equivalent strictly decreasing bid curves \( z_{Bi} (p; f_i (y_i, c_i), b_i) \). Fix some realization of \( y_i, c_i \) across agents, and consider the aggregate demand function,

\[
\sum_{i=1}^{n} z_{Bi} (p; f_i (y_i, c_i), b_i)
\]

Claim 6 states that the dynamic auction game always ends in finite time; thus, the dynamic auction must end at some time \( T \), at some price \( p(T) \), at which

\[
\sum_i z_{Bi} (p(T); f_i (y_i, c_i), b_i) = 0
\]

Then each agent purchases

\[
z_{Bi} (p; f_i (y_i, c_i), g_i (y_i, c_i))
\]

units of the good at price \( p(T) \), and is paid \( p(T) \) per unit contract she holds. This is exactly the same outcome as agents receive in a static auction game in which agents submit bid curves \( z_{Bi} (p; f_i (y_i, c_i), b_i) \).

The strategies in the dynamic auction game with naive linear strategies are a subset of their strategies in the static auction, since they are not allowed to condition the slopes of their demand on \( y_i, c_i \). However, all equilibria in the static auction game correspond to naive linear strategies, because agents’ bid slopes in equilibrium do not depend on \( y_i, c_i \), as shown by proposition 3 in the baseline model of section 3. Thus, if agents are restricted to playing naive linear strategies, equilibria in the dynamic auction game correspond to those of the static auction game. The following Claim shows that these strategies also constitute equilibria without the restriction to naive linear strategies:

Claim 8. If all agents other than \( i \) are playing naive linear strategies, it is weakly optimal for agent \( i \) to play a naive linear strategy.

Proof. Suppose all other agents are playing naive linear strategies, described by

\[
f_j (y_j, c_j), b_j
\]
From the perspective of agent $i$, there is a random residual supply curve:

$$z_{RSi}(p, y_{-i}, c_{-i}) \equiv -\sum_{j \neq i} z_{Bj}(p, y_j, c_j) = -\sum_{j \neq i} \left[ f_j(y_j, c_j) + b_j(p - \psi) \right]$$

Importantly, the slope of the residual supply curve is constant at $d = \sum_{j \neq i} b_j$. The dynamic auction game concludes when $\sum_{i=1}^n z_{Bi}(p) = 0$, that is, when

$$z_{Bi}(p) = z_{RSi}(p, y_{-i}, c_{-i})$$

Hence, the set of all attainable combinations of quantity and price available to the agent, for any realization of $(y_{-i}, c_{-i})$, are described by $z_{RSi}(p, y_{-i}, c_{-i})$. The agent can do no better, using arbitrarily complex strategies, than choosing the optimal point on $z_{RSi}(p, y_{-i}, c_{-i})$ point for every realization of $(y_{-i}, c_{-i})$. Proposition 3 shows that this can be accomplished in the static auction game by submitting the bid curve:

$$z_{Bi}(p; y_i, c_i) = -\frac{d}{\kappa + d} y_i + \frac{\kappa}{\kappa + d} c_i - \frac{\kappa d}{\kappa + d} (p - \psi)$$

Since the slope of $z_{Bi}$ does not depend on $y_i$ or $c_i$, agent $i$ can implement this bid curve by playing the naive linear strategy:

$$f_i(y_i, c_i) = -\frac{d}{\kappa + d} y_i + \frac{\kappa}{\kappa + d} c_i + \frac{\kappa d}{\kappa + d} \psi, \quad b_i = -\frac{\kappa d}{\kappa + d}$$

Hence, agent $i$ can do no better than playing a naive linear strategy. \hfill \Box

Claim 8 shows that there exists an equilibrium in the game in which agents play naive linear strategies corresponding to equilibrium strategies in the static auction game, as in proposition 3. However, this does not imply that naive strategies are the only equilibrium strategies in this game; the strategy space is rich, and it is possible that many collusive equilibria of the kind documented by, for example, Wilson (1979) and others may exist.

