Depreciating Licenses*

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Abstract

Many governments assign use licenses for natural resources, such as radio spectrum, fishing rights, and mineral extraction rights, through auctions or other market-like mechanisms. License design affects resource users' investment incentives, as well as the efficiency of asset allocation. No existing license design achieves first-best outcomes on both dimensions. Long-term licenses give owners high investment incentives, but impede reallocation to high-valued entrants. Short-term licenses improve allocative efficiency but discourage investment. We propose a simple new mechanism, the *depreciating license*, and we argue that it navigates this tradeoff more effectively than existing license designs.

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1 Introduction

Many natural resources, such as land, radio spectrum, fishing rights, pollutant emissions permits, and mineral extraction rights, are allocated to private parties by selling use licenses in auctions. A sizable literature has analyzed how auctions should be designed to optimally allocate these licenses. Comparatively little work studies how license design affects the efficiency of resource utilization.

Optimal resource utilization has two important dimensions. Efficient use often requires users to make costly investments. For example, to maximize the social value of a fishery, fishing firms must invest to prevent fishery pollution and preserve fish stocks. Efficient use also requires that the resource is allocated to the highest-value user. Some fishing firms may have lower operating costs than others, and efficient use requires that fishing licenses can be quickly reallocated to low-cost firms when they enter the market.

Resource administrators face a tradeoff: no existing resource license design achieves full efficiency in both allocation and investment. Suppose the resource administrator assigns the resource using perpetual, tradable use licenses. License holders then have high incentives to invest in the resource, because they are residual claimants on any increases in the resource's value. However, reallocation is distorted, because license holders have incentives to hold out for high prices in secondary markets. The administrator can improve allocative efficiency by using short-term licenses, so the resource can be reassigned to the highest-value user after the end of each license term. However, a license holder then has no incentives to make investments which produce payoffs after her license term ends.

In this paper, we introduce a new mechanism for assigning resource use rights, which we call the *depreciating license*. It is simple to describe and implement, and we argue that it navigates the allocation-investment tradeoff more effectively than existing license designs. A depreciating license lasts forever, but decays over time. At regular intervals, the resource administrator issues a fraction τ of new equity in the license, and sells the equity to potential resource users in a second-price auction. The auction bidders are the incumbent owner of the license, who owns a share $(1-\tau)$ of all equity in the license, and other potential buyers, who own no license equity.

The incumbent can either purchase the administrator's share τ of the license, or sell her share $(1-\tau)$ to buyers. If the incumbent outbids all buyers, she pays the administrator τ times the second highest bid, to buy out the administrator's equity stake. The incumbent then owns all equity in the license, and can continue using the asset in the next period. If a buyer bids above the incumbent, the buyer pays the second highest bid for the license. A fraction $(1-\tau)$ of the buyer's payment goes to the incumbent, to buy out her equity stake, and a fraction τ goes to the administrator. The license is then transferred to the buyer, who receives the right to use the resource in the next period, and becomes the incumbent in the next auction. In each auction, regardless of the winner, the administrator receives a fraction τ of the second highest bid. These revenues can be thought of as auction-based license fees.

Depreciating licenses are essentially a partial property rights system, which interpolates between the extremes of perpetual licenses and short-term rental licenses. When $\tau = 1$, depreciating licenses behave like short-term rentals: the administrator repossesses the entire license each period, and incumbents compete on equal footing with other buyers to buy the license. Allocation is fully efficient, but license owners have no incentives to make common-valued investments. When $\tau = 0$, there is no depreciation, and licenses are effectively perpetual. Investment is efficient, but license owners optimally bid higher than their true values, so allocation is distorted. As τ varies from 0 to 1, the license holder's investment incentives decrease, and allocative efficiency increases.

We show that the optimal depreciation rate is always strictly between 0 and 1: depreciating licenses can always improve upon the extremes of perpetual licenses and short-term rental licenses. This is because, under a perpetual license, investment is fully efficient. This implies that the marginal value of investment is equal to the marginal cost, so there is only a second-order welfare loss from decreasing investment slightly. On the other hand, license owners' optimal bids are well above their values, so the marginal buyer's value is higher than the license owner's value, and a reduction in markups creates a first-order welfare gain. Adding a small amount of depreciation creates a first-order improvement in allocative efficiency, at a second-order cost to investment welfare. Analogously, under a short-term rental license, increasing property rights slightly creates a first-order improvement to investment welfare, at a second-order cost to allocative efficiency.

Next, we analyze the effects of depreciating licenses on the administrator's revenue. Suppose the government sells the license in an initial auction, and then collects license fee payments in all future auctions. In a model without investment or private values, we show that depreciating licenses do not affect the government's revenue: higher values of τ increase license fee payments, but decrease the initial sale price of the license, and these effects exactly cancel each other out. Next, adding investment and private values back into the model, we show that depreciating licenses always decrease government revenue, relative to perpetual ownership licenses. Intuitively, this is because depreciating licenses decrease the investment value of the asset, and also cause license owners to lower markups: both forces decrease the administrator's revenue. However, the revenue loss is only second-order at $\tau = 0$. Thus, if an administrator is maximizing some weighted combination of revenue and social welfare, she would always find it optimal to set τ greater than 0.

We then analyze a number of extensions of the baseline model. Transactions costs and private-valued investments decrease the efficient amount of trade, and thus tend to decrease the optimal value of τ. When the number of buyers increases, the highest buyer's bid tends to be higher than the license owner's bid even under perpetual licenses, so the allocative welfare loss from perpetual licenses decreases, and the optimal value of τ tends to decrease. If investments take multiple periods to pay off, depreciating licenses distort investment more, so the optimal τ tends to decrease. If the value of the resource is risky, depreciating licenses also affect how this risk is divided between the government and market participants: when τ is higher, the government bears more risk, since license fee payments are paid after uncertainty in the asset's value is realized. However, none of these forces change the main result of our model, that the optimal value of τ is strictly between 0 and 1.

We then discuss some details of how depreciating licenses could be implemented in practice, and in what settings their benefits for welfare might outweigh their unfamiliarity and potential costs of implementation. We discuss how depreciating licenses relate to other mechanisms used for managing resource use, such as term-limited leases, contracts with forced investment clauses, value-sharing schemes, and market-based taxation schemes.

Related literature. The primary contribution of this paper is to propose a new mechanism: depreciating licenses, with license fee payments based on auction prices. We believe this is the first paper to formally analyze this mechanism. This paper is related to an earlier working paper by the same authors (Weyl and Zhang (2016)), which focuses on a different "self-assessment mechanism". Related mechanisms are discussed in a number of other works.[1] The current paper focuses on an auction-based pricing mechanism, which is different from the self-assessment mechanism. We compare these two mechanisms briefly in Section 7.

More broadly, this paper builds on a number of strands of theory literature. The first is a set of papers about how property rights affect investment incentives; see, for example, Grossman and Hart (1986) and Hart and Moore (1990). The second is a set of papers about property rights and their implications for bargaining efficiency in mechanism design settings; see, for example, Cramton, Gibbons and Klemperer (1987), Segal and Whinston (2011), and Segal and Whinston (2016). We discuss the relationship between our work and the mechanism design literature briefly in Section 7.

Outline. In Section 2, we discuss some institutional details of resource use license design. In Section 3, we introduce the model and derive our main results. Section 4 describes the effects of depreciating licenses on the administrator's expected revenue. Section 5 analyzes various theoretical extensions of our model. Section 6 discusses some implementation details of depreciating licenses. Section 7 discusses our results, and Section 8 concludes.

2 Institutional background

In 1959, Coase proposed to the Federal Communications Commission (FCC) that radio spectrum should be allocated by selling rights to use spectrum (Coase (1959)). At the time, the FCC granted spectrum use rights by discretion. The FCC's first question for Coase was simply: "is this all a big joke?" (Hazlett (2000)). Despite early resistance,

[1] See, for example, Harberger (1965), Tideman (1983), Tideman and Tullock (1977), Plassmann and Tideman (2008), Plassmann and Tideman (2011), Posner and Weyl (2017), Milgrom, Weyl and Zhang (2017), Posner and Weyl (2018), and Plassmann and Tideman (2019).

however, in 1994, the FCC sold spectrum in auctions for the first time. The auctions were generally viewed as a success (Cramton (1995)); nowadays, most spectrum use licenses in the US, and many other countries, are allocated using auctions.

Many other resource use rights are also allocated through tradable licenses. In some jurisdictions, pollutant emissions are regulated through "cap-and-trade" mechanisms, involving tradable emissions permits sold in auctions.[2] Fishing rights in many countries are also allocated using individual transferrable quotas, or ITQs, which can be traded on secondary markets (Arnason (2002)). In some countries, governments sell transferrable land use rights in auctions (Cai, Henderson and Zhang (2013)).

A large literature analyzes the optimal design of auctions for selling resource use licenses.[3] Comparatively little attention has been paid to the optimal design of licenses and secondary markets. For example, in a set of comments to the FCC in 2001, a group of prominent economists propose that the FCC should "seek not to create secondary markets directly but instead to institute rules permitting such markets to emerge."[4] In most settings where resource licenses are sold in auctions, secondary markets are not heavily regulated, and there is little academic work analyzing how regulators can redesign licenses and secondary markets to improve efficiency.

Property rights and investment incentives. One of the main arguments for privatizing natural resources is that property rights give asset users incentives for investment. A large literature analyzes the effects of property rights for investment incentives in agricultural and residential land use settings (Besley (1995), Goldstein and Udry (2008), Galiani and Schargrodsky (2010)). In oil drilling, there is evidence that limited-term licenses distort the timing of drilling investments Herrnstadt, Kellogg and Lewis (2020). In the context of fishing rights, Anderson, Arnason and Libecap (2011) argue that long-term, grandfathered rights may be more dynamically efficient than auctions, because they give asset users higher incentives to invest in maintaining the value of fisheries.

Property rights, transactions costs, and allocative efficiency. There is also substantial

[2]See, for example, the EU Emissions Trading System. In the US, emissions rights for sulfur dioxide and nitrogen oxides are administered using a similar cap-and-trade program: see the EPA website on Market-based mechanisms

[3]See, for example, Klemperer (2000) and Milgrom (2004).

[4]See Comments of 37 Concerned Economists (Federal Communication Commission 2001).

evidence that secondary markets for resource use licenses can be inefficient: once licenses have been allocated, secondary markets are relatively slow to reallocate them to higher-valued uses. From 1982 to 1993, radio spectrum in the US was allocated through lotteries: essentially any party could apply for license use rights, and the FCC ran lotteries to determine who would be allocated these rights. In principle, in the absence of transaction costs in secondary markets, the Coase theorem should hold, and the initial allocation of spectrum licenses should not influence the final efficiency of spectrum use. This was not the case in practice. Many spectrum lottery winners had no intention of actually developing spectrum, and held on to licenses for many years before reselling them to higher-valued users. This perceived failure of the lottery system was an important reason why the FCC moved to an auction system.[5]

Auctions lead to more efficient initial allocations, but inefficient spectrum reallocation in secondary markets seems to be an ongoing problem. Hazlett (2001, p. 318) argues that analog uses of spectrum are much less efficient than modern digital devices, but there has not been substantial reallocation of licenses from analog towards digital users in secondary markets. The recent FCC incentive auction succeeded in reallocating large amounts of spectrum, suggesting that government intervention can improve the efficiency of secondary markets.[6]

Secondary market frictions appear to be present in many other settings. A sizable literature analyzes transactions costs in emissions permit markets theoretically, empirically, and experimentally.[7] Transaction costs are also discussed in the literature on fishing ITQs (McCay (1995), Libecap (2007)). For land use rights, eminent domain law is aimed at resolving perceived holdup problems, suggesting that lawmakers believe there are barriers to efficient reallocation in unregulated secondary markets (Stoebuck (1972)).

[5]See the 1997FCC Report to Congress on Spectrum Auctions.

[6]See the FCC website on Broadcast Incentive Auction and Post-Auction Transition.

[7]Some papers in this literature are Hahn (1984), Foster and Hahn (1995), Stavins (1995), Kerr and Maré (1998), Gangadharan (2000), Muller et al. (2002), Cason and Gangadharan (2003), Jaraite and Kažukauskas (2012), Heindl (2017), Toyama (2019), and Baudry et al. (2020).

3 Model

3.1 Setup

Time is discrete, with periods $t = 0, 1, 2 \ldots \infty$. All agents are risk-neutral, and discount utility at rate β. There is a single resource, which is rivalrous in use: it can only be used by one agent at a time. An agent S_0 owns the use license at time 0. In each period, a single buyer B_t can potentially purchase the license from the period-t license owner; we will describe the trading mechanism below. We will use S_t to refer to the license owner at the start of period t, and we will use i to denote a generic agent.

Each time period has two stages.