One concern is that, if agents adopt dynamic bidding strategies, the estimated slopes of auction demand may depend on which auction rounds we use to estimate slopes. To gauge how much this would affect my results, in figure 5 I estimate auction demand slopes using only data from rounds 3 and above, for auctions which lasted 4 or more
Figure 5: Gold auction demand slope, all rounds vs late rounds only

Notes. Auction demand slopes by auction participant number $n_a$ in primary estimation sample, estimated using data from all auction rounds (blue) and only data from auction rounds 3 and above (red). Dotted lines denote 80th and 20th percentile values.

rounds. If, for example, agents shade bids in early rounds and bid more aggressively in late rounds, demand slopes estimated using late-round bidding data should be higher than demand slopes using the full sample. However, figure 5 shows that late-round demand slopes are statistically almost indistinguishable from demand slopes estimated using the full dataset, suggesting that differential bid shading across rounds does not have large effects on estimated demand slopes.

D.2 Data cleaning

Let auctions be indexed by $a \in \{1 \ldots A\}$, and suppose that auction $a$ lasts for $R_a$ rounds, indexed by $r$. For each round $r$ of each auction $a$, I observe the number of participants, $n_{ar}$, the round price, $p_{ar}$, and the total volume of gold that auction participants wish to buy and sell, respectively $b_{ar}$ and $s_{ar}$. For estimating my model, I filter to auctions with at least 3 rounds, $R_a \geq 3$, as I will estimate slopes of demand by regressing round buy and sell volume on round prices. I also filter to auctions in which the number of participants $n_{ar}$ is constant over the course of the auction, and to auctions with 5-10 participants, as there are too few auctions with $n = 4$ and $n = 11$ to reliably estimate the distributions of volumes and demand slopes. This reduces the estimation sample to 502 auctions.
Table 1 shows features of my sample. Most auctions have 6-9 participants and conclude in 3-6 rounds. The range of prices between rounds for any given auction is small, relative to variation in gold prices between auctions: the difference between the highest and lowest round prices in a given auction is around $1 USD/oz on average. An average of 167,577oz of gold is traded on average in each auction in my estimation sample.

Define the volume imbalance in round $r$ of auction $a$ as the difference between buy and sell volume, that is:

$$i_{ar} \equiv b_{ar} - s_{ar}$$

The auction clearing price, buy and sell volume, and volume imbalance at the final round $R_a$ of auction $a$ are:

$$p_{aR_a}, b_{aR_a}, s_{aR_a}, i_{aR_a}$$

Define the total trade volume at the end of the auction as the sum of buy and sell volume:

$$v_{aR_a} = b_{aR_a} + s_{aR_a}$$

For simplicity, when discussing final-round quantities, I will omit the round subscript $R_a$; thus, I will write $p_a, v_a, i_a$ to mean the final auction price, trade volume, and volume imbalance in auction $a$. Let $n_a$ represent the number of participants in auction $a$; this is uniquely defined within my estimation subsample, since I filter to auctions in which participation is constant.

To calculate gold futures open interest, I use CME group end-of-day data on gold futures from 2015-02-02 to 2016-12-30, provided by the Fama-Miller Center. COMEX gold futures position limits are publicly available on the CME group’s website. To calculate the monthly volatility of gold prices, I use a time series of front-month COMEX gold futures prices from November 2008 to September 2018, downloaded from Factset, and calculate the standard deviation of monthly differences in gold prices, calculated using the first trading day of each month.
D.3 Details on demand slope measurement

Suppose an auction has $n_a$ participants, with holding capacities $\kappa_{a1} \ldots \kappa_{an_a}$, and the risk factor is $\psi_a$. In a static auction, agent $i$ would bid:

$$z_{Bi} (p_{a}; c_{ai}, y_{ai}) = -\frac{b_{ai}}{\kappa_i} y_{ai} + \frac{b_{ai}}{\sum_{j \neq i} b_{aj}} c_{ai} - b_{ai} (p_a - \psi_a)$$  \hspace{1cm} (80)

where the bid slopes $(b_{a1} \ldots b_{an_a})$ satisfy expressions (23) and (24) of proposition 3. I assume that agents bid their static equilibrium bids in each round of the dynamic price-setting mechanism: if the round prices is $p_{ar}$, $i$ bids to buy net quantity $z_{Bi} (p_{ar}; c_{ai}, y_{ai})$. Appendix D.1 shows that this behavior is an equilibrium in a stylized model of the dynamic price-setting mechanism. Under this assumption, in round $r$ of auction $a$, the aggregate volume imbalance will be:

$$i_{ar} = \sum_{i=1}^{n_a} z_{Bi} (p_{ar}, y_{ai}) = \sum_{i=1}^{n_a} \left[ -y_{ai} \frac{b_{ai}}{\kappa_i} - (p_{ar} - \psi_a) b_{ai} \right]$$  \hspace{1cm} (81)