1. **Investment:** The license owner at the start of period t, S_t, chooses how much she invests in the common value ψ_t of the resource. Only S_t can invest in the asset.

2. **Allocation:** An auction is used to determine whether the license is allocated to S_t or B_t, the price of the license, and license fee payments to the administrator.

If an agent owns a use license for the asset, at the end of stage 2 of period t, she derives flow utility:

$$\psi_t + u_{it} \tag{1}$$

from the resource. Thus, each agent's utility for the asset has two components. u_{it} is a private-valued component of utility. We assume that u_{it} is private information, and that each u_{it} is randomly drawn at the start of period t, i.i.d. over time and across agents, from a smooth distribution $F(\cdot)$.

Differences in u_{it} reflect differences in owners' use values for the asset. For resources such as radio spectrum or emissions permits, different agents may have very different desired uses for the resource, creating dispersion in agents' willingness to pay. Even for resources such as oil drilling rights and fishing rights, which have relatively homogeneous uses, some owners may be more efficient than others at resource extraction, creating differences in willingness-to-pay. The social planner wants to allocate the resource to the agent who has the highest value of u_{it} in each period.[8]

[8] We implicitly assume that there are no externalities from ownership, so allocating the resource to the

The term ψ_t is a common-valued component: when ψ_t increases, it increases the flow utility of all agents for using the resource. ψ_t is determined through costly investment by the period-t license owner S_t during the first stage of period t. In particular, at the start of each period, the owner in period t chooses ψ_t, and pays cost:

$$c(\psi_t)$$

These investments are costly actions resource users can take to preserve or improve the common value of the resource. For example, fishing firms can preserve fish stocks by avoiding overfishing and polluting waters, but this may come at the expense of short-term profits. Oil leaseholders can drill exploratory wells to gauge the value of a given oilfield; information from the exploratory well benefits any future leaseholders.

Use rights for the resource are allocated using a *depreciating license*, which is characterized by a *depreciation rate* τ. The license effectively gives its owner property rights over the asset which decay by a share τ in each time period. Equivalently, we can think of license depreciation as the administrator printing a share τ of new equity in the license each year, diluting the stake of the incumbent owner. In order to use the resource, an agent must own 100% of all outstanding equity in the license. Thus, if the incumbent wishes to continue using the resource, she must repurchase a share τ of the license from the administrator each period. Alternatively, she can sell her share $(1-\tau)$ of the license to buyers and leave the market.

In each period, license fee payments, and license allocation, are determined in a second-price auction between the license owner S_t and the potential buyer B_t. Suppose the agents bid b_{St} and b_{Bt}, respectively. If $b_{St} > b_{Bt}$, the license owner wins the auction and keeps her license. She pays τb_{Bt} to the administrator, to purchase the administrator's share τ of the asset, and the buyer leaves the game and receives continuation utility 0. If $b_{Bt} > b_{St}$, the buyer wins the auction; she pays τb_{St} to the administrator and $(1-\tau) b_{St}$ to the incumbent, and becomes the asset owner S_{t+1} in the next period. The incumbent then leaves the game and receives continuation utility normalized to 0.

agent with highest willingness-to-pay is optimal. If there are differences in externalities from different use cases, the administrator could subsidize some agents' bids and tax or penalize others, in accordance with any perceived externalities.

The depreciating license mechanism interpolates between rental auctions and perpetual ownership. To see this, note that if $\tau = 0$, the license is essentially a perpetual license: the incumbent is paid b_{St} if she bids lower than the buyer, and keeps the license without paying anything if she bids higher than the buyer. The incumbent's bid b_{St} effectively functions as a price offer. On the other hand, if $\tau = 1$, the license is effectively a short-term rental license: the incumbent pays the buyer's bid to the administrator if she outbids the buyer, and is paid nothing if she loses.

3.2 Equilibrium

We begin by characterizing the first-best outcome. Social welfare is maximized when the license owner invests until the marginal cost and marginal values of common-valued investment ψ_t are equal, and when the owner sells the license whenever the buyer's private value is higher than the owner's private value. Formally, in the first-best outcome, the license owner chooses ψ_t such that:

$$c'(\psi_t) = 1 \tag{2}$$

And the license is sold to the buyer if:

$$u_{Bt} > u_{St} \tag{3}$$

Next, we study how investment and allocation behave in equilibrium, for different choices of the depreciation rate τ. Let V represent the expected value of being a license owner in period t, before observing u_{St}. The willingness-to-pay of a buyer with value u_{Bt} in period t for the asset is thus the sum of her private value u_{Bt}, the common value ψ_t, and the expected value V of being a license owner next period; that is,

$$u_{Bt} + \psi_t + \beta V$$

We will represent license owners' bids as markups over the common value of the asset plus the continuation value, that is:

$$m_t \equiv b_{St} - (\psi_t + \beta V)$$

The following proposition describes license owners' equilibrium investment and price-setting decisions in equilibrium.

Proposition 1. *Incumbent license owners invest until:*

$$c'(\psi_t) = 1 - \tau \tag{4}$$

Incumbent license owners' bids are:

$$m^*(u_{St}, \tau) - u_{St} = (1-\tau)\frac{1 - F(m^*)}{f(m^*)} \tag{5}$$

The optimal markup $m^*(u_{St}, \tau)$, and the optimal investment $\psi_t^*(\tau)$, are monotonically decreasing in τ.

We can specialize Proposition 1 to two extreme cases, perpetual licenses and pure rental licenses, to build intuition about the general case. First, suppose $\tau = 0$, so the license is effectively perpetual: the license owner in any given period can keep using the asset forever without paying any fees.

Claim 1. When $\tau = 0$, license owners' investment decisions are efficient, satisfying:

$$c'(\psi_t) = 1 \tag{6}$$

License owners' optimal markups satisfy:

$$m^*(u_{St}, \tau) - u_{St} = \frac{1 - F(m_t)}{f(m_t)} \tag{7}$$

Hence, license owners sell to all buyers with $u_{Bt} > m_t$.

Claim 1 shows that perpetual licenses fully align license owners' incentives to make common-valued investments: the first-order condition for investment, (6), is identical to the efficient FOC, (2). Intuitively, perpetual license owners have efficient investment incentives because they are full residual claimants on any changes in the common value of the asset.

However, under perpetual licenses, asset allocation is distorted. Expression (7) shows that license owners will always set m_t higher than u_{St}. This is the standard monopolist markup formula. In a second-price auction, if the buyer bids above the license owner, the owner receives her own bid, not the buyer's bid, as payment for selling the license. This gives license owners an incentive to bid above their values, to increase the payments they receive from buyers if they sell.

As a result of license owners' markups, from (7), the license owner only sells to buyers when:

$$u_{Bt} - u_{St} \geqslant \frac{1 - F\left(m^*\left(u_{St}, \tau\right)\right)}{f\left(m^*\left(u_{St}, \tau\right)\right)} \tag{8}$$

Comparing (8) to the efficient trade condition, (3), too few trades occur under perpetual licenses.

Now, suppose that $\tau = 1$, so the license is effectively a short-term rental contract. In each period, the new owner of the license is determined through a second-price auction, and the incumbent license owner must bid in the auction on an equal footing to all other bidders.

Claim 2. When $\tau = 1$, license owners' investment decisions satisfy:

$$c'\left(\psi_t\right) = 0 \tag{9}$$

Owners bid efficiently:

$$m^*\left(u_{St}, \tau\right) = u_{St} \tag{10}$$

Intuitively, short-term rental contracts exactly reverse the conclusions of perpetual licenses. S_t, who is the user of the asset in period $t-1$, has no property right over the asset: she competes in the period-t auction identically to all other buyers. Thus, S_t simply bids her true value, $m^*\left(u_{St}, \tau\right) = u_{St}$. Thus, trade happens whenever

$$u_{Bt} \geqslant u_{St}$$

so allocations are perfectly efficient.

However, (9) shows that short-term rental contracts give the incumbent no incentive to make common-valued investments. Intuitively, there are two possible outcomes from investing to increase ψ_t. If B_t buys the asset, S_t gains nothing from increasing ψ_t. Even if S_t successfully purchases the asset, she will pay a higher price, because B_t will bid more aggressively if ψ_t is higher. Without property rights, S_t is not a residual claimant on any investments that she makes in the asset, so she has no investment incentives whatsoever.

As we vary τ between 0 and 1, we essentially interpolate between these two extreme cases. Strong property rights – low values of τ, approximating perpetual licenses – give agents strong incentives to make common-valued investments, but also give license owners market power, inhibiting efficient reallocation of the asset. Weaker property rights – higher values of τ, approximating short-term rental licenses – allow the asset to be reallocated more efficiently in each period, but leave license owners with no incentives to make common-valued investments in the asset. Formally, Proposition 1 shows that, as τ increases from 0 towards 1, the incumbent's markup $m^*(u_{St}, \tau)$ decreases towards her true value u_{St}, but investment also monotonically decreases. As a result, allocative efficiency is monotonically increasing in τ, and investment welfare is monotonically decreasing.

Why might a resource administrator want to use depreciating licenses, rather than short-term rental licenses or perpetual licenses? The next proposition provides a first-order condition for the value of τ which maximizes total welfare, optimally trading off the allocative benefits from raising τ and the investment benefits from lowering τ.

Proposition 2. *A necessary condition for τ to maximize welfare is that the marginal allocative welfare gain from increasing τ is equal to the marginal investment welfare loss; formally:*

$$\int \left[(1-\tau)\frac{(1-F(m^*(u_{St},\tau)))\,h(m^*(u_{St},\tau))}{(1-(1-\tau)\,h'(m^*(u_{St},\tau)))}\right] dF(u_{St}) = \frac{\tau}{c''(\psi_t)} \tag{11}$$

where

$$h(m) \equiv \frac{1-F(m)}{f(m)}$$

If $f(\cdot)$ is everywhere positive, $c(\cdot)$ is strictly convex, and $F(\cdot)$ and $c(\cdot)$ have continuous and *bounded second derivatives, the optimal τ is strictly between 0 and 1.*

Proposition 2 shows that, under some smoothness assumptions on $F(\cdot)$ and $c(\cdot)$, the optimal τ is always strictly between 0 and 1. The extreme cases of pure rental licenses and full property rights, which are commonly observed in practice, are never optimal: some interior depreciation rate can always improve upon either extreme.

There is a price-theoretic intuition for this result. When $\tau = 1$, incumbents set prices equal to their reservation values. Asset allocation is perfectly efficient: the marginal buyer's value is exactly equal to the incumbent's value. This implies that a small increase in the incumbent's markup has no first-order effect on allocative welfare, because the marginal buyer's value is equal to the incumbent's. On the other hand, investment is very distorted, so the marginal units of investment is worth much more than its cost, and a small increase in investment incentives creates a first-order increase in investment welfare. Thus, decreasing τ from 1 always increases total welfare.

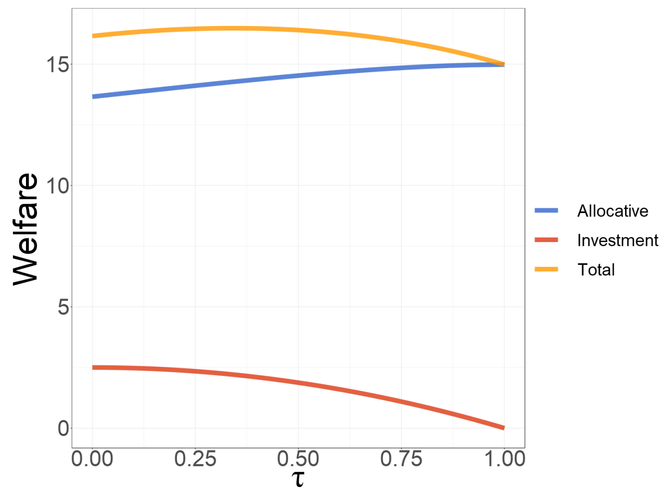
Analogously, when $\tau = 0$, incumbents' investment decisions are efficient. Thus, the marginal value of a unit of investment is equal to its marginal cost, implying that a small decrease in investment incentives has no first-order effect on investment welfare. On the other hand, incumbents set prices well above their private values, so a small increase in τ, which induces incumbents to decrease markups, creates a first-order allocative welfare gain. Thus, increasing τ from 0 always increases total welfare.

To illustrate our results, we study a simple numerical example. Assume that u_{St} and u_{Bt} are both drawn from a noncentered exponential distribution, with rate parameter λ, and minimum x_0. When λ is smaller, agents' private values are more disperse, so allocative efficiency matters more. Assume that the investment cost function is:

$$c(\psi_t) = \frac{\psi_t^2}{2\kappa}$$

The parameter κ determines the slope of the marginal cost curve. When κ is higher, investment is less costly, so total welfare from investment is larger. Figure 1 illustrates the result of Proposition 2: the allocative welfare function flattens as τ approaches 1, and the investment welfare function is flat as τ approaches 0, implying that the optimal τ is interior. In our example, the optimal τ is approximately 0.34, and the total welfare gain

Figure 1: Numerical example: allocative and investment welfare



Notes. Allocative, investment, and total welfare as a function of τ. We set $\lambda = 0.1, x_0 = 0, \kappa = 5$.

from the optimal τ is around 2%: however, these numbers are meant to be illustrative, and both values could vary substantially across different real-world settings, depending on the size of the allocative and investment components of welfare.