Thus, using any two rounds $r$ and $\tilde{r}$ of the same auction, we can measure the auction demand slope:

$$\frac{i_{ar} - i_{a\tilde{r}}}{p_{ar} - p_{a\tilde{r}}} = -\sum_{i=1}^{n_a} b_{ai} = -B_a$$

In any auction with more than two rounds, $B_a$ is overidentified. I estimate $\hat{B}_a$ by regressing volume imbalance on prices using all rounds of auction $a$:

$$\hat{B}_a = -\frac{\sum_{r=1}^{R_a} (i_{ar} - \bar{i}_a) (p_{ar} - \bar{p}_a)}{\sum_{r=1}^{R_a} (p_{ar} - \bar{p}_a)}$$  \hspace{1cm} (82)

where $\bar{i}_a$ and $\bar{p}_a$ are the averages of $i_{ar}$ and $p_{ar}$, respectively, within auction $a$. 

80
D.4 Descriptive evidence on model fit

Define the de-meaned volume imbalances and prices as:
\[
\bar{i}_{ar} \equiv i_{ar} - \bar{i}_a, \quad \bar{p}_{ar} \equiv p_{ar} - \bar{p}_a
\]

We can evaluate the fit of the model by estimating the following regression:
\[
\bar{i}_{ar} = B_a \bar{p}_{ar} + \epsilon_{ar}
\]
\[\text{(83)}\]
this evaluates how well volume imbalances can be fitted by linear functions of prices, separately for each auction. Estimating specification (83) for the primary estimation sample using OLS, we get an adjusted $R^2$ of 0.8295, and an F-statistic of 21.29, with a p-value below double precision, $2.2 \times 10^{-16}$. The estimated standard deviation of $\epsilon_{ar}$ is 11,080oz. This implies that, as the left panel of figure 4 suggests, volume imbalances are significantly associated with prices, and simple linear functions of prices explain around 83\% of variation in volume imbalances within auctions. Results of estimating (83) using the full dataset are very similar: we get a slightly lower $R^2$ of 0.7923, and we estimate the standard error of $\epsilon_{ar}$ to be 13,587oz.

D.5 Random sampling of $\kappa_i$ and $\sigma_{y_i}$

By inspection, (23) and (24) have constant returns to scale in $\kappa_i$: scaling all $\kappa_i$’s up by some factor increases the unique equilibrium $b_i$ and $B$ by the same factor. Hence, we can sample a vector $(\bar{\kappa}_1 \ldots \bar{\kappa}_n)$ consistent with some observed aggregate demand slope $B$, by sampling a random vector of $\bar{\kappa}_i$’s and then scaling them to match the desired $B$. Formally, I first draw some random vector:
\[
\bar{\kappa}_1 \ldots \bar{\kappa}_n
\]
where each $\bar{\kappa}_i$ is an i.i.d. random sample; to generate relatively high levels of concentration in my samples, I draw these from an exponential distribution with mean 100. I then
numerically solve (23) and (24) to find the unique equilibrium \( \tilde{b}_i \)’s and \( \tilde{B} \). I then set:

\[
(\hat{\kappa}_1 \ldots \hat{\kappa}_n) = \frac{B}{\tilde{B}} (\hat{\kappa}_1 \ldots \hat{\kappa}_n)
\]

Now, fix a given draw for holding capacities \( \hat{\kappa}_i \). If we assume that agents’ inventory shocks are independent normal random variables, appendix D.5.1 below shows that volume is a linear function of \( \sigma_{yi} \); thus, to simulate draws of \( \sigma_{yi} \) which match some target value for volume, we draw some random vector \( \tilde{\sigma}_{yi} \) and then scale it up to match the volume target.

D.5.1 Expected volume with independent normal inventory shocks

Setting the sum of agents’ bid curves in (80) to 0 and solving for \( p_a \), the auction clearing price is:

\[
p_a - \psi_a = \frac{1}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{i=1}^{n} -\frac{b_{ai}}{\kappa_i} y_{ai} + \frac{b_{ai}}{\sum_{j \neq i} b_{aj}} c_{ai} \right]
\]

(84)

Since I assume agents do not hold contract positions when estimating the model, set \( c_{ai} = 0 \). Demand for each agent at the auction clearing price is:

\[
z_{bai}(p_a) = -\frac{b_{ai}}{\kappa_i} y_{ai} - b_{ai} (p_a - \psi_a)
\]

\[
= -\left(1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}}\right) \left(\frac{b_{ai}}{\kappa_i} y_{ai}\right) + \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}} \left[ \sum_{j \neq i} b_{aj} \frac{b_{aj}}{\kappa_j} y_{aj} \right]
\]

Assuming \( y_{aj} \) are independent mean-0 normal random variables, \( z_{bi}(p_a) \) is also normal, with mean 0 and variance:

\[
\text{Var} (z_{bai}(p_a)) = \left(1 - \frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}}\right)^2 \left(\frac{b_{ai}}{\kappa_i}\right)^2 \sigma^2_{yai} + \left(\frac{b_{ai}}{\sum_{i=1}^{n} b_{ai}}\right)^2 \sum_{j \neq i} \left(\frac{b_{aj}}{\kappa_j}\right)^2 \sigma^2_{yaj}
\]

Thus, the expected total trade volume for participant \( i \) in equilibrium is:

\[
E [z_{bai}(p_a)] = \sqrt{\frac{2 \text{Var} (z_{bai}(p_a))}{\pi}}
\]
Expected volume for all participants is the sum of $E \| z_{Bai}(p_a) \|$ across all participants in auction $a$; this scales linearly with $\sigma_{yai}$.

### D.6 Target moments

Table 4 shows the 20th, 50th, and 80th percentile estimates of auction demand slopes and auction volumes, for auctions with different numbers of participants.

### D.7 Basis risk and manipulation rent expressions

I set the standard deviation of contract positions, $\sigma_{ci}$, to 150,000oz of gold per agent, and I set the standard deviation of inventory shocks, $\sigma_{yi}$, to match volume, as I describe in appendix D.5 above. Assuming $c_{ai}$ is independent of $y_{ai}$ and $c_{aj}, y_{aj}$, basis risk and manipulation rents are respectively:

$$\text{Var}(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right)^2 \sum_{i=1}^{n} \left( \frac{b_i}{d_i} \sigma_{ci} \right)^2$$

$$\mathbb{E} \left( \sum_{i} c_i \right)(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right)^n \sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}^2$$

If we assume $c_i$ can be correlated with $y_{ai}$, but is independent of $c_{aj}, y_{aj}$, basis risk and manipulation rents are maximized if $c_{ai}$ and $y_{ai}$ are perfectly negatively correlated, in which case we have:

$$\text{Var}(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right)^2 \sum_{i=1}^{n} \left( \frac{b_i}{d_i} \sigma_{ci} + \frac{b_i}{k_i} \sigma_{yi} \right)^2$$

$$\mathbb{E} \left( \sum_{i} c_i \right)(p - \psi) = \left( \frac{1}{\sum_{i=1}^{n} b_i} \right) \left( \sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}^2 + \frac{b_i}{k_i} \sigma_{ci} \sigma_{yi} \right)$$

If we assume $c_i$ is independent of $y_{ii}$, but contract positions can be correlated across agents, basis risk and rents are maximized if all agents’ contract positions are perfectly
Table 4: Target moments

<table>
<thead>
<tr>
<th>n</th>
<th>Demand slope</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>26.84 42.70 61.92</td>
<td>89.57 126.00 164.81</td>
</tr>
<tr>
<td>6</td>
<td>30.35 41.46 70.95</td>
<td>90.03 130.06 197.50</td>
</tr>
<tr>
<td>7</td>
<td>35.12 52.87 68.35</td>
<td>100.88 144.98 199.02</td>
</tr>
<tr>
<td>8</td>
<td>40.06 57.06 73.08</td>
<td>121.41 164.13 211.93</td>
</tr>
<tr>
<td>9</td>
<td>31.25 52.67 73.13</td>
<td>119.48 167.52 237.41</td>
</tr>
<tr>
<td>10</td>
<td>24.68 43.19 59.01</td>
<td>156.94 202.07 290.53</td>
</tr>
</tbody>
</table>