4 Revenue

In this section, we analyze the effects of depreciating licenses on the administrator's expected revenue.

Revenue without private values and investment. We begin by studying the effects of depreciating licenses in a very stylized setting, in which there is no private component of values, and agents make no investments. Suppose that the common value of the asset in each period t is ψ_t, which may be random, but whose distribution is common knowledge.

Claim 3. Suppose ψ_t is an exogeneous random variable, and its distribution is common knowledge. Agents have no private values, all agents are risk-neutral, and there are at least 2 potential buyers of the asset in each period. The administrator commits to a path of depreciation rates τ_t, which may vary over time, and then sells the license in an initial second-price auction, and collects license fees from future auctions. Then, the administrator's expected revenue is:

$$\underbrace{p_0}_{\text{Initial license price}} + \underbrace{\sum_{t=1}^{\infty} \beta^t E[\tau_t p_t]}_{\text{License fee payments}} = \underbrace{\sum_{t=1}^{\infty} \beta^t E[\psi_t]}_{\text{Asset use value}} \tag{12}$$

In words, the net present value of the administrator's revenue from selling the license is equal to the expected discounted sum of the future use values of the asset, ψ_t. In particular, the administrator's expected discounted revenue is independent of depreciation rates τ_t.

Expression (12) of Claim 3 states that the administrator's revenue is always equal to the discounted sum of the asset's expected future use values. Changes in the depreciation rates, τ_t, simply change the time pattern of the administrator's revenue: when the depreciation rates τ_t are higher, the administrator makes less revenue from the initial asset sale, but makes more from future license fee payments, and these effects exactly cancel out. Claim 3 serves primarily as a benchmark to demonstrate that, in our model, depreciating licenses only affect revenue through their effects on private values and allocation, and on agents' investment decisions. If we remove both of these components, the administrator can extract the entirety of market participants' utility from the asset – the right hand side of (12) – using any path of τ_t.

Revenue in the baseline model. Next, we analyze revenue in the baseline model, with private values and investment. As in Claim 3, suppose that, just before the first period, the administrator sells the license in a second-price auction, and there are at least two bidders. The administrator commits to a stationary path of depreciation rates, $\tau_t = \tau$. Bidders do not know their private values in $t = 1$, so all bidders value the license at $V(\tau)$ in the initial auction, where $V(\tau)$ is the stationary expected value of being a license owner, defined in Subsection 3.2. Thus, $V(\tau)$ will be the price of the license in the initial auction. The administrator's total expected revenue is thus:

$$R(\tau) = \underbrace{V(\tau)}_{\text{Auction revenue}} + \underbrace{\frac{\tau E_{u_{St},u_{Bt}}\left[p\left(u_{St}, u_{Bt}, \tau\right)\right]}{1-\beta}}_{\text{License fee revenue}} \tag{13}$$

where $p(u_{St}, u_{Bt}, \tau)$ is the price of the license, if the incumbent owner and buyers' values are u_{St} and u_{Bt} respectively. In words, the administrator is paid $V(\tau)$ in the initial auction, plus the discounted expected value of license fee payments, τp. The following claim characterizes how $R(\tau)$ varies as we change τ.

Proposition 3. *The administrator's expected revenue* $R(\tau)$ *is:*

$$R(\tau) = \underbrace{\frac{1}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[u_{St} F\left(m^*\left(u_{St}, \tau\right)\right) + m^*\left(u_{St}, \tau\right)\left(1 - F\left(m^*\left(u_{St}, \tau\right)\right)\right)\right]}_{\text{Allocative}} + \underbrace{\frac{\psi_t^*(\tau) - c\left(\psi_t^*(\tau)\right)}{1-\beta}}_{\text{Investment}} \tag{14}$$

$R(\tau)$ *is a monotonically decreasing function of* τ, *with derivative:*

$$R'(\tau) = \underbrace{\frac{\tau}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\frac{dm^*\left(u_{St}, \tau\right)}{d\tau}\left(1 - F\left(m^*\left(u_{St}, \tau\right)\right)\right)\right]}_{\text{Allocative}} - \underbrace{\frac{\tau}{(1-\beta)\, c''\left(\psi_t\right)}}_{\text{Investment}} \tag{15}$$

When $F(\cdot)$ *and* $c(\cdot)$ *have continuous and bounded second derivatives, we have:*

$$R'(0) = 0$$

Proposition 3 states that the administrator's expected revenue is always decreasing in τ: depreciating licenses pay the administrator less, in present value terms, than perpetual licenses. The marginal effect of τ on revenue, (15), can be decomposed into allocative and investment terms. The investment term,

$$\frac{\tau}{(1-\beta)\, c''\left(\psi_t\right)} \tag{16}$$

reflects the fact that, for higher values of τ, the value of the right to invest in the asset is lower. Taking the extreme case, when $\tau = 1$, the owner invests nothing, so the license cannot derive any value from its investment potential. However, when $\tau = 0$, (16) is 0: when investment is not distorted, a small change in τ only generates a second-order change in investment-associated revenues.

The allocative term in (15),

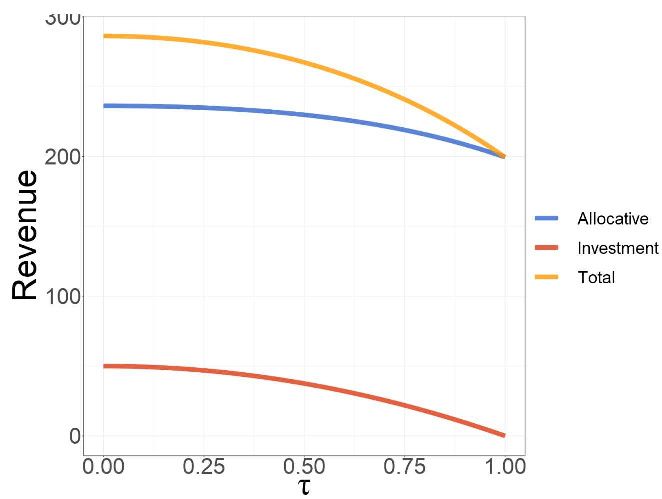
$$\frac{\tau}{1-\beta} E_{u_{St} \sim F(\cdot)} \left[\underbrace{\frac{dm^*\left(u_{St}, \tau\right)}{d\tau}}_{\text{Markup change}} \underbrace{\left(1 - F\left(m^*\left(u_{St}, \tau\right)\right)\right)}_{\text{Probability markup is binding}} \right] \tag{17}$$

reflects the fact that, when τ is positive, the administrator's revenue depends on the owner's markup. Small changes in τ lower markups, which causes the administrator's expected revenue to decrease. However, like the investment term, (17) is 0 when $\tau = 0$. Figure 2 illustrates the behavior of the allocative and investment components of revenue, as well as their sum, as τ varies. As Proposition 3 states, all three terms are monotonically decreasing in τ, with slope 0 when $\tau = 0$.

Proposition 3 implies that revenue is always maximized under perpetual licenses: any positive depreciation rate lowers the expected value of the administrator's revenue. However, when $\tau = 0$, increasing τ creates only a second-order revenue loss for the administrator. Proposition 2 showed that increasing τ from 0 creates a first-order increase in welfare. Thus, if an administrator is maximizing some weighted combination of revenue and social welfare, she would always set τ strictly greater than 0.

There are a number of factors which may cause Proposition 3 not to hold. If the initial auction is sufficiently uncompetitive, revenue could be much lower than the initial license buyer's willingness-to-pay. Increasing τ could potentially increase the administrator's revenue, since she would collect more revenue from future, more competitive auctions. As we discuss in Subsection 5.4 below, τ also affects the division of risk between the administrator and market participants. If there is substantial uncertainty in the common value of the asset, bidders may bid less aggressively in initial auctions due to risk aversion, and more aggressively in future auctions, after uncertainty about the asset's value is resolved. Increasing τ from 0 could potentially raise the government's revenue in these settings, since less revenue is collected from the initial auction and more from future auctions.

Figure 2: Numerical example: revenue



Notes. Allocative, investment, and total revenue as a function of τ. The allocative and investment components are defined by the underbraces in (14) of Proposition 3. We set $\lambda = 0.1, x_0 = 0, \kappa = 5, \beta = 0.95$.

5 Extensions

This section describes a number of extensions to our baseline model, and how they affect the conclusion of Proposition 2 about welfare-maximizing values of τ. Appendix 2 contains numerical simulations illustrating how these forces affect welfare and optimal depreciation rates.

5.1 Transactions costs

Suppose that, in additional to the assumptions of the baseline model, there is some one-time setup cost c which new buyers must pay to begin using the resource. The effective private value of buyers is then $u_{Bt} - c$. Buyers' willingness-to-pay for the asset then becomes:

$$V + \psi_t + u_{Bt} - c$$

It is socially efficient for the buyer to purchase the asset if:

$$u_{Bt} - c \geqslant u_{St}$$

Transactions costs are thus isomorphic to assuming that the distribution of buyers' private values shifts downwards by c: that is, buyers' private values are effectively drawn from $\tilde{F}$, where

$$\tilde{F}(u_{Bt}) = F(u_{Bt} - c) \tag{18}$$

The results of Proposition 1 and 2 still hold. That is, sellers still set prices above their private values u_{St}, and the optimal τ is still positive, since there is a first-order allocative efficiency gain from increasing τ from 0.

Thus, qualitatively, transactions costs do not change our main conclusion that the optimal τ is interior. However, quantitatively, these forces tend to make the incumbent's value higher than that of entering buyers. This makes the allocative losses from sellers' markups smaller, so the optimal τ tends to decrease as a result. To demonstrate this, in Appendix Figure A.1, we show how Figure 1 changes when we add transactions costs, and we find that higher transactions costs tend to lower the optimal value of τ.[9]

5.2 Competition

Next, we consider how the model changes when there is more than one buyer in each period. Suppose that there are N buyers in each period, with private values drawn i.i.d. from $F(\cdot)$. As in the baseline model, all buyers compete in a second-price auction with the incumbent license owner. If the asset owner submits the highest bid, she pays τ times the highest buyer bid to the administrator, and keeps the asset. If a buyer submits the highest bid, the buyer pays the incumbent $(1 - \tau)$ times the second highest bid (which may be the incumbent's bid or another buyer's), and an additional τ times the second highest bid to the administrator, and then becomes the new license owner.

[9] *Logistical costs* of transferring assets, which are borne by the license administrator, would change the conclusions: if the administrator must expend cost c whenever trade occurs, then buyers and sellers will tend to trade too much when $\tau = 1$, since they do not internalize costs borne by the administrator. However, the administrator can make market participants internalize these logistical costs by, for example, charging buyers these logistical costs whenever they win an auction.

From the buyers' perspective, the trading mechanism is simply a second-price auction, so buyers optimally bid their true values. The following claim characterizes the license owner's optimal bid when there are multiple buyers.

Claim 4. The first-order condition for optimal markups and investment are identical to Proposition 1:

$$(m - u_{it}) = (1 - \tau) \frac{1 - F(m)}{f(m)} \tag{19}$$

Somewhat surprisingly, the asset owner's optimal bid is identical to Proposition 1, and is independent of the number of bidders. This is related to a classic result from auction theory: in a second-price auction with N bidders with i.i.d. values, the optimal reserve price does not depend on N.[10] An intuition for this result is that, as N increases, it becomes more likely that the highest buyer's bid is above the incumbent's, which would seem to give incumbents incentives to raise their bids. However, it is also more likely that the *second* highest buyer bid is above the incumbent's bid. If the incumbent's bid is not the second highest bid, it does not affect the sale revenue the owner receives. These two effects turn out to exactly offset each other, so N does not affect the incumbent's bidding incentives.

In terms of welfare, as N increases, the incumbent sells the asset with probability approaching 1, so the allocative distortions from setting $\tau = 0$ decrease towards 0 as N becomes very large. Thus, the optimal level of τ approaches 0 as N increases towards ∞. However, the optimal τ is not necessarily monotonically decreasing in N. In Appendix Figure A.2, we show an example in which the optimal τ increases and then decreases as we increase N.

5.3 Long-term investments

In the baseline model, we assumed that all investment is short-term: the investment decision of the period-t owner only affects ψ_t. Now, suppose investments take χ periods to pay off. So an owner invests to determine the value $\psi_{t+\chi}$ of the asset $\chi > 0$ periods in the future, again at cost $c(\psi_{t+\chi})$. The following claim characterizes investment incentives

[10] See, for example, Bulow and Roberts (1989).

under depreciating licenses.