Notes. Estimated 20th, 50th and 80th percentile values for auction demand slopes and auction volumes, in the primary estimation sample of 502 auctions. Volume is in units of 1,000oz, and demand slopes are in units of 1,000 (oz²/USD).
correlated, in which case we have:

\[
\text{Var}(p - \psi) = \left(\frac{1}{\sum_{i=1}^{n} b_i}\right)^2 \left(\sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}\right)^2
\]

\[
E\left(\sum_{i} c_i\right) (p - \psi) = \left(\frac{1}{\sum_{i=1}^{n} b_i}\right)^2 \left(\sum_{i=1}^{n} \frac{b_i}{d_i} \sigma_{ci}\right)^2 \left(\sum_{i=1}^{n} \sigma_{ci}\right)
\]

### D.8 Full sample results

To ensure that my results are robust to the sample of auctions used, I repeat the analysis of the main text, using all 1,331 auctions which lasted more than one round. I cannot use auctions which lasted a single round for the estimation, because auction demand slopes are not identified. Since participation may change through the course of these auctions, I classify auctions by the maximum number of participants observed through the auction. Table 5 shows the estimated percentiles of demand slopes and volumes, which are the target moments used for estimation from the full sample, and tables 6 and 7 show estimated basis risk and manipulation rents. Results are quantitatively quite similar; in fact, auction demand slopes are actually somewhat higher in the full dataset than my primary estimation sample, implying that basis risk and manipulation rents estimated using the full sample are somewhat lower than those from the main estimation sample.

### D.9 Increasing participation holding fixed B

Suppose all agents have symmetric spot holding capacities \(k\), with independent contract positions \(c_i\) with variance \(\sigma_c^2\), and no inventory shocks. From proposition 1, price variance is:

\[
\frac{\sigma_c^2}{n (n-2)^2 \kappa^2}
\]

and the slope of aggregate auction demand is:

\[
B = \frac{n - 2}{n - 1} \kappa
\]
Table 5: Target moments, full sample

<table>
<thead>
<tr>
<th>n</th>
<th>Demand slope p20</th>
<th>Demand slope p50</th>
<th>Demand slope p80</th>
<th>Volume p20</th>
<th>Volume p50</th>
<th>Volume p80</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>24.69</td>
<td>39.34</td>
<td>65.96</td>
<td>79.76</td>
<td>118.79</td>
<td>171.27</td>
</tr>
<tr>
<td>6</td>
<td>30.16</td>
<td>50.00</td>
<td>81.48</td>
<td>77.81</td>
<td>122.00</td>
<td>181.88</td>
</tr>
<tr>
<td>7</td>
<td>46.27</td>
<td>65.05</td>
<td>93.31</td>
<td>86.55</td>
<td>126.45</td>
<td>179.56</td>
</tr>
<tr>
<td>8</td>
<td>44.85</td>
<td>68.73</td>
<td>96.89</td>
<td>105.74</td>
<td>146.21</td>
<td>200.51</td>
</tr>
<tr>
<td>9</td>
<td>36.12</td>
<td>62.75</td>
<td>98.38</td>
<td>121.31</td>
<td>166.99</td>
<td>244.60</td>
</tr>
<tr>
<td>10</td>
<td>34.81</td>
<td>54.92</td>
<td>84.14</td>
<td>144.26</td>
<td>191.07</td>
<td>260.52</td>
</tr>
</tbody>
</table>

Notes. Estimated 20th, 50th and 80th percentile values for auction demand slopes and auction volumes, in the full sample of 1,316 auctions. Volume is in units of 1,000 oz, and demand slopes are in units of 1,000 (oz²/USD).
Table 6: Gold auctions basis risk estimates, full sample

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>1.65 (1.27-2.63)</td>
<td>2.20 (1.86-2.87)</td>
<td>3.11 (2.84-3.95)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>2.77 (2.14-4.41)</td>
<td>4.20 (3.59-5.48)</td>
<td>5.22 (4.77-6.63)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>4.41 (3.40-7.03)</td>
<td>4.84 (3.96-7.12)</td>
<td>8.31 (7.60-10.56)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>1.20 (0.911-1.94)</td>
<td>1.59 (1.35-2.12)</td>
<td>2.41 (2.21-3.03)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>1.96 (1.48-3.17)</td>
<td>3.01 (2.54-3.83)</td>
<td>3.93 (3.61-4.93)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>3.24 (2.46-5.25)</td>
<td>3.63 (2.98-5.40)</td>
<td>6.51 (5.98-8.17)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.953 (0.713-1.58)</td>
<td>1.31 (1.15-1.75)</td>
<td>2.03 (1.88-2.53)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>1.37 (1.02-2.26)</td>
<td>2.15 (1.89-2.71)</td>
<td>2.92 (2.69-3.63)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>1.92 (1.44-3.18)</td>
<td>2.29 (1.92-3.32)</td>
<td>4.10 (3.79-5.11)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.854 (0.65-1.48)</td>
<td>1.26 (1.10-1.68)</td>
<td>1.91 (1.78-2.41)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>1.20 (0.916-2.09)</td>
<td>2.02 (1.76-2.53)</td>
<td>2.70 (2.51-3.40)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>1.84 (1.40-3.21)</td>
<td>2.18 (1.81-3.36)</td>
<td>4.13 (3.85-5.21)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.785 (0.588-1.32)</td>
<td>1.22 (1.08-1.57)</td>
<td>1.84 (1.72-2.20)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>1.23 (0.922-2.07)</td>
<td>2.21 (1.98-2.77)</td>
<td>2.89 (2.70-3.44)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>2.14 (1.60-3.60)</td>
<td>2.45 (2.01-3.71)</td>
<td>5.02 (4.70-5.98)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.862 (0.648-1.45)</td>
<td>1.45 (1.29-1.86)</td>
<td>2.12 (1.99-2.55)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>1.32 (0.993-2.23)</td>
<td>2.55 (2.27-3.17)</td>
<td>3.25 (3.05-3.91)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>2.08 (1.57-3.52)</td>
<td>2.38 (1.96-3.68)</td>
<td>5.12 (4.82-6.17)</td>
</tr>
</tbody>
</table>

Notes. Estimated standard deviations of basis risk for full sample of 1,316 auctions, in units of prices (USD/oz). Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. "B + v choice" refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 5. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
Table 7: Gold auctions manipulation rent estimates, full sample