Claim 5. The socially optimal sequence of investments satisfies:

$$\frac{\partial c}{\partial \psi_{t+\chi}} = \beta^{\chi} \tag{20}$$

In equilibrium, agents' investment decisions satisfy:

$$\frac{\partial c}{\partial \psi_{t+\chi}} = \beta^{\chi} (1-\tau)^{\chi+1} \tag{21}$$

Claim 5 shows that, the longer an investment takes to produce payoffs, the more investments are distorted by depreciating licenses. The investment wedge – the ratio between the marginal value of the equilibrium and optimal investments – is:

$$(1-\tau)^{\chi+1} \tag{22}$$

Intuitively, if the license owner makes an investment which only pays off χ periods in the future, bidders will increase their bids in each auction between now and time t. Thus, the owner must pay increased license fees $t+1$ times before she can receive the value of her investment. A related intuition is that, since the owner's stake in the asset is diluted by a share τ in each period, the period-t owner only owns a share $(1-\tau)^{\chi+1}$ of the asset in time $t+\chi$, so this share determines her investment incentives. Since the investment wedge, (22), decreases geometrically with t, even small depreciation rates can decrease incentives for long-term investment substantially. Appendix Figure A.3 shows that, as χ increases, depreciating licenses distort investment more, so the optimal value of τ becomes closer to 0.

5.4 Risk sharing

Property rights affect risk sharing: asset owners bear more price risk than asset renters. Depreciating licenses also affect the division of risk between the administrator and license users. We analyze this force in a very simple model: as in Claim 3, we assume there is no investment, and no private values. Instead, the asset has some exogeneous common value ψ per period, which is common to all asset users. There are at least two bidders in each period. Just before the first period, the administrator sells the license to bidders in a second-price auction. However, we assume ψ is unknown in the initial auction, to both the administrator and private parties; it is realized and commonly observed just after the initial auction. ψ has mean μ_ψ and variance σ_ψ^2. Uncertainty in ψ thus creates risk in market participants' utility from purchasing the license, as well as the administrators' revenue. We characterize this risk in the following claim.

Claim 6. If a buyer purchases the license in the initial auction, the variance of her utility is:

$$\frac{(1-\tau)^2 \sigma_\psi^2}{(1-\beta(1-\tau))^2} \tag{23}$$

The variance of the present value of the administrator's revenue is:

$$\left(\frac{\tau}{(1-\beta)(1-\beta(1-\tau))}\right)^2 \sigma_\psi^2 \tag{24}$$

Claim 6 shows that τ shifts risk from license buyers to the administrator. When $\tau = 0$, the administrator sells a long-term license for an asset with uncertain value to buyers at a fixed price. Buyers face risk, since the value of the asset may turn out to be low; the administrator faces no risk, since they are paid before uncertainty about ψ is realized. When $\tau = 1$, the administrator's revenue comes from fee payments, which are made after ψ is realized. Buyers face no risk, because the fee payments are high precisely when the asset is valuable. On the other hand, the administrator's revenue is much more risky, since the fee payments depend on the realized value of ψ.

Like other forms of property rights, if risk sharing is a concern, it will affect the optimal value of τ. If the government is less risk-averse than buyers, the optimal τ will tend to be higher, transferring more risk to the administrators. However, under most models of risk aversion – for example, if the administrator and market participants both have mean-variance preferences – the optimal τ will still be interior. This is because optimal solutions to risk-sharing problems are generally interior, because marginal variances are 0 at the boundary cases: the derivative of (23) is 0 when $\tau = 1$, and the derivative of (24) is 0 when $\tau = 0$. Thus, risk sharing considerations will affects the optimal choice of τ, but do not justify using the extreme cases of perpetual licenses or pure rental licenses.

6 Implementation details and variations

Depreciating licenses are straightforwards to implement. When the license is initially sold by the administrator, it can be sold in a standard auction, where all participants compete on equal terms. Future auctions, where there is an existing license owner, are essentially second-price auctions with modified payment rules. From the perspective of buyers, a depreciating license auction behaves identically to a standard auction: the buyer wins if her bid is highest, and she pays the second highest bid. For an incumbent license owner, the only difference is that she effectively begins the auction with a "foothold": she only pays τ times the second highest bid if she wins, and she is paid $(1-\tau)$ times the second highest bid if she loses. The intuition behind these payment rules is simply that the administrator starts each auction with a share τ of the asset to sell, and the previous license owner holds a share $(1-\tau)$ of the asset.

There are many ways to implement second-price auctions. A standard format, the ascending auction, would also work for depreciating licenses. Another possible implementation is sealed-bid second price auctions. License owners may feel that it is riskier, since they do not know, ex ante, how high they must bid to keep their licenses. One simple solution is to give the license owner the right to match the highest bid, once the auction has concluded. In theory, license owners should always bid above their true values, so license owners should in theory never want to exercise this option, but this may assuage some of participants' concerns about using a novel auction mechanism.

However, we cannot allow incumbents to back out of bids that they make. This is because incumbents have incentives to bid above their values, so there is a range of values between the incumbent's value u_{S_t} and her bid m, where the incumbent would be willing to sell to a buyer. However, if we allow incumbents to back out of their bids, they would have incentives to bid higher, knowing they can renege on their bids. The analysis of this paper would thus not necessarily apply.

Nonstationary depreciation rates. Depreciating licenses could be run with nonstationary depreciation rates: τ_t could change over time. For example, the administrator could assign a license with τ_t very low for a few periods, and then increasing thereafter. Under such a mechanism, investment incentives would be relatively high and allocative efficiency would be low in the early periods, and investment incentives would decrease, and allocative efficiency increase, in later periods when τ_t is high. Such mechanisms could be useful in settings where assets require relatively large investments in the first few years of ownership, and less investment in the long run.

Auction frequency. In the baseline model, it is optimal to set a constant depreciation rate for all periods. This is due to the assumption that values are independent and identically distributed over periods: the optimal τ_t can thus be calculated period-by-period, and is the same each period. There are two important ways in which our model assumptions do not hold, if auctions are run at high frequencies. First, our assumption that the incumbent seller's value is independent over time is violated: private values are likely to be persistent in the short run. While we have not formally analyzed this, intertemporal persistence in values may justify nonstationary depreciation rates: after an auction, the incumbent's value is likely to be relatively high, so reallocation to buyers is less of a concern, and τ_t should be lower for a period of time.

Second, when auctions are frequent, our assumption that each auction attracts new buyers may not hold: it is likely that a few buyers will interact repeatedly with the same license owner. As is known in the literature, such repeated-game settings can often support a rich variety of different equilibria, involving different kinds of group punishments and off-equilibrium threats. It is not clear that the analysis in this paper maps cleanly to these settings.

The analysis of this paper thus applies most closely when the time between auctions is sufficiently long that our core assumptions – independent values period-by-period, and new buyers each period – are at least approximately true. Depreciating licenses may also function well at higher auction frequencies as well, but we cannot reasonably conclude this based on the results of the current paper.

Calculating license fee payments. There is a simple way for asset users to calculate the effective cost of using depreciating licenses. Suppose the environment is stationary, so the expected price of the license does not change from one auction to the next. If a firm purchases a license in period t, uses it for a period, and sells it in period $t+1$, the present value of her expected payment is:

$$\underbrace{p}_{\text{Period}-t\text{ payment}} - \underbrace{\beta(1-\tau)p}_{t+1\text{ sale revenue}} = p(1-\beta(1-\tau)) \tag{25}$$

When there is little discounting, so β is close to 1, (25) is close to τp: the asset costs a fraction τ of the price to use for a single period. τp is thus the effective one-period rental price of the asset. Expression (25) can be modified assuming p grows at some rate: this could be the economy-wide growth rate, or industry-specific growth rates. The main difference from pure rental contracts is that depreciating licenses expose their holders to price risk: changes in p in the future affect sale revenue for the asset. If a firm purchases a license in period t and sells it in period $t+1$, she is exposed to risk equal to the standard deviation of sale revenue:

$$\beta(1-\tau)\,SD(p_{t+1}) \tag{26}$$

Firms could estimate (26), for example, based on the dispersion of other agents' bids in license auctions.

7 Discussion

Mechanism design. The allocative distortion from private property, fundamentally, is the Myerson and Satterthwaite (1981) distortion.[11] Depreciating licenses are a specific solution to this tradeoff, which we have focused on for its relative simplicity and ease of implementation. A mechanism-design approach to this problem would be to formally search for the optimal mechanism, under constraints such as budget balance for the government. There is no reason to believe that the mechanism we have proposed here is optimal among a broader class of mechanisms. We leave analysis of these mechanisms to future work.

The Vickrey-Clarke-Groves mechanism. A natural mechanism to consider in these settings is the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey (1961), Clarke (1971),

[11] There is a related body of work in mechanism design which analyzes the role of property rights in determining allocative efficiency (Cramton, Gibbons and Klemperer (1987), Segal and Whinston (2011), Segal and Whinston (2016)).

Groves (1973)). We briefly analyze this in Appendix D.2. We show that the VCG mechanism achieves full allocative and investment efficiency. However, it is not budget balanced in each period for the government: in each period where the buyer and seller trade, the government must subsidize an amount equal to the difference in the buyers' and sellers' values. In comparison, depreciating licenses guarantee the government positive revenue during each auction. Depreciating licenses may thus be more attractive in settings where it is not politically feasible for governments to commit to repeatedly subsidize trade.

First-price/self-assessment mechanisms. In the baseline mechanism we propose, license owners compete with potential buyers in a second-price auction. An alternative rule, more similar to a first-price auction, is that the license owner has to pay license fees based on her own bid, if she wins the auction. We will call this a "self-assessment" mechanism, since the incumbent essentially announces a price, and pays the administrator a license fee based on her announcement. This mechanism was the main focus of an earlier working paper by the same authors (Weyl and Zhang (2016)), and is also discussed in Posner and Weyl (2017) and Posner and Weyl (2018).[12]

In Appendix D.1, we briefly analyze this mechanism, and compare it to the main mechanism we propose. In short, relative to depreciating licenses, the self-assessment mechanism generates higher downward pressure on markups: for any given seller type, full allocative efficiency can be achieved with a value of τ strictly lower than 1. However, the optimal τ differs for different sellers, and if τ is set higher than its optimal value, allocative efficiency actually decreases. Qualitatively, however, the self-assessment mechanism behaves similarly to depreciating licenses, so it may be an attractive alternative in some settings.

Term-limited leases. A simple alternative to depreciating licenses is term-limited licenses. A term-limited license grants its owner full rights to use a resource for a limited time period; thereafter, the use right reverts to the administrator, who can then sell another term-limited license to another buyer. Within our model, term-limited licenses

[12] An early proponent of self-assessment was Harberger (1965), and other papers studying properties of self-assessment mechanisms include Tideman (1969), Plassmann and Tideman (2008), Plassmann and Tideman (2011), and Plassmann and Tideman (2019). Milgrom, Weyl and Zhang (2017) informally discusses both mechanisms.

can be thought of as licenses which depreciate at a nonstationary rate: depreciation rates are 0 outside of term-end periods, and 1 at each term-end period, so the administrator fully repossesses and resells the asset.

In Proposition 2, we showed that optimal depreciation rates are always interior. This suggests that term-limit licenses could generally be improved by introducing a small "foothold" in term-end periods, leaving incumbents with a small stake in the asset. This would increase welfare, without requiring the administrator to run auctions more frequently.

Depreciating licenses allow resource administrators to separate two parameters: the frequency of auctions, and the effective length of property rights. The administrator can organize auction events frequently but with low depreciation rates, creating a centralized market in which agents can trade, but without eliminating investment incentives in each auction. On the other hand, this flexibility also increases the complexity of depreciating licenses. Term-limited licenses only have one control parameter, the license term, whereas depreciating licenses require choosing both the auction frequency and the depreciation rate. Term-limited licenses may thus be preferred in some settings due to their simplicity.

Forced investment. In some settings, resource use licenses contain clauses which incentivize, or require, license owners to make certain investments. For example, in the US, firms who lease oil drilling rights are subject to a "prudent operation" standard. Nominally, this requires the lessee to operate the lease with consideration of the interests of both the lessor and the lessee. In practice, this standard has been interpreted as a requirement for lessees to actually develop wells, to operate safely, to take reasonable efforts to prevent drainage and thus maximize oil yields.[13] Radio spectrum licenses often include "buildout requirements", which require license holders to use the spectrum to meet specific cell service coverage requirements within a given time period.[14]

Forced investment clauses function as a substitute for property rights in providing incentives for investment. For example, if the administrator can compel market participants to make efficient investment decisions, τ_t does not affect investment welfare, so it is then efficient to set $\tau_t = 1$ each period: to sell the license as a pure rental license, in order

[13]See Conine (1993) for a discussion of these issues.