<table>
<thead>
<tr>
<th>N</th>
<th>B + v type</th>
<th>Ind. C</th>
<th>C + X</th>
<th>Corr. C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Low</td>
<td>0.467 (0.427-0.593)</td>
<td>0.688 (0.611-0.772)</td>
<td>2.333 (2.133-2.966)</td>
</tr>
<tr>
<td>5</td>
<td>Med</td>
<td>0.782 (0.715-0.995)</td>
<td>1.336 (1.167-1.503)</td>
<td>3.912 (3.577-4.974)</td>
</tr>
<tr>
<td>5</td>
<td>High</td>
<td>1.247 (1.140-1.585)</td>
<td>1.429 (1.309-1.718)</td>
<td>6.233 (5.698-7.924)</td>
</tr>
<tr>
<td>6</td>
<td>Low</td>
<td>0.362 (0.332-0.454)</td>
<td>0.536 (0.491-0.599)</td>
<td>2.170 (1.993-2.723)</td>
</tr>
<tr>
<td>6</td>
<td>Med</td>
<td>0.589 (0.541-0.739)</td>
<td>1.036 (0.932-1.147)</td>
<td>3.536 (3.248-4.437)</td>
</tr>
<tr>
<td>6</td>
<td>High</td>
<td>0.977 (0.897-1.226)</td>
<td>1.160 (1.078-1.372)</td>
<td>5.863 (5.385-7.355)</td>
</tr>
<tr>
<td>7</td>
<td>Low</td>
<td>0.305 (0.282-0.380)</td>
<td>0.475 (0.437-0.528)</td>
<td>2.136 (1.972-2.659)</td>
</tr>
<tr>
<td>7</td>
<td>Med</td>
<td>0.438 (0.404-0.545)</td>
<td>0.794 (0.719-0.870)</td>
<td>3.064 (2.828-3.814)</td>
</tr>
<tr>
<td>7</td>
<td>High</td>
<td>0.615 (0.568-0.766)</td>
<td>0.798 (0.746-0.923)</td>
<td>4.308 (3.976-5.362)</td>
</tr>
<tr>
<td>8</td>
<td>Low</td>
<td>0.287 (0.267-0.362)</td>
<td>0.487 (0.448-0.537)</td>
<td>2.295 (2.140-2.893)</td>
</tr>
<tr>
<td>8</td>
<td>Med</td>
<td>0.404 (0.377-0.510)</td>
<td>0.794 (0.722-0.862)</td>
<td>3.235 (3.017-4.079)</td>
</tr>
<tr>
<td>8</td>
<td>High</td>
<td>0.620 (0.578-0.781)</td>
<td>0.802 (0.750-0.942)</td>
<td>4.957 (4.623-6.250)</td>
</tr>
<tr>
<td>9</td>
<td>Low</td>
<td>0.277 (0.259-0.330)</td>
<td>0.503 (0.469-0.546)</td>
<td>2.490 (2.328-2.966)</td>
</tr>
<tr>
<td>9</td>
<td>Med</td>
<td>0.434 (0.406-0.517)</td>
<td>0.921 (0.848-0.986)</td>
<td>3.904 (3.650-4.649)</td>
</tr>
<tr>
<td>9</td>
<td>High</td>
<td>0.754 (0.704-0.897)</td>
<td>0.936 (0.888-1.068)</td>
<td>6.783 (6.340-8.077)</td>
</tr>
<tr>
<td>10</td>
<td>Low</td>
<td>0.318 (0.299-0.383)</td>
<td>0.632 (0.587-0.681)</td>
<td>3.180 (2.988-3.829)</td>
</tr>
<tr>
<td>10</td>
<td>Med</td>
<td>0.487 (0.458-0.587)</td>
<td>1.125 (1.033-1.201)</td>
<td>4.871 (4.578-5.866)</td>
</tr>
<tr>
<td>10</td>
<td>High</td>
<td>0.769 (0.722-0.926)</td>
<td>0.951 (0.909-1.094)</td>
<td>7.686 (7.224-9.256)</td>
</tr>
</tbody>
</table>

Notes. Estimated expected manipulation rents for full sample of 1,316 auctions, in units of millions of US dollars. Each cell reports the median value, then the minimum and maximum values over all samples in parentheses, as described in subsection 6.4.1. “B + v choice” refers to the low, medium and high manipulation risk choices for slope and volume target moments, as discussed in subsection 6.4.2. Target moment values are in table 5. Columns 3-5 use different assumptions on contract and inventory covariances, as described in subsection 6.4.3.
Suppose we fix the slope of aggregate auction demand at some value $B$, and the variance of individual spot traders’ contract positions $\sigma_c^2$, and increase the number of auction participants $n$. For any $n$, there is a unique value of $\kappa$ which rationalizes any $B$:

$$\kappa(B, n) = \frac{B}{n-2} \frac{n - 1}{n}$$  \hspace{1cm} (86)

Plugging (86) into (85) and rearranging, we have price variance as a function of $\sigma_c^2$, $B$, and $n$:

$$\sigma^2 = \frac{\sigma_c^2}{B^2 (n-1)^2} \frac{n}{n}$$

This is decreasing in $n$ for $n \geq 3$. Thus, if spot traders are symmetric, increasing the number of spot traders, fixing $B$ and $\sigma_c^2$, decreases basis risk.

Similarly, plugging (86) into expression (19) for manipulation rents, we get:

$$\tau = \frac{\sigma_c^2}{B} \frac{n}{n-1}$$

so manipulation rents are also decreasing in $n$, converging to the constant $\frac{\sigma_c^2}{B}$.

Intuitively, as we increase $n$, there are a number of counteracting effects: competition increases, so agents’ manipulation incentives decrease; however, since each agent holds a contract position with variance $\sigma_c^2$, the variance of contract positions summed across all agents increases. Basis risk is decreasing in $n$, because the competition effect dominates; for manipulation rents, the effects asymptotically offset each other exactly. Note also that the variance of total contract positions increases linearly in $n$, so manipulation rents per agent – or per contract – decrease towards 0 rapidly as $n$ increases.