[14]For example, see the FCC's website on Construction Requirements by Service, and Construction/Coverage Requirements.

to maximize allocative efficiency. In general, if forced investment clauses can partially alleviate investment distortions from relaxing property rights, they will tend to raise the optimal choice of τ_t. However, forced investment clauses face a Hayekian problem: efficient investment decisions are often very context-specific, and administrators may not have full knowledge of optimal investment decisions in any given setting. Property rights decentralize investment decisions: when asset users are residual claimants on investments, they can use their context-specific knowledge in determining optimal investments.

Depreciating licenses in the private sector. We have focused our discussion on the allocation of public resources. A natural question is whether they might also be useful in private settings, in which both perpetual ownership and rental contracts are prevalent. One reason that private-sector agents might not use depreciating licenses is the result of Proposition 3: under the assumptions of our model, revenue is maximized under perpetual ownership licenses. However, this does not explain why we observe rental contracts in private markets. Rental-like agreements could be motivated by forces such as risk-sharing, or asset users' liquidity constraints. Yet our theory suggests that total welfare could always be improved by setting depreciation rates slightly less than 1: adding a small amount of ownership to rental contracts is always welfare-improving, through its effects on investment incentives.

One reason why we might not observe these is that private asset owners may actually be fairly effective at monitoring asset users, and using contractual provisions to ensure that users invest in and maintain the value of assets. For example, consumer car or apartment rental contracts contain detailed provisions describing acceptable uses of assets, and specifying penalties contingent on violation. These provisions may be sufficiently effective that property rights are not necessary to incentivize investment.

Governments may be less effective than the private sector at monitoring and enforcing investment for a number of reasons: governments may have lower operational capacity, less local knowledge about efficient investments, lower incentives to monitor users, and greater vulnerability to capture by asset users. Property rights may be particularly useful for government resource allocation because they are relatively hands-off: under property rights, the market disciplines asset users' investment decisions, which lowers the need for explicit government intervention.

Secondary rental markets. Allocation and use of the asset is efficient as long as the asset is *used* by its highest-value user, regardless of whether it is *owned* by the user. Hence, efficient allocation could take place if secondary rental markets for assets were sufficiently efficient. In settings where assets have active rental markets, depreciating licenses could potentially harm allocative efficiency by interfering with the functioning of secondary rental markets: it is difficult to rent an asset out when its ownership is encumbered by license payments, and changes hands frequently.

However, rental markets are not a panacea for allocative efficiency, because the same information frictions which prevent the efficient sale of assets may be a barrier to efficient rental. License owners do not know the valuations of potential renters, and have incentives to hold out for overly high prices. Moreover, in settings where assets require substantial investments, there are hold-up problems with rental contracts in secondary markets: once a user has made sunk investments in using the asset, the asset owner may have an incentive to raise rent prices ex-post.

Market-based taxation schemes. In some contexts, governments impose taxes based on the market value of assets for reasons other than allocative efficiency. For example, in many US states, property taxes are used to raise funds for local governments. Many of these schemes are known to have adverse effects on market function.[15] Depreciating licenses are related to these schemes: license owners are charged periodic fees based on the market value of underlying assets. However, depreciating licenses actually improve allocative efficiency, relative to perpetual tax-free ownership. Hence, depreciating licenses may be a more efficient way to implement market value-based taxes in some settings. A logistical barrier to implementing depreciating licenses in these settings is that they require administrators to actively participate in markets, running periodic auctions to determine fee payments: taxes cannot simply be charged based on prices from decentralized secondary markets.

Nonrival use resources. Depreciating licenses are primarily designed for application to rival-use resources, for which use by one party essentially precludes use by others. For rival goods, allocative efficiency simply means assigning exclusive use rights to

[15]For example, taxes which reset based on sale prices create disincentives to sell property when house prices are rising (Wasi and White (2005)). Taxes based on non-market value assessments can be inaccurate, in ways that are vulnerable to regulatory capture (Hodge et al. (2017), Avenancio-León and Howard (2019)).

the highest-value single owner. For nonrival goods, allocative efficiency is much more complex: it requires determining what the optimal set of users and uses is, taking into account the externalities each user imposes on others. For example, parks, highways and spectrum face similar problems of congestion, and optimal policy should take these into account. It is not clear that depreciating licenses are applicable to these kinds of problems.

An important class of nonrival assets is intellectual property. The investment-allocation tradeoff for intellectual property rights is particularly clear: the socially optimal allocation is to allow all parties to use all innovations at no cost, but such a system gives innovators no incentives to develop innovations. A sizable literature has addressed the question of optimal ownership rights over intellectual property, largely finding that partial ownership systems, such as limited-term patents, are optimal.[16] In a sense, our argument in this paper is that a similar allocation-investment tradeoff is relevant, and thus partial property rights may be welfare-improving, for some rival-use assets.

8 Conclusion

In this paper, we have highlighted a tradeoff involved in resource license design. Long-term or perpetual resource use rights improve resource users' investment incentives, but decrease allocative efficiency. Short-term licenses improve allocative efficiency, but lower asset users' investment incentives. We have proposed a new mechanism, the *depreciating license*, which continuously interpolates between long-term and short-term licenses. The optimal depreciation rate is always interior: depreciating licenses can always improve upon the extremes of perpetual licenses and pure rental contracts.

We have abstracted away from a number of important issues. We have assumed a relatively simple model of agents' private values, with no intertemporal correlations. We assumed that the common value of assets is fully observed by all participants, ignoring "lemons" problems from asymmetric information about common values. We assume a single asset: for cases like land and radio spectrum, there are often multiple

[16]Some papers analyzing mechanisms for improving efficiency in intellectual property settings are Kremer (1998), Hopenhayn, Llobet and Mitchell (2006), and Weyl and Tirole (2012).

complementary assets, leading to multi-asset holdout problems. Depreciating licenses could be studied under richer models of strategic interactions between agents, including repeated relationships and collusion. We have also assumed rational expectations. Long-term property rights can enable speculation and lead to asset bubble, and an interesting question is whether such behavior is likely under depreciating licenses.

We hope that future research will analyze depreciating licenses under more general models relaxing these assumptions. This paper is also only a first step in examining the possibilities for improving resource license design, and we hope that future work continues to explore this area.

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Appendix

A Supplemental material for Section 3

A.1 Proof of Proposition 1

First, conjecture that the value of being an owner at the start of any period, with value u_{St}, is

$$V\left(u_{St}\right)$$

Let

$$V \equiv E_{u_{St} \sim F(\cdot)}\left[V\left(u_{St}\right)\right]$$

be the ex-ante value of an owner, before she learns the realization of u_{St}.

A.1.1 Incumbent profit

The incumbent first chooses ψ_t, and pays cost $c\left(\psi_t\right)$. After ψ_t is determined, the value of the asset owner for keeping the good is:

$$\psi_t + u_{it} + \beta V$$

Likewise, the value of a buyer is:

$$\psi_t + u_{Bt} + \beta V$$

Since the auction is second-price, the dominant strategy for the buyer is to bid her true value.

The objective function of the incumbent is thus the following. If the incumbent bids above the buyer, she gets:

$$\underbrace{\left(\psi_t + u_{St} + \beta V\right)}_{\text{Continuation value}} - \underbrace{\tau\left(\psi_t + u_{Bt} + \beta V\right)}_{\text{Fee payment}}$$

i.e. she pays the buyer's bid, keeps the asset and gets her continuation value. If the incumbent bids below the buyer's value, she gets paid her bid for her share $(1-\tau)$ of the asset, that is:

$$(1-\tau)(\psi_t + m + \beta V)$$

In either case, the investment cost $c'(\psi_t)$ is sunk. Hence, the incumbent's expected profit can be written as:

$$\Pi_t = \int_0^m [(\psi_t + u_{St} + \beta V) - \tau(\psi_t + u_{Bt} + \beta V)]\, dF(u_{Bt}) + \int_m^\infty (1-\tau)(\psi_t + m + \beta V)\, dF(u_{Bt}) - c(\psi_t)$$

$$\Pi_t = (1-\tau)(\psi_t + \beta V) - c(\psi_t) + \int_0^m [u_{St} - \tau u_{Bt}]\, dF(u_{Bt}) + (1-\tau)\, m \int_m^\infty dF(u_{Bt}) \quad (27)$$

A.1.2 Price setting

Differentiating (27) with respect to m, we have:

$$\frac{\partial \Pi}{\partial m} = (u_{St} - m) f(m) + (1-\tau)(1 - F(m)) \quad (28)$$

Rearrainging, we have:

$$m - u_{St} = (1-\tau)\frac{1 - F(m)}{f(m)}$$

This proves (5) of Proposition 1. Note that the derivative (28) is monotonically decreasing in τ, hence (27) has increasing differences in m and $-\tau$, hence the optimal m is monotonically decreasing in τ.

A.1.3 Investment

Differentiating (27) with respect to ψ_t, we have:

$$c'(\psi_t) = (1-\tau)$$

This proves (4) of Proposition 1.

A.1.4 The value function

The incumbent's value function satisfies:

$$V(u_{St}) = \Pi_t^* =$$
$$(1-\tau)(\psi_t^* + \beta V) + \int_0^{m^*(u_{St})} [u_{St} - \tau u_{Bt}]\, dF(u_{Bt}) + (1-\tau)\, m^*(u_{St}) \int_{m^*(u_{St})}^{\infty} dF(u_{Bt}) - c(\psi_t^*) \tag{29}$$

The expected value V is thus the expectation of (29) over u_{St}. In closed form, V is:

$$V = \frac{(1-\tau)\,\psi_t^* - c(\psi_t^*)}{1-\beta(1-\tau)} + \frac{E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St})} [u_{St} - \tau u_{Bt}]\, dF(u_{Bt}) + (1-\tau)\, m^*(u_{St}) \int_{m^*(u_{St})}^{\infty} dF(u_{Bt})\right]}{1-\beta(1-\tau)} \tag{30}$$

A.2 Proof of Claims 1 and 2

These are special cases of Proposition 1, with $\tau = 0$ and $\tau = 1$ respectively.

A.3 Proof of Proposition 2

In the following two subsections, we will show that marginal allocative welfare is:

$$\int \left[(1-\tau)\frac{(f(m^*(u_{St},\tau)))^2\, h(m^*(u_{St},\tau))}{(1-F(m^*(u_{St},\tau)))\,(1-(1-\tau)\, h'(m^*(u_{St},\tau)))}\right] dF(u_{St}) \tag{31}$$

where

$$h(m) \equiv \frac{1-F(m)}{f(m)}$$

and marginal investment welfare from changing τ is:

$$\frac{dW}{d\tau} = -\frac{\tau}{c''\left(\psi_t^*\left(\tau\right)\right)} \tag{32}$$

Equating the sum of these to 0 and rearranging, we have (11).

Now, if F has continuous second derivatives, $h'\left(m^*\left(u_{St},\tau\right)\right)$ and $h\left(m^*\left(u_{St},\tau\right)\right)$ exist everywhere. Moreover, if $f\left(\cdot\right)$ is everywhere positive, then $h\left(\cdot\right)$ is nonzero everywhere, so (31) is strictly positive at $\tau = 0$, if it exists. Now, (32) is always 0 at $\tau = 0$, hence (11) cannot hold when $\tau = 0$: the marginal allocative value of raising τ is positive, whereas the marginal investment loss is 0.

Similarly, when $\tau = 1$, if c is strictly convex, (32) is always negative, and (31) is always 0. Hence, (11) cannot hold when $\tau = 1$: the marginal investment gain from lowering τ is positive, whereas the marginal allocative loss is 0. This proves that the first-order condition (11) cannot hold at $\tau = 1$ or $\tau = 0$, proving that the optimal τ must be interior.

A.3.1 Allocative welfare

Allocative welfare, for type u_{St}, is:

$$\int_{m^*\left(u_{St},\tau\right)}^{\infty} u_{Bt} dF\left(u_{Bt}\right) + u_{St}\int_0^{m^*\left(u_{St},\tau\right)} dF\left(u_{Bt}\right)$$

Note that we have:

$$\frac{dW\left(u_{St},\tau\right)}{d\tau} = \frac{\partial W}{\partial m}\frac{\partial m}{\partial \tau}$$

Now,

$$\frac{\partial W}{\partial m} = \left(u_{St} - m\right) f\left(m\right)$$

Now, define the inverse hazard rate function $h\left(m\right)$ as:

$$h\left(m\right) \equiv \frac{1 - F\left(m\right)}{f\left(m\right)}$$

We can then write the markup FOC as:

$$m^* (u_{St}, \tau) - u_{St} - (1-\tau)\, h\, (m^* (u_{St}, \tau)) = 0$$

Applying the implicit function theorem, we have:

$$\frac{\partial m}{\partial \tau} = \frac{-h\,(m^*(u_{St},\tau))}{1-(1-\tau)\,h'\,(m^*(u_{St},\tau))} \tag{33}$$

Hence,

$$\frac{dW\,(u_{St},\tau)}{d\tau} = (m^*(u_{St},\tau) - u_{St})\,\frac{f\,(m^*(u_{St},\tau))\,h\,(m^*(u_{St},\tau))}{1-(1-\tau)\,h'\,(m^*(u_{St},\tau))}$$

Substituting for $m - u_{St}$ using (5) and simplifying, we have:

$$\frac{dW\,(u_{St},\tau)}{d\tau} = (1-\tau)\,\frac{(1-F\,(m^*(u_{St},\tau)))\,h\,(m^*(u_{St},\tau))}{(1-(1-\tau)\,h'\,(m^*(u_{St},\tau)))}$$

The change in total allocative welfare is just the integral of this:

$$\frac{dW\,(\tau)}{d\tau} = \int \left[(1-\tau)\,\frac{(1-F\,(m^*(u_{St},\tau)))\,h\,(m^*(u_{St},\tau))}{(1-(1-\tau)\,h'\,(m^*(u_{St},\tau)))}\right] dF\,(u_{St})$$

which is (31).

A.3.2 Investment welfare

Investment welfare is:

$$\psi_t^*(\tau) - c\,(\psi_t^*(\tau))$$

Differentiating, we have:

$$\frac{dW}{d\tau} = \frac{\partial W}{\partial \psi_t}\frac{\partial \psi_t^*}{\partial \tau}$$

Now,

$$\frac{\partial W}{\partial \psi_t} = 1 - c'\,(\psi_t)$$

And, applying the implicit function theorem to (4), we have:

$$\frac{\partial \psi_t}{\partial \tau} = \frac{1}{c''(\psi_t)}$$

Hence,

$$\frac{dW}{d\tau} = -\frac{1 - c'(\psi_t)}{c''(\psi_t)}$$

Since $c'(\psi_t) = 1 - \tau$, we have (32).

B Supplemental material for Section 4

B.1 Proof of Claim 3

Since we have assumed away private values, buyers and license owners are identical. In any period, the license owner can choose to sell for $(1 - \tau) p$, or buy for τp. Buyers and the license owner agree on the continuation value in period t, which is:

$$V_t$$

The license owner is willing to keep the asset if:

$$V_t - \tau p_t \geqslant (1 - \tau) p_t$$

$$\implies p_t \leqslant V_t$$

The buyer is willing to buy if $p_t \leqslant V_t$. Thus, the unique market clearing price is $p_t = V_t$ in each period, which makes license owners indifferent between buying and selling in each period. In the first period, where there is no license owner, bidders will bid until $p_0 = V_0$.

Since license owners are always indifferent between buying and selling, and there is no uncertainty, to calculate a license owner's utility, we can simply assume a single license owner purchases the license in period 0, and then keeps the good forever. Her expected utility is:

$$\underbrace{\sum_{t=1}^{\infty}\beta^{t}E\left[\psi_{t}\right]}_{\text{Asset use value}}-\underbrace{p_{0}}_{\text{Initial license price}}-\underbrace{\sum_{t=1}^{\infty}\beta^{t}E\left[\tau_{t}p_{t}\right]}_{\text{License fee payments}} \tag{34}$$

The license owner must be indifferent between purchasing and not purchasing the license in period 0. Setting (34) to 0 and rearranging, we have:

$$\underbrace{p_{0}}_{\text{Initial license price}}+\underbrace{\sum_{t=1}^{\infty}\beta^{t}E\left[\tau_{t}p_{t}\right]}_{\text{License fee payments}}=\underbrace{\sum_{t=1}^{\infty}\beta^{t}E\left[\psi_{t}\right]}_{\text{Asset use value}} \tag{35}$$

The left hand side of (35) is the expected net present value of the government's revenue, over the initial sale of the license and the future license fee payments. The right hand side is the expected net present value of the future common use values, ψ_t, of the asset; this is not affected by τ_t. Thus, (35) shows that the net present value of the government's revenue does not depend on τ_t.

B.2 Proof of Proposition 3

Prices in each auction are:

$$p=\min\left[u_{Bt}+\psi_{t}+\beta V\left(\tau\right),m^{*}\left(u_{St},\tau\right)+\psi_{t}+\beta V\left(\tau\right)\right]$$

$$=\beta V\left(\tau\right)+\psi_{t}+\min\left[u_{Bt},m^{*}\left(u_{St},\tau\right)\right]$$

Thus, total tax revenue is:

$$\frac{\tau}{1-\beta}p=\frac{\tau}{1-\beta}\left[\beta V\left(\tau\right)+\psi_{t}\right]+\frac{\tau}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt},m^{*}\left(u_{St},\tau\right)\right]\right]$$

We thus have:

$$R\left(\tau\right)=V\left(\tau\right)+\frac{\tau E\left[p_{t}\right]}{1-\beta}$$

$$= V(\tau) + \frac{\tau}{1-\beta}\left[\beta V(\tau) + \psi_t^*(\tau)\right] + \frac{\tau}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt}, m^*(u_{St},\tau)\right]\right]$$

$$R(\tau) = \frac{(1-\beta(1-\tau))}{1-\beta} V(\tau) + \frac{\tau}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt}, m^*(u_{St},\tau)\right]\right] + \frac{\tau\psi_t^*(\tau)}{1-\beta} \tag{36}$$

Now, note that from (30) of Appendix A, we have:

$$V(\tau) = \frac{(1-\tau)\psi_t^*(\tau) - c(\psi_t^*(\tau))}{1-\beta(1-\tau)} + \frac{E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)}\left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau) m^*(u_{St},\tau)\int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right]}{1-\beta(1-\tau)}$$

Hence,

$$\frac{(1-\beta(1-\tau))}{1-\beta} V(\tau) = \frac{(1-\tau)\psi_t^*(\tau) - c(\psi_t^*(\tau))}{1-\beta} + \frac{E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)}\left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau) m^*(u_{St},\tau)\int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right]}{1-\beta}$$

Hence,

$$R(\tau) = \frac{1}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)}\left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau) m^*(u_{St},\tau)\int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right] + \frac{\tau}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt}, m^*(u_{St},\tau)\right]\right] + \frac{(1-\tau)\psi_t^*(\tau) + \tau\psi_t^*(\tau) - c(\psi_t^*(\tau))}{1-\beta}$$

$$R(\tau) = \frac{1}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)}\left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau) m^*(u_{St},\tau)\int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right] + \frac{\tau}{1-\beta} E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt}, m^*(u_{St},\tau)\right]\right] + \frac{\psi_t^*(\tau) - c(\psi_t^*(\tau))}{1-\beta} \tag{37}$$

Now, note that we can write:

$$\frac{\tau}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\min\left[u_{Bt}, m^{*}\left(u_{St},\tau\right)\right]\right] =$$
$$\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\int_{0}^{m^{*}(u_{St},\tau)}\tau u_{Bt}dF\left(u_{Bt}\right)+\tau m^{*}\left(u_{St},\tau\right)\int_{m^{*}(u_{St},\tau)}^{\infty}dF\left(u_{Bt}\right)\right]$$

Hence, (37) becomes:

$$R\left(\tau\right)=\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\int_{0}^{m^{*}(u_{St},\tau)}\left[u_{St}-\tau u_{Bt}\right]dF\left(u_{Bt}\right)+\left(1-\tau\right)m^{*}\left(u_{St},\tau\right)\int_{m^{*}(u_{St},\tau)}^{\infty}dF\left(u_{Bt}\right)\right]+$$
$$\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\int_{0}^{m^{*}(u_{St},\tau)}\tau u_{Bt}dF\left(u_{Bt}\right)+\tau m^{*}\left(u_{St},\tau\right)\int_{m^{*}(u_{St},\tau)}^{\infty}dF\left(u_{Bt}\right)\right]+\frac{\psi_{t}^{*}\left(\tau\right)-c\left(\psi_{t}^{*}\left(\tau\right)\right)}{1-\beta} \tag{38}$$

Adding the two integrals, (38) simplifies to:

$$R\left(\tau\right)=\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[u_{St}\int_{0}^{m^{*}(u_{St},\tau)}dF\left(u_{Bt}\right)+m^{*}\left(u_{St},\tau\right)\int_{m^{*}(u_{St},\tau)}^{\infty}dF\left(u_{Bt}\right)\right]+$$
$$\frac{\psi_{t}^{*}\left(\tau\right)-c\left(\psi_{t}^{*}\left(\tau\right)\right)}{1-\beta}$$

$$R\left(\tau\right)=\underbrace{\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[u_{St}F\left(m^{*}\left(u_{St},\tau\right)\right)+m^{*}\left(u_{St},\tau\right)\left(1-F\left(m^{*}\left(u_{St},\tau\right)\right)\right)\right]}_{\text{Allocative}}+$$
$$\underbrace{\frac{\psi_{t}^{*}\left(\tau\right)-c\left(\psi_{t}^{*}\left(\tau\right)\right)}{1-\beta}}_{\text{Investment}} \tag{39}$$

This is (14) of Proposition 3. We can now differentiate $R\left(\tau\right)$ by differentiating each of the allocative and investment terms with respect to τ. Beginning with the allocative term, we have:

$$\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\frac{dm^*\left(u_{St},\tau\right)}{d\tau}\frac{\partial}{\partial m^*}\left[u_{St}\int_0^{m^*(u_{St},\tau)}dF\left(u_{Bt}\right)+m^*\left(u_{St},\tau\right)\int_{m^*(u_{St},\tau)}^{\infty}dF\left(u_{Bt}\right)\right]\right]$$

$$=\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\frac{dm^*\left(u_{St},\tau\right)}{d\tau}\left[1-F\left(m^*\left(u_{St},\tau\right)\right)+\left(u_{St}-m^*\right)f\left(m^*\left(u_{St},\tau\right)\right)\right]\right]$$

$$=\frac{1}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\frac{dm^*\left(u_{St},\tau\right)}{d\tau}\left(1-F\left(m^*\left(u_{St},\tau\right)\right)\right)\left[1-\left(m^*-u_{St}\right)\frac{f\left(m^*\left(u_{St},\tau\right)\right)}{\left(1-F\left(m^*\left(u_{St},\tau\right)\right)\right)}\right]\right] \tag{40}$$

Now, from rearraging (5) of Proposition 1, we have:

$$\frac{\left(m^*-u_{St}\right)f\left(m^*\left(u_{St},\tau\right)\right)}{1-F\left(m^*\left(u_{St},\tau\right)\right)}=1-\tau \tag{41}$$

Plugging (41) into (1), we get:

$$\frac{\tau}{1-\beta}E_{u_{St}\sim F(\cdot)}\left[\frac{dm^*\left(u_{St},\tau\right)}{d\tau}\left(1-F\left(m^*\left(u_{St},\tau\right)\right)\right)\right] \tag{42}$$

Note that $\frac{dm^*(u_{St},\tau)}{d\tau}$ is always weakly negative, and all other terms in (42) are positive, hence (42) is always weakly negative.

Now, the investment term in (39) is:

$$\frac{\psi_t^*\left(\tau\right)-c\left(\psi_t^*\left(\tau\right)\right)}{1-\beta}$$

Note that $\psi_t^*\left(\tau\right)-c\left(\psi_t^*\left(\tau\right)\right)$ is just investment welfare. This is monotonically decreasing in τ. From (A.3.2) of Appendix A.3.2, the derivative is:

$$\frac{d}{d\tau}\frac{\psi_t^*\left(\tau\right)-c\left(\psi_t^*\left(\tau\right)\right)}{1-\beta}=-\frac{\tau}{\left(1-\beta\right)c''\left(\psi_t\right)} \tag{43}$$

Expression (43) is always negative. Combining (42) and (43), we get (15). Now, when c has continuous second derivative, c'' exists everywhere. When F has continuous and bounded second derivatives, from (33), $\frac{\partial m}{\partial\tau}$ is finite and exists everywhere. Thus, all terms in $R'(\tau)$ exist. When $\tau = 0$, both terms are 0, so $R'(0) = 0$.

C Supplementary material for Section 5

C.1 Numerical simulations

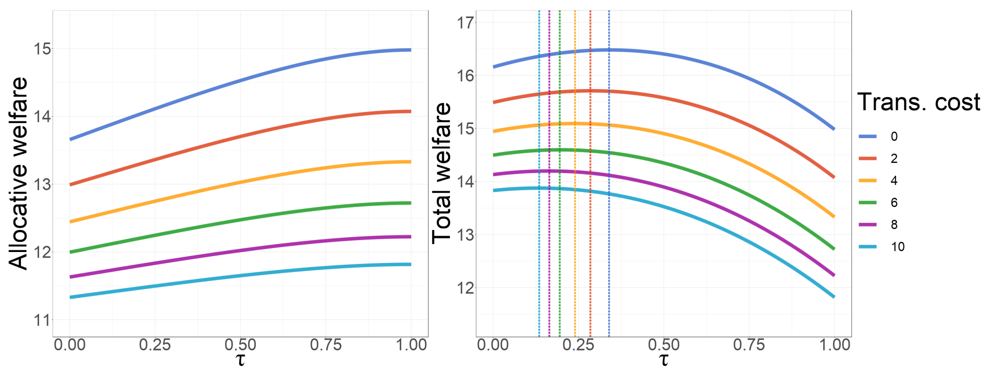
In Figures A.1, A.2, and A.3, we simulate outcomes as we vary, respectively, transactions costs, the number of buyers, and the time period before investments pay off.

Figure A.1 varies transactions costs c. As we show in Subsection 5.1, this is equivalent to shifting buyers' values uniformly downwards by c, which for the exponential distribution, means shifting x_0 to $x_0 - c$. The left panel of Figure A.1 shows that, as we decrease transactions costs, allocative welfare decreases, as well as the allocative welfare gain from increasing τ. Intuitively, when transactions costs are high, buyers' values tend to be lower than sellers', so the welfare distortions from markups are quantitatively smaller. Allocative welfare is still monotonically increasing in τ, but the gains are smaller the smaller x_0 is. The right panel shows that, taking investment welfare into account, the optimal τ tends to be smaller when transactions costs are high, though it is always interior.

Figure A.2 varies the number of buyers. The left panel shows that, as the number of buyers N increases, allocative efficiency tends to increase, since the highest bidder's private value tends to increase. As N gets very large, allocative welfare is also a flatter function of τ. Thus, when N is very large, the optimal τ approaches 0. This can be seen in the right panel: the optimal τ starts decreasing for large N. However, note that in this example, the optimal τ does not vary monotonically with N: the optimal τ is somewhat higher for $N = 2, 3, 4$ than it is with $N = 1$.

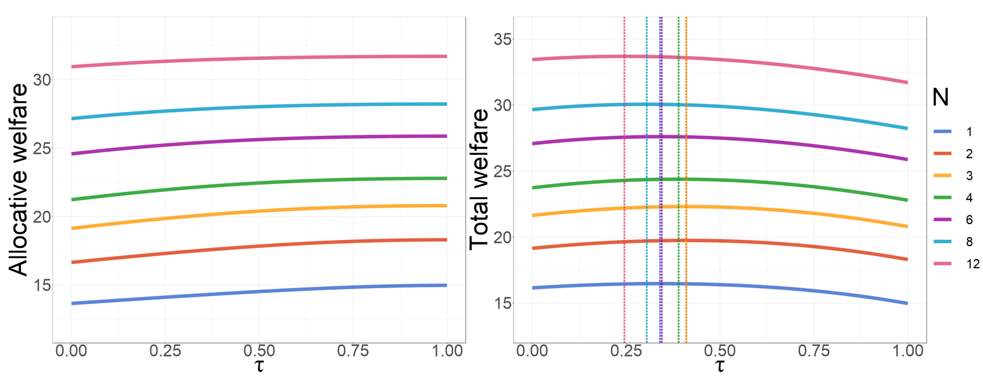
Figure A.3 shows results from varying χ, the time horizon on which investments pay off. The left panel shows that, when χ is large, τ decreases investment incentives much more rapidly. The right panel shows that this leads to a decrease in the optimal level of τ. However, the slope of investment welfare is always 0 when $\tau = 0$, so the optimal τ is always interior.

Figure A.1: Transactions costs



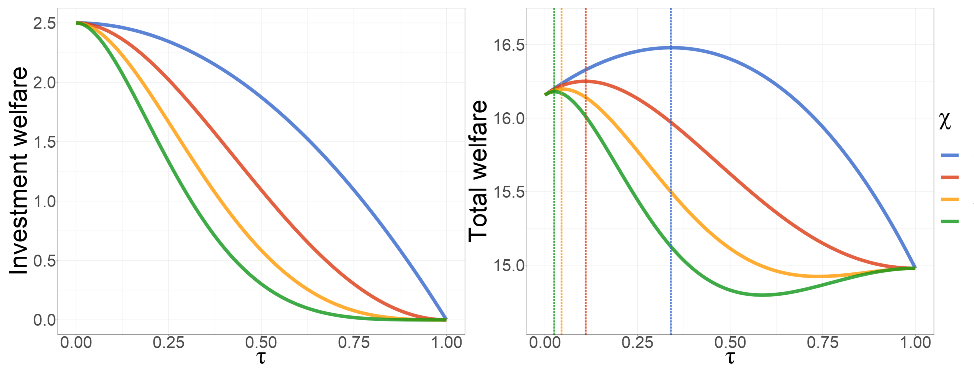
Notes. The left plot shows allocative welfare as a function of transactions cost c. The right plot shows total welfare. Vertical dotted lines represent the welfare-maximizing values of τ, for different values of c. Throughout, values are exponential with minimum 0. The exponential rate parameter is $\lambda = 0.1$. The investment cost parameter is $\kappa = 10$.

Figure A.2: Competition



Notes. The left plot shows allocative welfare as a function of N, the number of buyers arriving each period. The right plot shows total welfare. Vertical dotted lines represent the welfare-maximizing values of τ, for different values of N. Throughout, sellers' and buyers' values are exponential with minimum $x_0 = 0$, and rate parameter $\lambda = 0.1$, and the investment cost parameter is $\kappa = 10$.

Figure A.3: Long-term investments



Notes. The left plot shows investment welfare as a function of χ, the number of periods that investments take to pay off. The right plot shows total welfare. Vertical dotted lines represent the welfare-maximizing values of τ, for different values of χ. Throughout, sellers' and buyers' values are exponential with minimum $x_0 = 0$, and rate parameter $\lambda = 0.1$. We set the investment cost parameter, κ, equal to $\frac{10}{2\beta^{2\chi}}$, which ensures that total attainable investment welfare does not vary with χ.

C.2 Proof of Claim 4

The payoff to the license owner of bidding p in the auction can be described as follows.

- With probability $N(1-F)F^{N-1}$, the owner's bid is the second highest bid, so she sells her share $(1-\tau)$ of the license at p, and gets $p(1-\tau)$.
- With probability $1 - N(1-F)F^{N-1} - F^N$, the owner's bid is lower than the second highest bid, so she sells her share $(1-\tau)$ at the conditional expectation of second highest bid.
- With probability F^N, the owner keeps the asset, and pays τ times the conditional expectation of the highest bid.

We can thus write the incumbent's profit as:

$$(1-\tau)\left[\int_{p}^{\infty}(b_2-p)\,dF_2(b_2)+p\,(1-F_1(p))\right]+$$
$$(\psi_t+\beta V+u_{it})\,(F_1(p))-\tau\int_0^p(\psi_t+\beta V+b_1)\,dF_1(b_1)$$

where F_1 and F_2, respectively, are the distribution of the highest and second highest buyers' bids. In markup terms, this is:

$$(1-\tau)\,(\beta V+\psi_t)+(1-\tau)\int_{m}^{\infty}(b_2-m)\,dF_2(b_2)+$$
$$(1-\tau)\,m\,(1-F_1(m))+u_{it}F_1(m)-\tau\int_0^m b_1 dF_1(b_1)$$

Differentiate with respect to m, to get: and rearrange, to get:

$$(m-u_{it})\,f_1(m)=(1-\tau)\,(F_2(m)-F_1(m))$$

Now, the distributions of the first and second highest buyers' bids satisfy:

$$F_1=F^N,\ f_1=NF^{N-1}f$$

$$F_2=F^N+NF^{N-1}(1-F)$$

Plugging in, we have:

$$(m-u_{it})\left(NF^{N-1}(m)\,f(m)\right)=(1-\tau)\left(NF^{N-1}(1-F)\right)$$

Rearranging, we have (19).

C.3 Proof of Claim 5

An increase in ψ_t affects the value of any asset owner in period t. The social planner sets the marginal cost of investment in period t, $\frac{\partial c}{\partial \psi_{t+\chi}}$, equal to its discounted marginal value in period $t+\chi$. A unit of investment is always worth 1 in period $t+\chi$, so it is worth β^k

in period-t dollars. This is (20).

To solve for equilibrium investment, let V_t be the expected value of an owner in any period t, before her private value is known. We have:

$$\Pi_t(u_S) = \int_0^{m^*(u_{St},\tau)} \left[\left(\psi_t + u_{St} + \beta V_{t+1}\right) - \tau\left(\psi_t + u_{Bt} + \beta V_{t+1}\right)\right] dF(u_{Bt}) + \int_{m^*(u_{St},\tau)}^{\infty} (1-\tau)\left(\psi_t + \beta V_{t+1} + m^*(u_{St},\tau)\right) dF(u_{Bt})$$

$$= (1-\tau)\left(\psi_t + \beta V_{t+1}\right) + \int_0^{m^*(u_{St},\tau)} \left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau)\, m^*(u_{St},\tau) \int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})$$

Since the second piece is stationary, we have:

$$E\left[\Pi(u_S)\right] = E\left[(1-\tau)\left(\psi_t + \beta V_{t+1}\right)\right] + E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)} \left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau)\, m^*(u_{St},\tau) \int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right]$$

Define

$$W = E_{u_{St}\sim F(\cdot)}\left[\int_0^{m^*(u_{St},\tau)} \left[u_{St} - \tau u_{Bt}\right] dF(u_{Bt}) + (1-\tau)\, m^*(u_{St},\tau) \int_{m^*(u_{St},\tau)}^{\infty} dF(u_{Bt})\right]$$

Then we have:

$$E\left[\Pi_t(u_S)\right] = E\left[W + (1-\tau)\left(\psi_t + \beta V_{t+1}\right)\right] \tag{44}$$

Now, suppose during period t that that $\psi_t \ldots \psi_{T-1}$ have been determined by past license owners' investments. We can expand the recursion in (44) to get:

$$E\left[\Pi_t(u_S)\right] = E\left[W + (1-\tau)\left(\psi_t + \beta\left(W + (1-\tau)\left(\psi_{t+1} + \beta V_{t+2}\right)\right)\right)\right]$$

Expanding further to χ periods in the future,

$$E\left[\Pi_t\left(u_S\right)\right] =$$

$$E\left[W + \left[\sum_{\tilde{t}=t}^{t+\chi}(1-\tau)^{\tilde{t}-t}\beta^{\tilde{t}-t}W\right] + \left[\sum_{\tilde{t}=t}^{t+\chi}(1-\tau)^{\tilde{t}-t+1}\beta^{\tilde{t}-t}\psi_{\tilde{t}}\right] + (1-\tau)^{\tilde{t}-t+1}\beta^{\tilde{t}-t}V_{\tilde{t}-t+1}\right]$$

Hence differentiating with respect to $\psi_{t+\chi}$, we have:

$$\frac{\partial c}{\partial \psi_{t+\chi}} = \beta^{\chi}(1-\tau)^{\chi+1}$$

This proves Claim 5.

C.4 Proof of Claim 6

First, we wish to calculate the price of the license once ψ is known, $p(\psi)$. Since all agents are identical, the auction price must make the license owner in each period indifferent between holding the asset and selling the asset. This means:

$$\underbrace{p(1-\tau)}_{\text{Sale revenue}} = \underbrace{\psi}_{\text{Use value}} - \underbrace{\tau p}_{\text{Fee payment}} + \underbrace{\beta(1-\tau)p}_{\text{Next-period sale revenue}}$$

Rearranging, we have:

$$p(\psi) = \frac{\psi}{1-\beta(1-\tau)} \tag{45}$$

Since agents are indifferent between holding and selling, and there is no uncertainty after ψ is realized, agents' utility from owning the license is equal to their revenue from selling the license, $(1-\tau)p(\psi)$. The price in the initial auction has no uncertainty, so the variance in the utility of an agent who buys the license in the first period is thus simply the variance of $(1-\tau)p(\psi)$ over uncertainty in ψ, that is, the variance of:

$$\frac{(1-\tau)\psi}{1-\beta(1-\tau)}$$

This is (23).

Now, note that agents' expected utility for owning the asset, after ψ is known, must be equal to its price, since the price makes agents indifferent between owning the asset and selling it in each period. The price in the initial auction has no uncertainty, so the variance in the utility of an agent who buyers the license in the first period is thus simply the variance of $p(\psi)$ over uncertainty in ψ. This is (23).

Now, conditional on ψ, the administrator's fee revenue is:

$$\frac{\tau p(\psi)}{1-\beta} = \frac{\tau\psi}{(1-\beta)(1-\beta(1-\tau))} \tag{46}$$

The price in the initial auction has no uncertainty, so the variance of the administrator's revenue is just the variance of (46). This is (24).

D Supplementary materials for Section 6

D.1 Self-assessment mechanisms

In this appendix, we analyze self-assessment mechanisms. We assume the same preference and investment structure as the baseline model. However, suppose that the incumbent license owner must announce some price p. She pays τp to the government regardless of what the buyer bids. If the buyer's bid is higher than the incumbent's price, the incumbent is paid p, and the buyer takes the asset. This is a "self-assessed tax" mechanism, in the sense that the incumbent must pay taxes based on her self-assessed price p, which is also a binding reserve price for buyers.

The following claim characterizes license owners' optimal price-setting and investment decisions under this mechanism.

Claim 7. The license owner bids:

$$V + \psi_t + m$$

where V is the continuation value, which is common to buyers and the license owner, and the optimal markup satisfies:

$$m - u_{St} = \frac{(1 - F(m)) - \tau}{f(m)} \tag{47}$$

Investment satisfies:

$$c'(\psi_t) = 1 - \tau \tag{48}$$

Intuitively, Claim 7 says that, for the same τ, from (48), investment incentives are identical to those in Proposition 1, the baseline mechanism. However, the incumbent's optimal markup is different. From (47), a incumbent with type u_{St} sets m equal to u_{St}, so allocative efficiency is achieved, when we have:

$$\tau = 1 - F(u_{St})$$

that is, when τ is equal to the probability that the incumbent sells the asset. This has a number of implications.

For any given incumbent type, full allocative efficiency can be achieved with a τ which is strictly lower than 1. Intuitively, this is because the self-assessed tax mechanism produces stronger incentives to set lower prices than the second-price mechanism. Under the second-price mechanism, the incumbent's price announcement only indirectly affects the taxes she pays if she wins, by increasing the set of buyer bids that she wins over; under the self-assessment mechanism, incumbents always pay the price they announce, so they have stronger incentives to announce low prices. As a result, for a single incumbent type, the optimal τ is more efficient, because it achieves full efficiency at the cost of sacrificing less investment welfare.

The self-assessment mechanism has two weaknesses relative to the second-price mechanism we discuss in the main text. The optimal τ differs for different incumbents: any given fixed τ will be too high for some incumbents and too low for others. Since the license designer must commit to a sequence of τ's when she allocates the license, it is not possible to adjust τ depending on the realized incumbent types.[17] Relatedly, with the self-assessed tax mechanism, it is possible for the administrator to overshoot the optimal

[17] Any mechanism which attempted to elicit license owners' types and use them to set τ would generate further incentive problems; license owners who recognize their type announcements affect τ's will have further incentives to distort their types.

τ, and actually decrease allocative efficiency. If τ is higher than $1 - F(m)$, license owners will announce markups m below their private values u_{St}, and will sell too often, relative to the social optimum.

D.1.1 Proof of Claim 7

Under the self-assessment mechanism, if the incumbent bids above the buyer, she gets:

$$\underbrace{(\psi_t + u_{St} + \beta V)}_{\text{Continuation value}} - \underbrace{\tau(\psi_t + m + \beta V)}_{\text{Fee payment}}$$

That is, the incumbent pays the buyer's bid, keeps the asset and gets her continuation value. If the incumbent bids below the buyer's value, she gets paid her bid for her share $(1 - \tau)$ of the asset, that is:

$$(1-\tau)(\psi_t + m + \beta V)$$

In either case, the investment cost $c'(\psi_t)$ is sunk. Hence, the incumbent's expected profit can be written as:

$$\Pi_t = \int_0^m [(\psi_t + u_{St} + \beta V) - \tau(\psi_t + m + \beta V)]\, dF(u_{Bt}) + \int_m^\infty (1-\tau)(\psi_t + m + \beta V)\, dF(u_{Bt}) - c(\psi_t)$$

$$\Pi_t = (1-\tau)(\psi_t + \beta V) - c(\psi_t) + \int_0^m u_{St} dF(u_{Bt}) + m \int_m^\infty dF(u_{Bt}) - \tau m \qquad (49)$$

Differentiating with respect to m, we have:

$$(u_{St} - m) f(m) + (1 - F(m)) - \tau = 0$$

Rearranging, we have (47). Differentiating (49) with respect to ψ_t and rearranging, we have (48).

D.2 The Vickrey-Clarke-Groves mechanism

Suppose the administrator allocates the asset by running a Vickrey-Clarke-Groves (VCG) mechanism in each period. Suppose the common value of the asset is ψ_t. Suppose the seller's private value is u_{St}, and the buyer's is u_{Bt}. Let V represent the stationary value of being a seller at the start of any period (which will generally be different under the VCG mechanism than in the baseline model). Under the VCG mechanism, if $u_{Bt} > u_{St}$, then the buyer's payment to the administrator increases by the buyer's value,

$$u_{St} + \psi_t + \beta V \tag{50}$$

and the buyer receives the license. If $u_{St} > u_{Bt}$, then the seller's net payment to the administrator increases by the buyer's value,

$$u_{Bt} + \psi_t + \beta V \tag{51}$$

and the seller receives the license. Now, we impose individual rationality period-by-period for the seller: the seller must achieve higher utility than she would get from refusing to participate in period t, then participating from $t+1$ onwards, in any period. This implies that the seller cannot make any payment to the administrator if she keeps the license: payments can only be made if the license is actually traded. Thus, in order for the seller's net payment to the administrator to be $u_{Bt} + \psi_t + \beta V$ if she keeps the license, the seller must get paid $u_{Bt} + \psi_t + \beta V$ if she sells to the buyer. Intuitively, the VCG mechanism with two agents is a second-price auction for the buyer – the buyer must pay the seller's true value to keep the asset. It is a first-price auction for the seller: if she sells to the buyer, she is paid the buyer's true value for the asset.

Under this mechanism, the expected utility for a seller with type u_{St} in period t is:

$$E_{u_{Bt}} \max\left[u_{Bt} + \psi_t + \beta V, u_{St} + \psi_t + \beta V\right]$$

$$= \psi_t + \beta V + E_{u_{Bt}} \max\left[u_{Bt}, u_{St}\right]$$

Hence, the seller's utility increases one-for-one with increases in ψ_t. The seller's optimal investment decision is thus:

$$c'(\psi_t) = 1$$

which is socially efficient. Thus, the VCG mechanism achieves both allocative and investment efficiency.

However, the VCG mechanism is not budget-balanced, period by period. In fact, it is guaranteed to makes a budget deficit for the administrator, whenever trade occurs. When trade does not occur, there are no payment. Trade occurs whenever $u_{Bt} > u_{St}$, and the buyer pays (50), whereas the seller is paid (51). Thus, whenever trade occurs, the administrator must subsidize the difference between the buyer and sellers' values, $u_{Bt} - u_{St}$.

We note that, when considering the upfront revenue V that the administrator makes from the initial sale of the license, the VCG mechanism may still be budget-balanced ex ante for the administrator: the present value of the upfront revenue may be enough to cover the costs of future auctions. However, administrators may not be able to commit to a mechanism which requires repeated, and in principle unbounded, subsidies throughout the life of the mechanism.

Another weakness of the VCG mechanism is that it is vulnerable to collusion: the buyer and seller can collude to extract subsidy revenue from the increases. If the buyer raises her bid, her payment does not change, but the subsidy revenue the seller receives from the administrator increases. This collusion strategy is also possible when there are more bidders. Suppose, for example, that there are multiple buyers, and suppose at least two buyers have values higher than the seller's value. The VCG mechanism then requires that the highest valued buyer pays the second highest valued buyer's value, whereas the seller is paid the highest buyer's value. This implies that market participants can collude to extract revenue from the administrator: if the highest-valued buyer raises her bid, this increases the payment the administrator makes to the seller, without changing the price that the highest-valued buyer pays. In comparison, the depreciating license mechanism always creates positive revenue for the government, every time an auction is run.

E Numerical simulations

In this appendix, we calculate results for the noncentered exponential distribution, which are used in all numerical simulations: Figures 1, 2, A.1, A.2, and A.3. Suppose buyers' values u_{Bt} are drawn from the noncentered exponential distribution, with minimum x_0:

$$F(u_{Bt}) = 1 - e^{-\lambda(u_{Bt} - x_B)}, \; f(u_{Bt}) = \lambda e^{-\lambda(u_{Bt} - x_B)}$$

Where x_0 is the minimum of u_{Bt}. Sellers' values are also noncentered exponential, with a possibly different minimum value x_S.

Allocative welfare. We will solve for markups and welfare conditional on u_{St}. Applying (7) of Proposition 1, markups are:

$$m^*(u_{St}, \tau) = u_{St} + (1-\tau)\frac{e^{-\lambda(x - x_B)}}{\lambda e^{-\lambda(x-x_B)}}$$

$$m^*(u_{St}, \tau) = \max\left[u_{St} + \frac{(1-\tau)}{\lambda}, x_B\right] \tag{52}$$

where, it is never optimal to set $m(u_{St})$ lower than x_B, the minimum of buyers' values. Allocative welfare for a seller with type u_{St} is thus:

$$W(u_{St}, \tau) = u_{St} F(m^*(u_{St}, \tau)) + E[u_{Bt} \mid u_{Bt} > m^*(u_{St}, \tau)](1 - F(m^*(u_{St}, \tau))) \tag{53}$$

Now, for exponential distributions,

$$E[u_{Bt} \mid u_{Bt} > m^*(u_{St}, \tau)] = m^*(u_{St}, \tau) + \frac{1}{\lambda}$$

Hence, plugging in expressions for F, (53) becomes:

$$W(u_{St}, \tau) = u_{St}\left(1 - e^{-\lambda(m^*(u_{St}, \tau) - x_B)}\right) + \left(m^*(u_{St}, \tau) + \frac{1}{\lambda}\right) e^{-\lambda(m^*(u_{St}, \tau) - x_B)} \tag{54}$$

We can compute expected welfare by numerically integrating (54) over u_{St}.

Investment. From (4) of Proposition 1, agents invest until:

$$\frac{\psi_t}{\kappa} = 1 - \tau$$

$$\implies \psi_t^*(\tau) = (1-\tau)\,\kappa$$

Investment welfare is:

$$\psi_t^*(\tau) - \frac{\psi_t^*(\tau)^2}{2\kappa} = (1-\tau)\,\kappa - \frac{((1-\tau)\,\kappa)^2}{2\kappa}$$

$$= \left(1-\tau^2\right)\frac{\kappa}{2}$$

Revenue. We can calculate revenue using expression (14) of Proposition 3. Revenue for type u_{St} is:

$$\frac{1}{1-\beta}\left(u_{St}\left(1-e^{-\lambda(m^*(u_{St},\tau)-x_B)}\right) + m^*(u_{St},\tau)\,e^{-\lambda(m^*(u_{St},\tau)-x_B)}\right) + \frac{\psi_t^*(\tau) - c\,(\psi_t^*(\tau))}{1-\beta} \tag{55}$$

We compute expected revenue by numerically integrating (55) over u_{St}.

Transaction costs. If buyers must pay some transaction cost c, then their private values are effectively drawn from a noncentered exponential distribution with minimum $x_0 - c$. All calculations are otherwise unchanged.

Long-term investment. If investment takes χ periods to pay off, the first-order condition is (21). Agents invest until:

$$\frac{\psi_t^*(\tau)}{\kappa} = \beta^{\chi}(1-\tau)^{\chi+1}$$

$$\implies \psi_t^*(\tau) = \beta^{\chi}(1-\tau)^{\chi+1}\,\kappa$$

Investment welfare generated in each period is:

$$\beta^{\chi}\psi - \frac{\psi^2}{2\kappa}$$

$$= \beta^{2\chi} (1-\tau)^{\chi+1} \kappa \left(1 - \frac{(1-\tau)^{\chi+1}}{2}\right) \tag{56}$$

In the simulations of Figure A.3, for each χ, we set

$$\kappa = \frac{10}{2\beta^{2\chi}}$$

The result is that total investment welfare – (56) when $\tau = 0$ – does not vary as we change χ.

Auctions. Suppose there are n buyers. Let $F_1(\cdot)$ denote the CDF of the highest buyer's value. This has distribution:

$$F_1(u_{Bt}) = F^n(u_{Bt}) = \left(1 - e^{-\lambda(u_{Bt} - x_B)}\right)^n$$

From Claim 4, sellers' optimal markups are still described by (52). To calculate welfare, note that the license is transferred to the highest-valued buyer if her value is higher than $m^*(u_{St}, \tau)$, and kept by the seller otherwise. Hence, welfare for a seller of type u_{St} is:

$$W(u_{St}, \tau) = u_{St} F_1(m^*(u_{St}, \tau)) + \int_{m^*(u_{St}, \tau)}^{\infty} u_{Bt} dF_1(u_{Bt}) \tag{57}$$

For the simulations in Figure A.2, we calculate (57) numerically for each u_{St}, and then integrate over u_{St}.